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THE

# ACCOMPLISHED TUTOR;

OR,

COMPLETE SYSTEM

OF

## *LIBERAL EDUCATION.*

VOL. II.

S. Gosnell, Printer, Little Queen Street, Holborn.

THE

# ACCOMPLISHED TUTOR;

OR,

## Complete Syſtem

OF

## LIBERAL EDUCATION:

CONTAINING THE

*MOST IMPROVED THEORY AND PRACTICE*

OF THE

FOLLOWING SUBJECTS:

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THE

# THE

# ACCOMPLISHED TUTOR.

## CHAP. VII.

## OF ALGEBRA.

### SECT. I.

#### OF NOTATION.

*Definitions.*

1. ALGEBRA is the art of refolving difficult queftions more readily than by the rules of common arithmetic.

In algebra, the value of quantities is expreffed by fome letters of the alphabet, which have fometimes figures, and certain characters added to them, whereby their value is increafed, or diminifhed; and each letter may reprefent any quantity at pleafure. But, generally, the firft letters in the alphabet, $a$, $b$, $c$, $d$, &c. are ufed to fignify quantities, the value whereof is known; and the latter letters, as $w$, $x$, $y$, $z$, &c. are ufed for quantities which are unknown: the letters are then managed according to the rules of art.

2. The fign + fignifies *addition*, and in algebra it is called *plus*; it denotes that the characters or letters placed on each fide of it are to be added together, thus, $a+b$ fignifies that

the quantity expreſſed by $a$ is to be added to that repreſented by $b$. Thus, if $a$ ſtand for 3, and $b$ for 6; then $a+b$ will be equal to 9.

3. The ſign — ſignifies *ſubtraction*; and ſhows that the quantity following it is to be ſubtracted from the quantity preceding it: thus, $a—b$ ſignifies that the quantity repreſented by $b$ is to be ſubtracted from that repreſented by $a$; as, if $a$ was 8, and $b$ was 3, then $a—b$ would be equal to 5: this ſign is called *minus*.

4. The ſign + repreſenting addition, is called a *poſitive*, or an *affirmative* ſign. The ſign —, ſignifying ſubtraction, is called a *negative* ſign.

5. Like ſigns are when ſeveral quantities have all the ſign + or —; and unlike ſigns are quantities where ſome have the ſign +, and others the ſign —.

6. The ſign = denotes *equality*, and is placed between two quantities, to ſhow they are equal: thus, $a=b$ ſignifies that $a$ and $b$ are equal to each other.

7. The ſign × ſtands for *multiplication*, and ſignifies that the quantities placed on each ſide are to be multiplied together: thus, $a \times b$ ſignifies the quantity $a$ is to be multiplied by the quantity $b$; as, if $a$ be equal to 5, and $b$ equal to 6, they will, with the product, ſtand thus $a \times b=30$, which ſignifies that $a$ multiplied by $b$ is equal to 30. But the product of two or more ſimple quantities is generally ſignified by merely joining the letters. Thus, the product of the above quantity is expreſſed $a\,b$, and if there be three or more quantities to be mutiplied together, as $a \times b \times c$, they will be expreſſed thus, $a\,b\,c$.

8. The ſign ÷ expreſſes *diviſion*: thus, $a \div b$ ſignifies that $a$ is to be divided by $b$; but this ſign is not much uſed, for diviſion is generally expreſſed in the manner of a fraction: thus, $\frac{a}{b}$ and $\frac{a—b}{c+d}$ ſignifies that $a$ is to be divided by $b$, and $a—b$ divided by $c+d$.

9. The ſign ∽ ſignifies the *difference* between two quantities;

ties: thus, $a \backsim b$ ſtands for the difference between $a$ and $b$. Thus if $a$ ſtand for 9, and $b$ for 4, $a \backsim b$ repreſents 5.

10. The ſign $\sqsubset$ or $\daleth$ are ſigns of *majority*, and ſhow that the quantity placed before the ſign is greater than that which follows it: thus, $a \sqsubset b$, or $a \daleth b$, ſhows that $a$ is greater than $b$.

11. The ſign $\sqsupset$ or $\angle$ ſignifies *minority*, and ſhows that the quantity placed before the ſign is leſs than that which follows it; thus, $a \angle b$, or $a \sqsubset b$, ſignifies that $a$ is leſs than $b$.

12. The ſign $\surd$ is the ſign of the *ſquare root*. It alſo expreſſes the cube root, biquadrate root, &c. by placing 3 or 4, &c. over it; thus, $\sqrt{a}$, or $\sqrt[3]{a}$, or $\sqrt[4]{a}$, denote the ſquare root, cube root, and biquadrate root of $a$ reſpectively.

13. *Involution* is the raiſing of a quantity to any power, according as it is joined to the figures 2, 3, 4, &c. reſpectively.

14. The ſign *lu* ſignifies *evolution*, and denotes that the quantity to which it is joined is the ſquare or cube root, &c. as it is joined to the numbers 2, 3, &c. reſpectively.

The *power* of a quantity is often expreſſed in algebra by placing a figure over the quantity; thus, $a^2$, $a^3$, and $a^4$, denote the ſquare, cube, and biquadrate, of $a$ reſpectively; or the ſecond, third, and fourth power; and the figures 2, 3, and 4, placed over $a$, are called the indices or exponents of $a$.

15. *Like quantities* are thoſe that conſiſt of the ſame letters, as $a$, $4a+2a$, or $b-2b+3bb$, &c.

16. *Unlike quantities* conſiſt of different letters; as $a$, $2b$, $3c$; or $2a$, $cd-d$.

17. *Simple quantities* conſiſt of one term only; as $4b$, or $3a^2$, or $12d$, &c.

18. *Compound quantities* conſiſt of ſeveral terms; as $a+c$, $2b-d$, &c.

19. A vinculum is a line drawn over ſeveral quantities, and ſhows that they are to be taken as a compound quantity; as $\overline{a+b-c}$.

20. The *coefficient* of a quantity is the number prefixed

to it; as $6d$; here 6 is the coefficient, and fignifies that the quantity $d$ is multiplied thereby.

21. A *binomial quantity* confifts of two terms; as $b+c$. A *trinomial quantity*, of three terms; as $a+b+c$. A *quadrinomial quantity*, of four terms; as $a+b+c+d$.

22. A *refidual quantity* is a binomial, where one of the terms is a negative one; as $a-b$.

23. A *rational quantity* has no radical fign.

24. A *furd quantity* is that which has not a proper root; as the fquare root of $b$ ($\sqrt{b}$), the biquadrate root of $bb$ ($\sqrt[4]{bb}$).

25. The fign : :: : fignifies *proportion*; as 5 : 10 :: 40 : 80, that is, as 5 to 10, fo is 40 to 80.

26. An *equation* is the comparifon of two quantities which are equal to one another, having the fign of equality between them, as $5, 9=6, 8$, which fignifies that 5 and 9 are equal to 6 and 8, or 14. Of equations there are feveral forts:

1. A *dependant equation* is that which may be deduced from fome others.—2. An *independent equation*, that which cannot be deducible from another.—3. A *pure equation*, that which contains but one power of the unknown quantity.—4. An *affected equation*, that which has feveral powers of the unknown quantity.

## *Axioms.*

1. If equal quantities be added to equal quantities, the fums will be equal. And if equal quantities be taken from equal quantities, the remainders will be equal.

2. If equal quantities be multiplied by equal quantities, the products will be equal. And if equal quantities be divided by equal quantities, the quotients will be equal.

3. If equal quantities be raifed to equal powers, the products will be equal.

4. Quantities equal to any other quantity are equal to one another.

5. The whole is equal to all its parts taken together.

SECT.

## SECT. II.

### OF THE FOUR SINGLE RULES OF ALGEBRA.

### OF ADDITION.

RULE 1. When like quantities having like ſigns are to be added together, add together the coefficients (if there be any), and to the ſum prefix the ſign, and ſubjoin the common quantity.

2. When the quantities are like, but have unlike ſigns, take the difference between the ſum of the affirmative coefficients, and the ſum of the negative ones; to which difference prefix the ſign of the greater ſum, and annex the common quantity.

3. But when the quantities are all unlike, they cannot be brought into one ſum, but muſt be written down one after another, prefixing to each its proper ſign; as in the following examples:

| | | |
|---|---|---|
| $7ab$ | $12a^2c+2ab$ | $-10x^2y^2-2a^2$ |
| $12ab$ | $9a^2c+5ab$ | $-7x^2y^2-7a^2$ |
| $9ab$ | $10a^2c+2ab$ | $-4x^2y^2-2a^2$ |
| $2ab$ | $4a^2c+3ab$ | $-31x^2y^2-a^2$ |
| $30ab$ Sum. | $35a^2c+12ab$ Sum. | $-52x^2y^2-12a^2$ Sum. |

Where there is no coefficient prefixed to a quantity, the coefficient is 1. And when there is no ſign prefixed, the quantity is affirmative; as in the quantities of the firſt and ſecond of the foregoing examples.

*Examples*

*Examples of like Quantities, and unlike Signs.*

| | |
|---|---|
| $-3ac-7cd+de$ | $+12a^2b^2-de-12x^2$ |
| $+7ac+4cd-de$ | $+\ 4a^2b^2+de+\ 6x^2$ |
| $-6ac-2cd+de$ | $+\ 5a^2b^2-de-10x^2$ |
| $-2ac-5cd+de$ | $+21a^2b^2-de-16x^2$ |

*Unlike Quantities, and unlike Signs.*

$$\begin{array}{l} +15a-6a^2-2b^2+5cd \\ -2de+4xy-x^2+xy \\ +3b^2-2x+y^2 \\ \hline +15a-6a^2+b^2+5cd-2de+5xy-x^2-2x+y^2 \end{array}$$

## SUBTRACTION.

RULE. Place the quantities one under the other, and change all the ſigns of the ſubtrahend; that is, where there is an affirmative ſign, place a negative one; and *vice verſa*. Then add the quantities together, as in addition.

*Examples.*

| | |
|---|---|
| From $+2a^2b^2-cd$ | From $-\ ab+6a^2-9cd$ |
| Take $+\ a^2b^2+cd$ | Take $+4ab-2cd+x^2$ |
| $+\ a^2b^2-2cd$ | $5-ab+6a^2-7cd-x^2$ |

Subtraction, as well as each of the other four rules in algebra, is proved in the ſame manner as in common arithmetic *.

* The reaſon of this rule is evident from hence, that if a decrement or a negative quantity be taken away from an affirmative quantity, the remainder will be the ſame as if an increment or an affirmative quantity of equal quantity be added to the original quantity. For every negative quantity always decreaſes the value of any quantity with which it is joined. Thus, if $-b$ be taken from $a-b$, there will remain $a$, and if $+b$ be added to $a-b$, the ſum is likewiſe $a$.

MULTI-

## MULTIPLICATION.

RULE. Multiply each term of the multiplier into every term of the multiplicand: that is, the coefficients into the coefficients, and the letters into the letters; and to each prefix its ſign; viz. + to like ſigns, and — to unlike ſigns.

*Examples.*

| | | | | | |
|---|---|---|---|---|---|
| Multiply | $7a$ | Multiply | $3d^2$ | Multiply | $-5c^2d^2$ |
| By | $3b$ | By | $5c^2$ | By | $+7c^2d^2$ |
| Product | $21ab$ | Product | $15d^2c^2$ | Product | $-35c^4d^4$ |

| | | | |
|---|---|---|---|
| Mult. | $7a-3c$ | Multiply | $a^2-ab+c^2$ |
| By | $12a+4c$ | By | $-a+2b^2$ |
| | $84aa-36ac$ | | $-a^3+a^2b-ac^2$ |
| | $+28ac-12cc$ | | $+2a^2b^2-2ab^3+2c^2b^2$ |
| Pro. | $+84aa-8ac-12cc$ | P. | $-a^3+a^2b+2a^2b^2-ac^2-2ab^3+2c^2b^2$ |

The foregoing examples may be proved by diviſion, as in common arithmetic *.

* This rule depends upon the ſame principle as multiplication in common arithmetic. And that two quantities having like ſigns, ſhould give a product with the ſign +, and two quantities of unlike ſigns the ſign —, may be proved from hence: viz. 1. If an affirmative quantity be multiplied by an affirmative quantity, the product muſt of courſe be an affirmative quantity.—2. If a negative quantity be multiplied by an affirmative one, the negative quantity muſt be taken as often as there are units in the affirmative one, and the ſum of any number of negative quantities will be negative. And if an affirmative quantity be multiplied by a negative one, the affirmative quantity muſt be ſubtracted as often as there are units in the negative one; and the ſum of any number of negatives will be negative.—3. Again, if a negative quantity be multiplied by a negative quantity, the multiplicand is to be ſubtracted, as often as there are units in the multiplier: but, to ſubtract a negative quantity is the ſame thing as to add an equal affirmative one: therefore, the product will be affirmative. From hence the general rule, that like ſigns produce +, and unlike ſigns —.

DIVISION.

## DIVISION.

RULE. If the quantities be ſimple, divide the coefficient of the dividend, by the coefficient of the diviſor, and place the anſwer in the quotient; annexing thereto thoſe letters in the dividend, which are not found in the diviſor: obſerving that like ſigns produce +, and unlike ſigns —.

2. But if the quantities be compound, divide the firſt term of the dividend by the firſt term of the diviſor, and place the reſult in the quotient. Then multiply the whole diviſor thereby, and ſubtract the product from the dividend, and to the remainder bring down the next term in the dividend. And repeat the operation as in common arithmetic. But the terms in the dividend ſhould be ranged in a proper order, that is, according to the dimenſions of ſome letter; the quantities repreſented by $a$ being generally placed firſt; thoſe by $b$ or $a\,b$ next, as follows;

*Examples.*

$$\begin{array}{l} 9ac)36abcd(4bd \\ \quad\;\; 36abcd \\ \hline \qquad\qquad 0 \end{array} \qquad\qquad \begin{array}{l} 4a^2b^2)-24a^2b^3d(-6bd \\ \qquad\quad\;\; 24a^2b^3d \\ \hline \qquad\qquad\qquad 0 \end{array}$$

$$\begin{array}{l} 4a-6c)16abb-24cbb(4bb \\ \qquad\quad\;\; 16abb-24cbb \\ \hline \qquad\qquad\qquad\quad 0 \end{array}$$

$$\begin{array}{l} 3a-b)3a^3-12a^2-ba^2+10ab-2b^2(a^2-4a+2b \\ \qquad\;\; 3a^3 \qquad\qquad\; ba^2 \\ \hline \qquad\quad -12a^2 \qquad +10ab \\ \qquad\quad -12a^2 \qquad +\;\, 4ab \\ \hline \qquad\qquad\qquad\quad +\;\, 6ab-2b^2 \\ \qquad\qquad\qquad\quad +\;\, 6ab-2b^2 \\ \hline \qquad\qquad\qquad\qquad\quad 0 \end{array}$$

When

When the divifor will not exactly divide the dividend, as is often the cafe, the dividend is to be placed over the divifor, and a line drawn between them, like a fraction, throwing out fuch letters as are found in both the divifor and dividend. Thus, If $ab+c^3$ was to be divided by $ab-c^2$, it would ftand thus, $\frac{ab+c^3}{ab-c^2}=\frac{+c^3}{-c^2}$. When the power of a quantity is to be divided by any other power of the fame quantity, it is done by fubtracting the exponent of the divifor, from that of the dividend:

Thus, $\frac{b^8}{b^4}=b^4$*.

## SECT. III.

### OF FRACTIONAL QUANTITIES.

BEFORE the ftudent proceed to equations, it is neceffary that he know how to manage fractional quantities; and to raife a quantity to any given power; and, on the contrary, to extract the root of any quantity; to manage furd quantities, &c.

The rules for managing Algebraic Fractions are exactly the fame as thofe for Vulgar Fractions in arithmetic, and therefore need not be repeated; as few perfons would attempt Algebra, till they were fufficie , fkilled in common arithmetic. An example or two may, however, be of fervice.

* To prove the reafon of this rule, that like figns give +, and unlike figns −, it is only neceffary that the divifor be multiplied by the quotient, and the product will be equal to the dividend.

EXAMPLE 1. Reduce the mixed quantity $a \frac{-c}{d}$, to a fraction. Here $\frac{da-c}{d}$, is the fraction required.

EXAMPLE 2. Let the fraction $\frac{bd-b^2}{d}$, be reduced to a mixed quantity. Here dividing $bd-b^2$ by $d$, the quantity will be $b$, and the remainder $-b^2$, and therefore the mixed quantity will be $b-\frac{-b^2}{d}$.

EXAMPLE 3. Reduce the fractions $\frac{a}{b}\frac{c}{d}$ and $\frac{e}{f}$ to fractions of the same value, having a common denominator. Here $\frac{adf}{bdf}\frac{cbf}{bdf}\frac{ebd}{bdf}$ are the fractions required.

## *Involution; or, to find any given Power of any given Quantity.*

RULE. Multiply the quantity into itself as often as the index contains units, except one, and the last product will be the required power; or, which is more convenient, multiply the index of the quantity by the index of the power.

Thus, let it be required to raise the quantities $b+c$ and $b-c$ to the third power, or the cube.

$$\begin{array}{ll}
b\ +c & \text{Root} \\
\underline{b\ +c} & \\
b^2+bc & \\
\underline{\quad +bc+c^2} & \\
b^2+2bc+c^2= & \text{Square} \\
\underline{b\ +c} & \\
b^3+2b^2c+bc^2 & \\
\underline{\quad b^2c+2bc^2+c^3} & \\
b^3+3b^2c+3bc^2+c^3= & \text{Cube.}
\end{array}$$

$b-c$

$$\begin{array}{l} b-c \quad \text{Root} \\ b-c \\ \hline b^2-bc \\ \quad -bc+c^2 \\ \hline b^2-2bc+c^2= \quad \text{Square} \\ b-c \\ \hline b^3-2b^2c+bc^2 \\ \quad -b^2c+2bc^2-c^3 \\ \hline b^3-3b^2c+3bc^2-c^3 \quad \text{Cube.} \end{array}$$

Theſe quantities are raiſed to the third power only; but by the ſame method of proceeding, quantities may be raiſed to any higher power.

If a trinomial, or quadrinomial, &c. be required to be raiſed to any power, it will be beſt done by taking the firſt or the laſt term of the quantity, and for all the other terms ſubſtitute any ſingle term. Theſe two terms being raiſed to the required power, the anſwer will be obtained by replacing, inſtead of the ſubſtituted term, the proper value.

Thus, if it be required to raiſe $b-c+d-ef+g$ to the fourth power, for the terms $b-c+d-ef$, ſubſtitute the term $a$, then the quantity will be $a+g$, which being raiſed to the fourth power, the quantity $a$ may be taken away, and the proper value placed inſtead thereof in the product, which the learner may prove at his leiſure.

In involving a fractional quantity, both the numerator and denominator muſt be raiſed to the required power, by which a new fraction will be obtained, being the anſwer of the queſtion. Thus, the third power $\frac{a^2}{b}=\frac{a^6}{b^3}$.

The *Binomial Theorem*, invented by *Sir Iſaac Newton*, is the moſt elegant and concise method of raiſing a quantity to any power, and is as follows: let $n$ denote any number at

 pleaſure,

pleaſure, and let $a+b$ be a binomial, then the *nth* power thereof will be as follows:

$$\text{by } a^n + na^{n-1}b + \frac{n \,.\, n-1}{1 \,.\, 2}\,a^{n-2}b^2 + \frac{n \,.\, n-1 \,.\, n-2}{1 \,.\, 2 \,.\, 3}\,a^{n-3}b^3$$

$$+ \frac{n \,.\, n-1 \,.\, n-2 \,.\, n-3}{1 \,.\, 2 \,.\, 3 \,.\, 4}\,a^{n-4}b^4 + \frac{n \,.\, n-1 \,.\, n-2 \,.\, n-3 \,.\, n-4}{1 \,.\, 2 \,.\, 3 \,.\, 4 \,.\, 5}\,a^{n-5}b^5, \text{ \&c.}$$

And the *nth* power of $a-b$, is expreſſed in the ſame manner, except that the ſigns of every other term will be negative.

To illuſtrate this theorem, let $a+b$ be involved to the third power. In this caſe, the index is 3, which muſt be placed in the theorem inſtead of $n$, then the firſt term will be $a^3$, the ſecond term $3a^{3-1}b = 3a^2b$; the third term, $\frac{3 \times 2}{2}a^{3-2}b^2 = 3ab^2$, the fourth term $\frac{3 \times 2 \times 1}{2 \times 3}a^{3-3}b^3 = b^3$: the fifth and following terms are equal to nothing. Therefore theſe four terms together, or the third power of $a+b$, is $a^3 + 3ab^2 + 3ab^2 + b^3$.

It may here be obſerved that the coefficients increaſe till the indices of the two letters $a$ and $b$ become equal or change values; then they return or decreaſe again in the ſame order: thus, having the coefficients of half the terms, the reſt are known.

## *Evolution; or, to extract the Root of a given Power.*

Rule 1. If the quantities be ſimple, extract the root of the coefficient for the new coefficient, and divide the index of the letters by the index of the power, and the quotient will be the root required.

Thus, the ſquare root of $25b^2$ will be $5b^{\frac{2}{2}} = 5b$; and the cube root of $27x^3 = 3x^{\frac{3}{3}} = 3x$.

Rule

RULE 2. If the quantities be compound, after ranging the terms according to the dimenſions of ſome letter, ſo that the higheſt power of that letter may ſtand firſt in order, and the lower powers of the ſame letter follow, according to the dimenſions of their power: take the root of the firſt term, and place it in the quotient, and if it be the ſquare, or cube root, ſubtract the ſquare, or cube thereof, from the firſt term, bring down two, or three of the next terms for a dividend, according as the caſe ſhall be the ſquare, or cube root; then proceed to find the diviſor as in extracting the ſquare, or cube root, in common arithmetic. And if the root of a higher power be to be extracted, it is performed in the ſame manner as in common arithmetic.

*Examples.*

EXAMPLE 1. What is the ſquare root of

$$\begin{array}{rl} & 36x^4+108x^2+81(6x^2+9 \\ & 36x^4 \\ \hline 12x^2+9) & +108x^2+81 \\ & +108x^2+81 \\ \hline \end{array}$$

EXAMPLE 2. What is the ſquare root of $9x^4+16a^4+4b^4-24a^2x^2+12b^2x^2-16a^2b^2$. Anſwer $3x^2-4a^2+2b^2$.

EXAMPLE 3. What is the cube root of

$$\begin{array}{rl} & x^3-6x^2y+12xy^2-8y^3\ (x-2y \text{ Root} \\ & x^3 \\ \hline 3x^2 6xy+4y^2) & -6x^2y+12xy^2-8y^3 \\ & -6x^2y+12xy^2-8y^3 \\ \hline \end{array}$$

## *Surd Quantities.*

When a quantity has not a perfect root, it is called a ſurd quantity; and the root cannot be expreſſed any other way, than by either inſerting the quantity with its proper radical ſign, or throwing it into an infinite ſeries. Thus, the ſquare root

root of $a$, can be expreffed in no other way than by $\sqrt{a}$, or $a^{\frac{1}{2}}$; the cube root of $b^2$ by $\sqrt[3]{b^2}$ or $b^{\frac{2}{3}}$; the cube root of $\frac{a^2b}{c^2}$ by $\sqrt[3]{\frac{a^2b}{c^2}}$.

### *To reduce a rational Quantity to the Form of a Surd.*

RULE. Multiply the index of the quantity by the index of the furd, and over the product place the radical fign, and it will be the form required. Thus, let 3 be reduced to the form of $\sqrt{5}$. Here $3^{1} \times {}^{2}$, or $3^2 = 9$, therefore, $\sqrt{9}$ is the furd required.

Again, let $a^2$ be reduced to the form of a cube furd of the form of $\sqrt[3]{b}$ Here $a^2 \times {}^3 = a^6$ and $\sqrt[3]{a^6}$ is the furd quantity.

### *To reduce Quantities of different Indices to other Quantities equal in Value, and having one given Index.*

RULE. Divide the indices of the quantities by the given index, and the quotients will be the new indices of thofe quantities. Then over the faid quantities with their new indices place the given index, and they will be the equivalent values required.

EXAMPLE. Let $12^{\frac{1}{2}}$, and $9^{\frac{1}{4}}$, be reduced to equal quantities, having the common index $\frac{1}{3}$. Here $\frac{1}{2} \div \frac{1}{3}. = \frac{3}{2} =$, the index of the firft quantity, and $\frac{1}{4} \div \frac{1}{3} = \frac{3}{4} =$, the index of the fecond quantity, therefore, $\overline{12^{\frac{3}{2}}}\rvert^{\frac{1}{3}}$ and $\overline{9^{\frac{3}{4}}}\rvert^{\frac{1}{3}}$, are the quantities required.

### *To reduce a Surd Quantity to its moft fimple Terms.*

RULE. Divide the furd by the greateft power which it contains, and place the root of fuch power before the quotient with the radical fign between them.

Thus,

Thus, let $\sqrt{32}$ be reduced to its moſt ſimple terms. Here 16 is the greateſt ſquare number that will divide 32, which being divided by 16, the quotient is 2, therefore, $4\sqrt{2}$ is the ſurd required.

Again the moſt ſimple term of the ſurd $\sqrt{81a^2b}$ is $9a\sqrt{b}$.

### *To find whether Surds are commenſurable, or not.*

RULE. Reduce the ſurds to the leaſt common index; and the quantities, if fractions to a common denominator, except when like terms are commenſurable; then divide them by the greateſt common diviſor, or by ſuch a diviſor as will give one rational quotient, and if both the quotients are rational, the ſurds are commenſurable; otherwiſe, not.

EXAMPLE. Let $\sqrt{27}$ and $\sqrt{12}$ be given to find whether they are commenſurable; theſe two ſurds have already one common index, and are equal to $\sqrt{3 \times 9}$, and $\sqrt{3 \times 4}$, reſpectively. Therefore, divide 27 and 12 by 3, and the quotients are $\sqrt{9}$ and $\sqrt{4}$, that is, 3 and 2; therefore, they are commenſurable.

### *To add, or ſubtract Surd Quantities.*

RULE. If the quantities have unlike indices, reduce them to quantities with like indices; and fractional quantities muſt be reduced to a common denominator, or to other fractions that have rational denominators or numerators; then reduce the quantities to their ſimpleſt terms, and if the ſurd part be the ſame in all, annex it to the ſum or difference with the ſign $\times$; but if the ſurd part is not the ſame in all, the quantities muſt be added or ſubtracted by joining them together with the ſign $+$ or $-$.

EXAMPLE

Example 1. Let $\sqrt{32}$ $\sqrt{72}$ be added together. Here $\sqrt{32} = \sqrt{\overline{16\times2}} = 4\sqrt{2}$; and $\sqrt{72} = \sqrt{36\times2} = 6\sqrt{2}$, and the ſum $= \overline{4+6}\times\sqrt{2} = 10\sqrt{2}$.

Example 2. Let $\sqrt{4a}$, and $\sqrt[4]{a^6}$ be added together. Here $\sqrt{4a} = \sqrt[4]{\overline{16a^2}} = 2\sqrt[4]{a^2}$. And $\sqrt[4]{a^6} = \sqrt[4]{\overline{a^4\times a^2}} = a\sqrt[4]{a^2}$. Therefore, their ſum $= \overline{a+2}\times\sqrt[4]{a^2} = \overline{a+2}\times\sqrt{a}$. If it were required to have ſubtracted $\sqrt{4a}$ from $\sqrt[4]{a^6}$, the remainder would have been $\overline{a-2}\times\sqrt{a}$. Alſo if $\sqrt[3]{a^2} - \sqrt{a^3} + \sqrt{7}$ be added to $2\sqrt[3]{a^2} + \sqrt{a} - \sqrt{3}$, the ſum will be $3\sqrt[3]{a^2} - \sqrt{a^3} + \sqrt{7} + \sqrt{a} - \sqrt{3}$.

## *To multiply and divide Surds.*

Rule. Reduce the ſurds to the ſame index; and the product or quotient of the rational quantities being annexed to the product or quotient of the ſurds, will give the product or quotient required.

Example 1. Multiply $2\sqrt{2}$ by $3\sqrt{3}$, theſe ſurds have the ſame index already, therefore, $7\times3\times\sqrt{\overline{2\times3}} = 6\sqrt{6}$, thus, $6\sqrt{6}$ is the product required.

Example 2. Multiply $a^{\frac{1}{3}}$ by $b^{\frac{1}{4}}$. Here $a^{\frac{1}{3}} = \overline{a^4}\rceil^{\frac{1}{12}}$, and $b^{\frac{1}{4}} = \overline{b^5}\rceil^{\frac{1}{12}}$, therefore, their product is $\overline{a^4b^3}\rceil^{\frac{1}{12}}$.

Example 3. Let $x^{\frac{1}{2}}$ be divided by $x^{\frac{1}{3}} + y^{\frac{1}{2}}$, this is the ſame as if the dividend $x^{\frac{1}{2}}$ was multiplied by $\frac{1}{x^{\frac{1}{3}}+y^{\frac{1}{2}}}$; therefore, the quotient $\overline{\frac{x^3}{x^2+y3}.}\rceil^{\frac{1}{6}}$

## *Involution or Evolution of Surd Quantities.*

Rule. If the ſurd be a ſimple quantity, multiply the index of the quantity by the index of the power, to which the ſurd is to be involved; or by the fraction, expreſſing the root to which it is to be evolved; and if there is a rational part, its proper power or root is to be prefixed thereto.

Compound

Compound furds are involved and extracted as integers or rational quantities, having regard to the operation of fimple furds.

EXAMPLE 1. What is the fquare of $a\sqrt{x}$? Here the fquare of $a=aa=a^2$ and $\sqrt{x}=x$; therefore the fquare of $a\sqrt{x}=a^2x$.

EXAMPLE. What is the cube of $a\sqrt{x^{\frac{5}{2}}}$? Here the cube of $a=a^3$, and the cube of $\sqrt{x^{\frac{5}{2}}}=\sqrt{x^{15}}$, therefore the cube of $a\sqrt{x^{\frac{5}{2}}}=a^3\sqrt{x^{15}}$.

EXAMPLE 3. What is the cube root of $\overline{a+x}|^{\frac{1}{2}}$? Here $\overline{a+x}|^{\frac{1}{2}}\times\frac{1}{3}=\overline{a+x}|^{\frac{1}{6}}$.

## SECT. IV.

### OF EQUATIONS.

AN Equation is the mutual comparing of two equal quantities, having the fign $=$ between them. Thus, if $a$ be equal to 3, and $b$ to 6, and $c$ to 4, and $d$ to 13, then, $a$ added to $b$, will be equal to $d$ made lefs by $c$; and is thus expreffed in algebra $a+b=d-c$.

## *To reduce an Equation.*

When a queſtion is brought to an Equation, in order to underſtand the value thereof, the quantity or quantities ſought muſt be placed on one ſide of the equation, and the known quantities on the other ſide. For this purpoſe, the following rules muſt be attended to :—

Firſt. When any quantity is expreſſed on both ſides of the equation, it may be entirely rejected, or thrown out of both. Thus, if $3d+7x=4c-2b+7x$. Here $7x$ ſhould be rejected from both ſides of the equation, then it will ſtand thus, $3d=4c-2b$.

Second. When known and unknown quantities are both on the ſame ſide of the equation, the known quantities muſt be brought to one ſide of the equation, and the unknown quantities to the other ſide, and thoſe quantities ſo tranſpoſed muſt have their ſigns changed. That is, thoſe which have the ſign $+$, muſt, after they are tranſpoſed to the other ſide, have the ſign $-$. And thoſe which have the $-$, muſt, after tranſpoſition, have the $+$. Thus, if $10+5=x-5$: here if $x$ be the quantity ſought, $-5$ muſt be tranſpoſed to the other ſide of the equation with the ſign $+$; it will then ſtand thus, $+5+10+5=x$; therefore $x=20$. Again, what is the value of $x$ in this equation? $24-4x+10=60-12x$. Here $+24$ and $+10$ on the firſt ſide of the equation muſt be transferred to the other ſide, and $+12x$ on the ſecond ſide of the equation being transferred to the firſt ſide, it will ſtand thus, $-4x+12x=60-24-10$. And by ſubtracting $4x$ from $12x$, and 24 and 10 from 60, the equation will be $8x=26$: therefore $x=3\frac{1}{4}$.

Third.

Third. If there be fractions in the equation, multiply both ſides of the equation by the denominators of the fractions; and the product will be the true integral quantities.

EXAMPLE 1. Reduce the fractional equation $a+\frac{b^2}{x}=c$ to integral quantities. Here, by multiplying the whole by $x$, we ſhall have $ax+b^2=cx$.

Again, If there be given $\frac{a^2}{a+x}+\frac{b^2}{x}=c$, then will $a^2x+b^2\,\overline{a+x}=c\overline{a+x}\times x$.

Fourth. If in the unknown quantity there be a ſurd, all the other terms muſt be tranſpoſed to the contrary ſide, and each ſide of the equation involved according to the index of the ſurd; and if there be more ſurds than one, the operation muſt be as often repeated as there are ſurd quantities. Thus, if $\sqrt{x^2+ax}+d=c$, by tranſpoſing $d$, the equation will be $\sqrt{x^2+ax}=c-d$, and by ſquaring both ſides, the equation is $x^2+ax=c^2-2cd+d^2$, thus the equator is freed from the ſurd.

Fifth. When any quantity is multiplied into both ſides of the equation, or into the higheſt term of the unknown quantity, divide the whole equation thereby. Thus, the equation $5bx^2=3bc$ is divided by $b$, and it becomes $5x^2=3c$; again if it were divided by 5 it would be $x^2=\frac{3c}{5}$.

Sixth. When the ſide of the equation containing the unknown quantity is a pure power, or when being affected it has a rational root, extract the root from both ſides of the equation. Thus in the equation $a^2=b^2+ax$, the equation will be $a=\sqrt{b^2+ax}$. Again, if $x^2+4x+9=25c$ be given by taking the ſquare root we have $x+2x+3=5c$.

Each or all of the foregoing rules are to be uſed as may be neceſſary, till the equation be brought to a proper form.

*Examples, wherein the foregoing Rules appear.*

EXAMPLE 1. What is the value of $x$, in the equation $10+\frac{36}{12-x}=16$.

By ſubtracting 10 from each ſide of the equation, we have $\frac{36}{12-x}=6$, both ſides of which divided by 6, the quotient is $\frac{6}{12-x}=1$, this multiplied by $12-x$, gives $6=12-x$, whence by tranſpoſing $x$ and 6, we have $x=12-6$, or $x=6$.

EXAMPLE 2. What is the value of $x$ in the equation $\frac{ax^2+ac^2}{a+x}=ax+b^2$. Here multiplying by $a+x$, there comes out $ax^2+ac^2=\overline{ax+b^2}\times\overline{a+x}$, or $ax^2+ac^2=a^2x+ab^2+ax^2+b^2x$, which tranſpoſed and ordered according to the foregoing rules, is $a^2x+b^2x=-ab^2+ac^2$, wherefore, $x=\frac{-ab^2+ac^2}{a^2+b^2}$, ſo that if $a=1$, $b=2$, $c=3$, then will $x=\frac{-4+9}{1+4}=1$.

EXAMPLE 3. What is the value of $x$ in the equation $\sqrt{a^2+x^2}=\sqrt[4]{b^4+x^4}$. Both ſides of this equation being raiſed to the fourth power, we have $a^4+2a^2x^2+x^4=b^4+x^4$, which by tranſpoſition, &c. becomes $2a^2x^2=b^4-a^4$, which divided by $2a^2$, becomes $x^2=\frac{b^4-a^4}{2a^2}$, therefore $x=\overline{\frac{b^4-a^4}{2a^2}}\Big|^{\frac{1}{2}}$.

EXAMPLE 4. What is the value of $x$ in the following equation: $x=\sqrt{c^2+x\sqrt{b^2+x^2}}-c$. In this equation $x+c=\sqrt{c^2+x\sqrt{b^2+x^2}}$, which ſquared gives $x^2+2cx+c^2=c^2+$

$x\sqrt{}$

$x\sqrt{b^2+x^2}$, or $x^2+2cx=x\sqrt{b^2+x^2}$; dividing this by $x$, it quotes $x+2c=\sqrt{b^2+x^2}$; this ſquared gives $x^2+4cx+4c^2=b^2+x^2$, and by tranſpoſition it becomes $4cx=b^2-4c^2$; and dividing by $4c$ we have $x=\frac{b^2-4c^2}{4c}$.

*To exterminate an unknown Quantity out of ſeveral Equations; or, to reduce two or more Equations to a ſingle one.*

Rule. If the quantity to be exterminated has but one dimenſion in the equation, find the value of it in two equations, and put thoſe values equal to each other; or having found the value in one equation, ſubſtitute it in the room of the quantity in the other equations. Proceed in the ſame manner with every unknown quantity. But if the quantity to be exterminated be of ſeveral dimenſions, find the value of its higheſt power in two equations. Then if the coefficients are not the ſame, multiply the leſs quantity, ſo that it may become equal to the greater. Put theſe values equal to each other, and there will ariſe a new equation, with a leſs power of the unknown quantities: and the operation muſt be repeated till the quantity be exterminated.

*Examples.*

Example 1. What is the value of $x$ and $y$ in theſe two equations, $7x-5y=28$ and $3x+4y=55$? By tranſpoſing 28 in the firſt equation, and $5y$, we have $7x-28=5y$, therefore the value of $y$ is $\frac{7x-28}{5}$.

In

In the ſecond equation by proceeding in the ſame manner, viz. :—By tranſpoſing $3x$, the value of $y$ is found to be $\frac{55-3x}{4}$; therefore, theſe two quantities being put equal to each other, we have the equation $\frac{7x-28}{5}=\frac{55-3x}{4}$. In this equation only $x$ is concerned. Multiply this equation by 20, which is the production of 4 and 5; or, which is the ſame thing, multiply the numerators and the denominators croſsways, and there will ariſe the equation $28x-112=275-15x$, which by tranſpoſition becomes $43x=387$, or $x=\frac{387}{43}$; therefore, $x=9$; therefore, 9 being ſubſtituted in either of the given equations inſtead of $x$, the value of $y$ will be found. Thus, in the firſt equation if 9 be ſubſtituted for $x$, it will be $63-5y=28$, which tranſpoſed is $\frac{63-28}{5}=y$, or $7=y$.

EXAMPLE 2. Required the value of $x$, $y$, and $z$, in the three following queſtions:
$x+100=y+z$; $y+100=2x+2z$; $z+100=3x+3y$, by tranſpoſing 100 in the firſt equation, $x=y+z-100$ ariſes, which value ſubſtituted in the other two equations, inſtead of $x$, we have the two following:—

$y+100\ (=2y+2z-200+2z)=2y+4z-200$

$z+100\ (=3y+3z-300+3y)=6y+3z-300$, then by tranſpoſing $y$ and $4z-200$, in the firſt of theſe two equations we have $300-4z=y$, which ſubſtituted for the $y$ in the laſt equation, is $z+100=1800-24z+3z-300$, that is $z+100=1500-21z$; wherefore $22z=1400$, or $z=\frac{1400}{22}=63\frac{7}{11}$: therefore, $y=300-4z=45\frac{5}{11}$, and $x=y+z-100=9\frac{1}{11}$.

Of

### *Of the Nature and Compoſition of Equations, containing different Dimenſions of the ſame unknown Quantity.*

It often happens that the unknown quantity will be of ſeveral different dimenſions; then ſuch equation is called a quadratic, a cubic, a biquadratic equation, &c. according as the dimenſion of the higheſt power is a ſquare, cube, or biquadrate: in ſuch equations we muſt diſcover the root or value of the unknown quantities.

All equations are derived (or may be conſidered ſo) from thoſe of a more ſimple form. Thus, if $x-b=0$, which is a ſimple equation, be raiſed to the ſecond power, there ariſes $x^2-2bx+b^2=0$, which is called a quadratic equation; if the former equation be raiſed to the third power, we have $x^3-3bx^2+3b^2x-b^3=0$, which is called a cubic equation, and ſo on. It is but ſeldom that equations occur in this regular form, for the coefficients of the terms will generally be more or leſs than thoſe produced by the involution of one quantity, as $x-b$, and therefore a quadratic equation is a compound one, generally derived from $\overline{x-b}\times\overline{x-c}$. A cubic equation is derived from $\overline{x-b}\times\overline{x-c}\times\overline{x-d}$. A biquadratic equation from $\overline{x-b}\times\overline{x-c}\times\overline{x-d}\times\overline{x-e}$, or from a quadratic ſquared, &c. But, the letters $b$, $c$, $d$, &c. may have either affirmative or negative ſigns.

In equations of this nature, as the whole is equal to nothing, it is obvious that ſome or other of the factors muſt be equal to nothing. It is alſo evident that any ſuch equation may be divided by its factors, till there remain only one factor; and as each of the inferior equations obtained by ſuch diviſion muſt ſtill be equal to nothing, it muſt follow that each of theſe factors themſelves are equal to nothing; therefore, $b$, $c$, $d$, $e$, &c. exhibit ſo many different values of $x$ with contrary ſigns; therefore every equation has as many

roots

roots as there are dimenfions of the unknown quantity in its higheft power. And where $b$, $c$, $d$, $e$, &c. are found negative, $x$ is affirmative; and where any of thefe are affirmative, $x$ is negative. By multiplying the factors or roots together, under different figns, it is obfervable than when $b$, $c$, $d$, &c. are all negative, or, which is the fame thing, when all the values of $x$ are affirmative, the figns in the equation are +, and — alternately. But when there is a negative root, one affirmative quantity will follow another; therefore, there will be as many affirmative roots in the equation as there are changes of the figns from + to —, and from — to +, and all the reft will be negative.

What is here delivered, concerns only poffible roots. An impoffible root is when $b$, $c$, $d$, &c. denote the fquare or any other even root of a negative quantity: an equation derived from fuch roots is an impoffible or imaginary one: if there be one poffible root, the equation will admit of one poffible anfwer.

In the multiplication of the roots of fuch equations, the coefficient of the fecond term is the fum of all the roots with contrary fines; the coefficient in the third term is equal to the fum of the rectangles of thofe roots; or, of all the products that can poffibly arife by combining them two and two: the coefficient of the fourth term is equal to the fum of all the products that can poffibly arife by the combination of them three and three, &c.; and the laft term is always equal to the product of all the roots with contrary fines.

### *The Refolution of Quadratic Equations.*

If it be a pure quadratic, as $x^2=b^2$, or $x^2-b^2=o$, it is produced from the rectangle of $x-b$ and $x+b$, and therefore has one affirmative, and one negative root, and the affirmative root is equal in number to the negative. The root in this cafe is found by extracting the fquare root of the number dented by $b^2$. Thus, if $x^2+=576$, then $x=+\sqrt{576}=+24$.

All

All other quadraties are comprehended under fome of the following forms: viz. $x^2+2bx=\pm d=o$; or, $x^2-2bx\pm d=o$; or, $2bx-x^2\pm d=o$; and this laft form, by tranfpofition, becomes the fame as the fecond form, only the negative roots are changed into affirmative roots, and the affirmative into negative; therefore we may confider the two other forms as applicable to all cafes, and in the folution of them it will be more commodious to tranfpofe $d$, and then they will ftand thus: $x^2+2bx=\pm d$ and $x^2-2bx\pm d$. Now if to each of thefe equations be added $b^2$ (the fquare of half the coefficient of the fecond term) we fhall have in the former cafe $x^2+2bx+b^2=\pm d+b^2$, and in the latter cafe $x^2-2bx+b^2=\pm d+b^2$, and by extracting the fquare roots, the equations become $x+b=\sqrt{\pm d+b^2}$ and $x-b=\sqrt{\pm d+b^2}$ refpectively; and $x$ in the former cafe $=\sqrt{\pm d+b^2}-b$, and in the latter $=\sqrt{\pm d+b^2}+b$ which expreffions give the affirmative values of $x$: but the fquare roots of the above equations may alfo be $-x-b=-\sqrt{\pm b+b^2}$ and $-x+b=-\sqrt{\pm d+b^2}$ refpectively; and therefore in the former cafe $x=-\sqrt{\pm d+b^2}+b$, and in the latter $=-\sqrt{\pm d+b^2}-b$. That is, if $B=\sqrt{\pm d+b^2}$, where $d$ is + or —, according as it is + or — in the fecond fide of the given equation; then in the firft cafe, where $x^2+2bx=\pm d$ the values of $\pm x$ are $\pm B+b$, where $b$ is — or + according as $x$ is affirmative or negative; and in the fecond cafe, where $x^2-2bx=\pm d$, the values of $\pm x$ are $\pm B\pm b$, where $b$ is + or —, as $x$ is + or —.

*Examples.*

EXAMPLE 1. What are the two values of $x$ in this equation, $x^2+6x=2295$? Here $b=3$, and $2295=+d$, and $\sqrt{+d+b^2}=\sqrt{2295+9}=48=B$, and $\pm B\mp b=\pm 48\mp 3=+45$, and $-51$ for the two values of $x$.

EXAMPLE 2. What are the two values of $x$ in $x^2-11x=-28$? Here $b=\frac{11}{2}$, and $-28=-d$, alfo $b^2=30\frac{1}{4}$, and

[illegible]; and [illegible], and $+1$ for the two values of $x$, which are here both affirmative.

### *To increase or diminish the Roots of Equations.*

RULE. Substitute a new letter for the unknown quantity ± the given increment, or − the given decrement; and substitute the powers thereof in the equation, instead of the unknown letter.

EXAMPLE. Increase the roots of the following equation by 2, $x^3+x^2-10x+8=0$. Let $x+2=z$, or $z-2=x$, then $x^3=\overline{z-2}^3$, $x^2=\overline{z-2}^2$ and $-10x=-10\times z-2=-10z+20$ then the powers of $x$ being actually involved, and the several terms collected, we have $x^3+x^2-10x+8=z^3-5z^2-2z+24=0$, where the roots of $z$ are greater than those of $x$ by 2.

Thus all the negative roots of an equation may be made affirmative, by increasing them with a proper quantity.

### *To complete a deficient Equation.*

RULE. Increase or diminish the roots of the equation, by some given quantity, as shown in the last example.

### *To multiply or divide the Roots of any Equation, by a given Quantity.*

RULE. Multiply or divide any new letter by the given number, and substitute its powers in the equation for the unknown quantity.

EXAMPLE. Divide the roots of the equation $x^3-2x+\sqrt{3}=0$ by $\sqrt{3}$. Here by putting $x=y\sqrt{3}$, and substituting it for $x$, we have $3y^3\sqrt{3}-2y\sqrt{3}+\sqrt{3}=0$, which, by dividing by $\sqrt{3}$, is $3y^3-2y+1=0$ for the equation required.

By this rule, fractions and surds may be taken out of an equation, viz. by dividing the new letter by the common deno-

denominator; or by multiplying the new letter by the ſurd quantity.

### *To take away any Term out of an Equation.*

RULE. Add an unknown quantity to a new letter, and ſubſtitute this ſum and the powers thereof, for the root in the given equation; then any term, or any of thoſe quantities wherein the new letter is of the ſame power, being put into an equation, and made equal to nothing, will give the value of the unknown quantity, which being put into the equation with the new letter, the power of the new letter, which was equated, will vaniſh.

*Example.*

Suppoſe $x^4-3x^3+3x^2-5x-2=0$

Put $y+e=x$ then $x^4=y^4+4y^3e+6y^2e^2+4ye^3+e^4$

$-3x^3=-3y^3-9y^2e-9ye^2-3e^3$

$+3x^2=+3y^2+6ye+3e^2$

$-5x=-5y-5e$

$+2=+2$

If the ſecond term is to be taken away, we have $4y^3e-3y^3=0$; and dividing by $y^3$, $4e-3=0$, or, $e=\frac{3}{4}$, which ſubſtituted for $e$, the ſecond term will vaniſh. If the third term is to be taken away, we have $6y^2e^2-9y^2e+3y^2=0$, and dividing by the $y^2$, we have $6e^2-9e+3=0$, from which quadratic equation $e$ may be determined. In like manner the fourth term may be taken away by ſolving the cubic equation; and the fifth term by ſolving a biquadratic equation, &c.

### *To reſolve or extract the Root of a cubic Equation.*

RULE. Take the ſecond term of the equation away, as taught in the laſt example; then the equation will be in this form, $x^3+ax=b$, and the following general expreſſion will give the value of $x$.

 $\frac{1}{3}a$

$$\overline{\frac{b}{2}+\sqrt{\frac{b^2}{4}+\frac{a^3}{27}}}\Big|^{\frac{1}{3}} \qquad \frac{\frac{1}{3}a}{\overline{\frac{-b}{2}+\sqrt{\frac{b^2}{4}+\frac{a^3}{27}}}\Big|^{\frac{1}{3}}}$$

There are ſeveral other particular rules for the ſolution of biquadratic and other higher equations; but the method of approximating the roots of equations has ſuperſeded all the other methods, on account of its diſpatch.

### *To approximate the Roots of Equations, in general.*

Rule. By ſeveral trials, chooſe ſome number to repreſent the unknown quantity, and ſuch a number that approaches pretty near to the true value. Then aſſume ſome letter, as $v$, to denote the effect or exceſs of the number ſo found, and put that number $+$ or $-v$, inſtead of the unknown quantity in the equation; by which a new equation will ariſe affected with $v$ only, and known quantities; wherein all the terms that contain two or more dimenſions of $v$ may be rejected as inconſiderable in reſpect to the reſt. This being done, the value of $v$ will be found by a ſimple equation, which, added to or ſubtracted from the ſaid number, according as it was taken, too little or too big, will give a number ſtill nearer the truth. Then with this number and the letter $v$, proceed as before to find another value of $v$, which muſt be applied as above: repeat the operation till the unknown quantity be diſcovered to a ſufficient degree of exactneſs.

Example. Let it be required to find the value of $x$, in this equation, $x^3+24x=587814$. Here by a few trials it will be found that $x$ is ſomething above 80; wherefore, let $80+v=x$; then, $x^3=512000+192000v+1680v+v^3$, and $24x=1920+24v$. Therefore (rejecting thoſe terms affected with

with $v^2$ and $v^3$) we have $513920+19224v=587914$, and $v=\frac{587914-513920}{19224}=\frac{73994}{19224}=3.8$: this, added to 80, gives 83 . 8 for the approximate value of $x$; this being fubftituted in the equation, it will be found too great; therefore, for another operation take $83.8-v=x$, and $x^3=588480.472-21067.32v$, and $24x=2011.2-24v$, and confequently $590491.672-21091.32v=587914$, and $v=\frac{2577.072}{21091.32}=0.1222$ nearly. Hence $x=83.8-0.1222.=83.6778$. true to five figures, and the fixth being too much by only two.

If the operation be repeated, it will give the anfwer true to eleven figures. But when five or fix figures of the root have been obtained, and more exactnefs is ftill required, it will fhorten the work to feek a correction for $v$, inftead of one for the whole root, which may be had by fubftituting the laft value of $v+$ or $-$ (new) $v$, inftead of the laft $v$ in the equation, including all the powers of the laft $v$, but rejecting thofe of the new $v$ (in the equation, thence arifing), as before.

Thus, in the laft example, the whole equation, including the terms affected with $v^2$ and $v^3$, is $513920+19224v+1680v^2+v^3=587914$, or $19224v+1680v^2+v^3=73994$, and in this equation putting $3.8-v$, inftead of $v$, and rejecting the terms of $v^2$ and $v^3$ as before; a correction will be obtained for the laft found $v$, which will give the anfwer as above. In like manner, a fecond or third correction may be found, and the operation carried on to any degree of exactnefs. This rule doubles the number of figures true in the root at each operation. If the term wherein $v^2$ is found be retained, $v$ will be had by folving a quadratic, and then treble the number of figures will be had each time; therefore, if the firft figure only be taken true, nine or ten figures will be had at two operations. This rule affords various theorems for folving particular equations as well as general

general formulas adapted to all, of which I ſhall give two examples:

EXAMPLE 1. $x^2+6x=61495=c$. Here, by trial, it will be found that $x$ is above 200; therefore, let $200=r$, then $-b=r-1200$ and $c-r^2-br=20295$. The diviſion is as follows:

*The Operation.*

$$\begin{array}{rll}
2r+b=406 & & (-45=245=x) \\
v=40 & \Big)\, 20295\,(45=v, \text{ and } r+v-=200 & \\
2r+b+v=446 & & \\
40 & 1784 & \\
2r\ (\text{new})\ +b=486 & \Big)\ 2455 & \\
v^2=\ \ 5 & \Big)\ 2155 & \\
2r+b+v^2=491 & &
\end{array}$$

EXAMPLE 2. Again, for a cubic equation, let $x^3+bx^2+dx=c$. Put $r\pm v=x^3$, ſo will $x^3=r^3\pm 3r^2v+\pm 3rv^2\pm v^3$; $bx^2=br\pm 2brv+bv^2$, and $dx=dr\pm dv$. Hence $r^3+br^2\pm\overline{3r^2+2br+d}\times v+\overline{3r+b}\times v^2+dr=c$, ($v^3$ being rejected, as ſmall in compariſon of the reſt,) and by tranſpoſition $\pm\overline{3r^2+2br+d}\times v+\overline{3r+b}\times v^2=c-r^3-br^2-dr$; and $v=\pm\dfrac{c-r^3-br^2-dr}{\overline{3r+2br+d}+\overline{3r+bv}}$, which is wrought after the ſame manner as the laſt example.

## *To find the Limits of Equations.*

When an equation contains ſeveral unknown quantities, it will admit of an infinite number of ſolutions, when both fractional and negative numbers are admitted; for all of them but one may be taken at pleaſure, and their value ſubſtituted in the equation, which quantity will de determined. But ſometimes both fractional and negative quantities will be excluded from an equation, and ſuch equation will be confined to a determinate number of ſolutions. I ſhall, therefore,

fore, aſſign the limits of ſuch equations in the following caſes:

## CASE I.

### *When ſeveral unknown Quantities are in one Equation; to find the Limits.*

Rule. Tranſpoſe all the negative quantities to the contrary ſide, that all the terms may be affirmative; then to find the limits of any one quantity, ſuppoſe all the reſt to vaniſh in the equation, then the value of that one will become determinate, and will be one limit thereof. And to know which limit it is, ſuppoſe the other quantities to increaſe, and become of ſome certain value: then if the value of the unknown quantity under conſideration, increaſe, it is the leaſt limit; but if it decreaſe, it is the greateſt.

When fractional quantities are to be excluded, inſtead of ſuppoſing the other quantities to vaniſh, put each of them $=1$, and an equation will ariſe, from which the limits of the remaining quantity will be found as before. Proceed in the ſame manner to find the limits of the other unknown quantities.

Example. What are the limits of $x$ and $y$, in the equation $4x+5y=67$? Let $y=0$ or be ſuppoſed to vaniſh, and then $4x=67$, and $x=16\frac{3}{4}$. Now let $y$ be ſuppoſed to be equal to ſome quantity; then it is evident, that as $y$ increaſes, $x$ decreaſes, therefore $16\frac{3}{4}$ is the greater limit; wherefore $x$ is leſs than $16\frac{3}{4}$.

If $x=0$ then $5y=67$ and $y+13\frac{2}{5}$. Now if $x$ be ſuppoſed to increaſe, $y$ will decreaſe; and therefore $13\frac{2}{5}$ is the greater limit of $y$; whence $y$ is leſs than $13\frac{2}{5}$: and the leſs limit of both $x$ and $y$ is $0$.

CASE

## CASE II.

*To determine the Limits of three or more unknown Quantities, when they are in two Equations.*

RULE. Fix upon a quantity to be limited, and expunge one of the other quantities; then there will be had one limiting equation. Do the ſame with another unknown quantity, and there will be had another limiting equation; from each of which equations find a limit for the quantity fixed on.

EXAMPLE. What is the limit of $x$ in the two following equations? $x+y+z=56$, and $32x+20y+16z=1232$. To exterminate $y$, multiply the firſt equation by 20, and there ariſes $20x+20y+20z=1120$; ſubtract this from the ſecond equation, and we have $12x-4z=112$; then excluding the fractions, the leſs limit of $x$ in this equation is $9\frac{2}{3}$.

Again, to exterminate $z$, multiply the firſt equation by 16, and ſubtract the product from the ſecond equation, and there remains $16x+4y=336$. And the greater limit of $x$ in this equation is $20\frac{3}{4}$. Hence $x$ is greater than $9\frac{3}{4}$, and leſs than $20\frac{3}{4}$. In the ſame manner may $y$ and $z$ be limited; and ſo alſo in any other equation.

### *Of indeterminate Problems.*

EXAMPLE. What is the leaſt integer for the value of $x$, that will alſo cauſe the value of the following fraction to be an integer? $\frac{ax+b}{c}$

RULE. Divide the denominator ($c$) by the coefficient ($a$) of the indeterminate quantity; then divide the diviſor by the remainder, and the laſt diviſor again by the laſt remainder, and continue this operation till an unit only remains. Write down all the quotients in a line, as they riſe, under the firſt quotient,

quotient write an unit, and under the fecond quotient write the firft quotient; then multiply thefe two together: to the product add the firft term of the lower line, or an unit, and place the fum under the third term of the upper line; multiply in like manner the next two correfponding terms of the two lines together, and add the fecond term of the lower line to the product; put down the fum under the fourth term of the upper line; proceed in the fame manner till you have multiplied by every number in the upper line. Then multiply the laft number, thus found by the abfolute number ($b$), in the numerator of the fraction, and divide the product by the denominator; then the remainder will be the true value of $x$ required, provided the number of terms in the upper line be even, and the fign of $b$ be negative; or that the number be odd, and the fign of $b$ affirmative: but if the number of terms be even, and the fign of $b$ affirmative, or *vice verfa*, then the difference between the faid remainder and the denominator of the fraction will be the true anfwer.

*Operation.*

The given fraction $\frac{ax+b}{c}=\frac{71x+10}{89}$

| | |
|---|---|
| 71)89(1 | 1.3.1 Total quotients |
| 18)71(3 | 1.1.4.5 |
| 17)18(1 | $10=b$ |
| 1 | 50 Product. |

Here, if the product 50 be divided by 89, the remainder is 50= the leaft value of $x$.

In this rule it is always fuppofed, that $a$ is lefs than $c$; and that they are prime to each other; for if they were to admit of a common meafure, whereby $b$ is not divifible, no integer could be affigned for $x$, fo as to give the value of the fraction $\frac{ax+b}{c}$ an integer.

Example 2. What is the leaſt value of $x$ and $y$ in whole numbers, in the equation $24x-13y=16$? Here by tranſpoſing $13y$, and dividing by 24, we have $x=\frac{13y+16}{24}$; and by tranſpoſing $-13y+16$, we have $y=\frac{24x-16}{13}$; therefore the leaſt value of $y=8$, and the leaſt value of $x=5$.

*To find the Value of a Fraction in an infinite Series.*

Rule. Divide the numerator by the denominator, and continue the operation as far as is neceſſary. For in many caſes, after the quotient is continued to a few terms, it may be ſeen how the terms converge, and thus any number of terms may be aſſigned at pleaſure.

Example 1. What is the value of $\frac{1}{1+x^2}$

$$
\begin{array}{l}
1+x^2)\,1\,(1-x^2+x^4-x^6+x^8-\&c. \\
\quad\ \ \underline{1+x^2} \\
\qquad -x^2 \\
\qquad \underline{-x^2-x^4} \\
\qquad\qquad x^4 \\
\qquad\qquad \underline{x^4+x^6} \\
\qquad\qquad\quad -x^6 \\
\qquad\qquad\quad \underline{-x^6-x^8} \\
\qquad\qquad\qquad x^8 \\
\qquad\qquad\qquad \underline{x^8+x^{10}} \\
\qquad\qquad\qquad \underline{\underline{\quad -x^{10}}}
\end{array}
$$

If a quantity, which is not a fraction, is to be thrown into an infinite ſeries, it muſt be brought into a fraction, by placing one underneath it, as the denominator.

After a few terms are found in the ſeries, the law by which it converges will ſoon be diſcovered, and the terms may be continued to any number.

Sometimes the ſeries cannot eaſily be diſcovered by reaſon of the coefficients; then it will be neceſſary to aſſume a ſeries

ſeries with unknown coefficients to repreſent it; which being multiplied or involved as the queſtion requires, and the quantities of the ſame dimenſion being put equal to each other, new equations will be had, wherein the coefficients may be diſcovered.

EXAMPLE 2. Suppoſe $\frac{1}{a-x}$ be the given quantity, and the aſſumed ſeries be $A+Bx+Cx^2+Dx^3+Ex^4$, &c. $=\frac{1}{a-x}$ Multiply both by $a-x$, and there ariſes $1=aA+aBx+aCx^2+aDx^3+aEx^4$, &c. And $-Ax-Bx^2-Cx^3-Dx^4$, &c. and by equating the coefficients of the ſame powers of $x$, $aA=1$, $aB-A=0$, $aC-B=0$, $aD-c=0$, $aE-D=0$, &c. Thus, in the firſt equation $A=\frac{1}{a}$; in the ſecond equation $B=\frac{A}{a}=\frac{1}{a^2}$; in the third $C=\frac{B}{a}=\frac{1}{a^3}$; in the fourth $D=\frac{C}{a}=\frac{1}{a^4}$, and in the like manner $E=\frac{1}{a^5}$; therefore, $\frac{1}{a-x}$ brought to a ſeries, is $\frac{1}{a}+\frac{x}{a^2}+\frac{x^2}{a^3}+\frac{x^3}{a^4}+\frac{x^4}{a^5}$, &c.

## *Some of the Properties of ſquare Numbers.*

1. All even ſquare numbers are diviſible by 4; therefore, if a number conſiſts of two even ſquare numbers, it will be diviſible by 4.

2. Any odd ſquare number divided by 4 leaves a remainder of 1; therefore, if a number conſiſting of two odd ſquare numbers be divided by 4, there will be a remainder of 2.

3. Therefore, if a number conſiſting of an odd and an even ſquare number, be divided by 4, there will be a remainder of 1.

4. From hence it follows, that if any number compoſed of two ſquare numbers be divided by 4, it cannot leave a remainder of 3; therefore, a number compoſed of two ſquare numbers, cannot fall within this progreſſion 3, 7, 11, 15, 19, 23, &c.

5. Any number ending in 2, 3, 7, or 8, is not a ſquare number.

6. The ſum of any number of terms of the ſeries 1, 3, 5, 7, 9, 11, &c. beginning with the firſt, is a ſquare number, whoſe root is equal to the number of terms.

7. The difference between any two ſquare numbers is equal to the product of the ſum and difference of their roots. Thus, if $a$ and $b$ be the roots, then $a+b\times a-b=a^2-b^2$, and the ſame is alſo equal to the ſum of the two roots, together with twice the ſum of the roots of all the intermediate ſquare numbers. Thus, the difference between 36 and $9=\overline{6+3}+2\times\sqrt{16+25}=9+18=27$.

To reſolve queſtions of this nature, the chief point is, to make ſuch aſſumptions for the root of the required ſquare, or cube, as ſhall, when involved, cauſe either the given number, or the higheſt power of the unknown quantity, to vaniſh from the equation; whereby at length there will be only one dimenſion of the unknown quantity, and ſo the queſtion will be ſolved by reducing the equation.

## SECT. V.

### THE RESOLUTION OF SEVERAL ALGEBRAIC PROBLEMS.

Qu. 1. What are thoſe two numbers, the ſum whereof is 108, and the proportion of the leſs to the greater is as 5 to 7?

Let $x$ repreſent the greater number, then $108-x$ is equal to the leſs number; and the proportion of the numbers will be as follows: $108-x : x :: 5 : 7$, theſe four quantities being in a direct proportion, the product of the two means $5x$ is equal to the product of the two extremes $756-7x$, therefore, we have this equation $5x=756-7x$; and by tranſpoſing $7x$, we have $12x=756$. Hence, by dividing 756 by 12, $x$ is found equal to 63, which is the greater number, therefore, $108-63=45$, the leſs number.

Qu. 2. Bought apples at ſix for a penny, and pears at five for twopence. The number of apples and pears together was 100; the money given for the whole was 2*s*. 2*d*; how many were there of each ſort?

Let $a$ repreſent the number of apples, then, $100-a$ will be the number of pears: and as $6 : 1d. :: a : \frac{a}{6}=$, price of the apples. Alſo, as $5 : 2d. :: 100-a : \frac{200-2a}{5}=$, price of the pears; then $\frac{a}{6}+\frac{200-2a}{5}=26$. This equation multiplied by 30, gives $5a+1200-12a=780$. By tranſpoſing and dividing $7a=1200-780=420$, and by diviſion $a=60$, the number of apples; and $100-60=40$, the number of pears.

Qu. 3. It is required to divide the number 128 into four ſuch parts, that if the firſt part be added to 7, the ſecond made

made lefs by 7, the third multiplied by 7, and the fourth divided by 7, the refults may be equal among themfelves.

Let the four parts into which the number is divided be reprefented by the letters $v$, $x$, $y$, $z$. Then $v+7=x-7=7y=\frac{z}{7}$ will reprefent the four quantities; and from the equality of the two firft equations we have $x=v+7+7$; from the equality of the firft and third equations $y=\frac{v+7}{7}$, and from the equality of the firft and fourth $z=\overline{v+7}\times 7=7v+49$; therefore, by collecting thefe together, there arifes $\overline{v+14}+\frac{v+7}{7}+\overline{7v+49}=128$. By collecting the terms and tranfpofition $9v+\frac{v+7}{7}=65$; and multiplying this by 7, collecting the terms, tranfpofing and dividing, we have $v=7$. And hence $x=7+14=21$; $y=\frac{7+7}{7}=2$, and $z=\overline{7+7}\times 7=98$, the feveral parts required.

QU. 4. There are two cubical pieces of marble, the fide of one exceeding the fide of the other by three inches, the folid inches of both are 2457 inches; what is the length of the fide of each piece?

Let the fide of each piece be reprefented by $x$, then the fide of the greater will be $x+3$, and $x^3+\overline{x+3}^3=2457$ inches; therefore, $2x^3+9x^2+27x=2430$; this equation folved, gives $x=9$, and confequently $y=12$.

QU. 5. A gentleman left a fum of money to be divided among three fervants, in fuch proportion, that one half of the fhare of the firft, one third of the fecond fhare, and one quarter of the fhare of the third, fhould be equal to 62*l.*; and one third of the firft, one fourth of the fecond, and one fifth of the third, equal to 47*l.*; and one fourth of the firft, one fifth of the fecond, and one fixth of the third, equal to 38*l.*;—what is each fervant's fhare?

Put $a=62$, $b=47$, and $c=38$, and let the three fhares required, be denoted by $x$, $y$, and $z$; then the conditions of

of the queſtion will ſtand thus: $\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=a$. $\frac{x}{3}+\frac{y}{4}+\frac{z}{5}=b$. $\frac{x}{4}+\frac{y}{5}+\frac{z}{6}=c$. Theſe equations brought out of fractions, give $6x+4y+3z=12a$. $20x+15y+12z=60b$. $15x+12y+10z=60c$. Here by ſubtracting the ſecond equation from four times the firſt, in order to exterminate $z$, there ariſes $4x+y=48a-60b$; then, by taking three times the third equation from ten times the firſt, we have $15x+4y=120a-180c$, which, ſubtracted from four times the laſt equation, leaves $x=72a-240b+180c=24$; wherefore $y=48a-60b-4x=60$ and $z=\frac{12a-6x-4y}{3}=120$. demonſtrated thus:

$$\frac{24}{2}+\frac{60}{3}+\frac{120}{4}=12+20+30=62$$

$$\frac{24}{3}+\frac{60}{4}+\frac{120}{5}=8+15+24=47$$

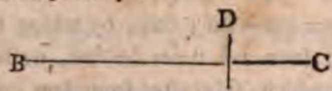
$$\frac{24}{4}+\frac{60}{5}+\frac{120}{6}=6+12+20=38$$

Qu. 6. A grocer bought 120 pounds of tea, and as many pounds of coffee; he had one pound of coffee more for 20 ſhillings than of tea, and the whole price of the tea exceeded that of the coffee by 6*l.*; I demand how many pounds of tea he had for 20 ſhillings, and how many pounds of coffee?

Let the number of pounds of tea bought for 20 ſhillings be repreſented by $x$, then the number of pounds of coffee, for 20 ſhillings, will be $x+1$, and the whole price of the tea will be $\frac{120}{x}$ pounds, and that of the coffee $\frac{120}{x+1}$ pounds; therefore, $\frac{120}{x}-\frac{120}{x+1}=6$; wherefore, $120x+120-120x=6x^2+6x$, therefore $x^2+x=20$; which reſolved gives $x=4$ the pounds of tea for 20 ſhillings, and $x+1=5$ the pounds of coffee for 20 ſhillings.

Qu.

QU. 7. Two travellers ſet out on a journey at the ſame time; the one ſets out from C to go to B, the other from B to go to C; they both travel uniformly, and in ſuch proportion that he that ſet out from B, four hours after meeting the other, arrives at C, and the other arrives at B, nine hours after meeting. How many hours did each perſon take to perform his journey?




In this figure, D is the place of their meeting; put $a=4$, $b=9$, and $x$ for the number of hours which they travel before they meet. Then the diſtances they have travelled, with the ſame uniform pace, will be to each other as the times in which they are deſcribed; therefore, B D : D C :: $x$, (the time in which the traveller who ſet out from B goes the diſtance B D,) : $a=$ the time in which he travels from D to C; and by the ſame manner as B D : D C :: $b$, (the time the other traveller goes from D to B,) : $x=$ the time he goes from C to D; now as $x$ is to $a$ in the ratio of B D to D C, and $b$ to $x$ in the ſame ratio, it will follow, that as $x : a :: b : x$, whence $x^2=ab$ and $x=\sqrt{ab}=6$, therefore, $a+\sqrt{ab}=10$, and $b+\sqrt{ab}=15$, are the two numbers required.

QU. 8. The ſum of three numbers in geometrical proportion, and the ſum of the ſquares of three numbers being given; to find the numbers themſelves.

Put $a$ for the ſum of the three numbers, and $b$ for their ſquares, and $x$, $y$, and $z$ for the numbers themſelves, then we ſhall have $x+y+z=a$, and $x^2+y^2+z^2=b$ and $xz=y^2$, whence, by tranſpoſing $y$ in the firſt equation, and involving both ſides to the ſecond power, there ariſes $x^2+2xz+z^2=a^2-2ay+y^2$, from which ſubtracting the ſecond equation, we have $2xz-y^2=a^2-2ay+y^2-b$; but $2xz$ by the third equation is $=2y^2$, therefore, $2y^2-y^2=a^2-2ay+y^2-b$, or $a^2-2ay-b=0$, whence, $y=$

$\frac{a}{2}$

$\frac{a}{2}-\frac{b}{2a}$. Now to find $x$ and $z$, we muſt look upon $y$ as a known quantity, and then by the ſecond equation we ſhall have $x^2+z^2=b-y^2$, from which ſubtracting $2xz=2y^2$, we have $x^2-2xz+z^2=b-3y^2$, and by taking the root we have $x-z=\sqrt{b-3y^2}$, but by the firſt equation $x+z=a-y$, therefore, $x=\frac{a-y+\sqrt{b-3y^2}}{2}$ and $z=\frac{a-y-\sqrt{b-3y^2}}{2}$.

QU. 9. A farmer ſold as many ſheep and oxen as brought him 100*l.*; for the ſheep he received 17 ſhillings each, and for the oxen 7*l.* each. It is required to know how many he ſold of each?

Let the number of ſheep be $x$, and that of the oxen $y$; then we have this equation $17x+140y=2000$, and conſequently $x=\frac{2000-140y}{17}=117-8y+\frac{11-4y}{17}$ which being a whole number $\frac{11-4y}{17}$ or $\frac{4y-11}{17}$ muſt therefore be a whole number likewiſe; whence by proceeding as above, we have $y=7$, and $x=60$, and this is the only anſwer the queſtion will admit of.

QU. 10. What are the dimenſions of a cubical block of marble, whoſe ſide in inches is expreſſed by two digits; the ſuperficies of the block is equal to 864 times the ſum of the ſaid digits; and its ſolidity is equal to 576 times the ſquare of the ſum of the ſaid digits?

Put $x$ for the digit in the place of tens, and $y$ for the digit in the place of units, then $10x+y$ is equal to the ſide of the cube and $\overline{10x+y}|^2\times 6=864\times\overline{x+y}$; or $\overline{10x+y}|^2=144\times\overline{x+y}$. Alſo, $\overline{10x+y}|^3=576\times\overline{x+y}|^2$ per queſtion; and multiplying theſe equations croſs ways, we have $\overline{10x+y}|^2\times 576\times\overline{x+y}|^2=144\times\overline{x+y}\times\overline{10x+y}|^3$. Then dividing both ſides by $144\times$

$\overline{x+y}\times\overline{10x+y}|^2$, we have $\overline{x+y}\times 4=10x+y$, or $4x+4y=10x+y$, and by tranſpoſition $3y=6x$, therefore, $y=2x$. This being ſubſtituted for $y$ in the former equation, we have $\overline{10x+2x}|^2=144\times\overline{x+2x}$ or $144x^2=144\times 3x$, and dividing by $144x$ we have $x=3$ and $y=2x=6$; therefore, the ſide of the cube is 36.

# CHAP. X.

# OF THE VALUE OF LIVES;

OR,

# *DOCTRINE OF ANNUITIES.*

## SECT. I.

### THE VALUE OF AN ANNUITY FOR A SINGLE LIFE.

AN Annuity is a ſum of money payable yearly, half-yearly, or quarterly; to continue either for life, for a certain number of years, or for ever.

When an annuity remains unpaid after it is due, it is ſaid to be in *arrear*. When the purchaſer of an annuity does not immediately enter upon poſſeſſion, the annuity is ſaid to be in *reverſion*.

The

The intereſt upon annuities in arrear may be computed either in the way of ſimple or compound intereſt. But compound intereſt being found moſt equitable, both for buyer and ſeller, is in moſt general uſe.

Annuities may be divided into certain and uncertain.

A certain annuity is that which continues for a certain time, or for ever. An uncertain annuity depends upon one or more lives.

Before I proceed to give the doctrine of contingent annuities, it will be neceſſary to deliver the rules for calculating of annuities certain.

## PROBLEM I.

### *To find the Amount of an Annuity for a given Term of Years, at a given Rate of Intereſt.*

EXAMPLE. What will an annuity of 50*l.* amount to, at the end of 8 years, at the rate of 5 *per cent. per annum*, ſimple intereſt?

In this example, the intereſt being at 5 per cent. multiply the rate of intereſt of 1*l.* for 1 year, or .05 by 50 the annuity, and the product by 8, the number of years, and the product hence ariſing is 20; the half whereof (10) multiplied by the number of years, made leſs by one, (7,) produces 70, the ſimple intereſt; which added to the product of 50, and 8, (400,) give 470, the amount required.

## PROBLEM II.

### *To find the Amount of an Annuity, at compound Intereſt.*

RULE. Multiply the amount of 1*l.* for 1 year, as often
there are years, except one; or, which is the
to the power whoſe index is equal to the num-
from the reſult ſubtract 1; then divide

the remainder by the intereſt of 1*l.* for 1 year, and multiply the quotient by the annuity, and the product will be the amount required.

Example. What is the amount of an annuity of 50*l.* for 3 years, at 5 *per cent. per annum*, *compound intereſt?* Here the amount of 1*l.* for 1 year is 1.05, which multiplied twice into itſelf, produces 1.157625, and 1 ſubtracted from this, the remainder is .157625, which divided by .05, the quotient is 3.1525; this multiplied by 50, produces 157.625, or 157*l.* 12*s.* 6*d.* the anſwer required.

Note. If the payments are half-yearly or quarterly, the amount, and intereſt of 1*l.* muſt be taken for a half, or a quarter of a year. And then the double or quadruple of the time muſt be taken. And the amount of 1*l.* for half a year at compound intereſt is equal to the ſquare root of the amount for a year; and the amount for a quarter of a year is equal to the ſquare root of that for half a year.

## PROBLEM III.

### *To find the preſent Value of an Annuity, having the Time and Rate.*

Rule. Multiply the amount of one year as often into itſelf as there are years, leſs 1; or involve it to the power denoted by the time: by this reſult, divide 1, and ſubtract the quotient from 1, divide the remainder by the intereſt of 1*l.* for a year; then multiply this laſt quotient by the annuity, and the product will be the preſent value.

Example. What is the preſent value of an annuity of 40*l.* for 5 years, diſcounting 5 *per cent. per annum*, *compound intereſt?* Here 1.05 involved to the fifth power is 1.27628. By which dividing 1, the quotient is .78353, which ſubtracted from 1, leaves .21647; this divided by .05 gives 4.3294, which multiplied by 40 is 173.176, or 173*l.* 3*s.* 6½*d.* the preſent worth.

PROBLEM

## PROBLEM IV.

### *Having the preſent Worth, Rate, and Time, to find the Annuity.*

Rule. Find the preſent value of 1*l.* annuity at the given rate and time; and then by the rule of three, ſay, as the preſent worth, thus found, is to 1*l.* annuity, ſo is the preſent worth given to its annuity; that is, divide the given preſent worth by that of 1*l.* annuity.

Example. What annuity will 173*l.* 3*s.* 7*d.* purchaſe to continue 5 years, allowing *compound intereſt at 5 per cent. per annum?*

.05:1::1:20*l.*

$1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05 = 1.2762815625$

1.2762815625)20.00000000(15.6705

$$\begin{array}{r} 20 \\ \underline{15.6705} \\ \underline{\underline{4.3295}} \end{array}$$ preſent worth of 1*l.* annuity.

4.329)173.179(40*l.* annuity, Anſwer.

### *Annuities for ever, or Freehold Eſtates.*

In calculating the value of an annuity for ever, commonly called an *Annuity in fee ſimple*, three things are to be conſidered: 1. The annuity, or yearly rent. 2. The price, or preſent worth. 3. The rate of intereſt.

## PROBLEM I.

### *Having the Rent and Rate of Intereſt, to find the Price or Value.*

Rule. As the intereſt of 1*l.* is to 1*l.* ſo is the rent to the price or value.

Example.

EXAMPLE. What is the prefent worth of an annuity of 40*l.* per annum in fee fimple, *compound intereft* $3\frac{1}{2}$ *per cent. per ann.?* As .035 the intereft of 1*l.* for a year is to 1*l.* fo is 40*l.* the rent of the annuity to 1142.857142 or 1142*l.* 17*s.* $1\frac{1}{2}$*d.*

## PROBLEM II.

### *Having the Price and Rate of Intereft, to find the Annuity.*

RULE. As 1*l.* is to its intereft, fo is the price to the annuity.

EXAMPLE. What annuity will 4000*l.* purchafe, at $4\frac{1}{2}$ *per cent. per ann. compound intereft?*

As 1*l.* is to .045, fo is 4000*l.* to 180*l.* the annuity.

## PROBLEM III.

### *Having the Price and Rent of the Annuity, to find the Rate of Intereft.*

RULE. As the price is to the rent, fo is 1*l.* to the rate of intereft.

EXAMPLE. If an annuity of 180*l.* coft 4000*l.* what is the rate of intereft compound *per ann.?*

As 4000:180::1: .045 or $4\frac{1}{2}$ per cent. rate of intereft.

## PROBLEM IV.

### *Having the Rate of Intereft, to find how many Years Purchafe an Eftate is worth.*

RULE. Divide 1 by the rate of intereft, and the quotient is the anfwer.

EXAMPLE. How many years purchafe is an annuity, when the purchafer has $2\frac{1}{2}$ *per cent.* for his money?

.025)1.000( 40 years purchafe.

PROBLEM

## PROBLEM V.

### *Having the Number of Years Purchafe, to find the Rate of Intereft.*

Rule. Divide 1 by the number of years purchafe, and the quotient is the rate of intereft.

Example. What intereft has a purchafer, who gives 40 years purchafe for an annuity?

40)1.000(.025 intereft required.

Though the foregoing examples are moftly performed by a fingle divifion or multiplication, yet they give the anfwers at compound intereft; but in cafes where there is a reverfion, recourfe muft be had to the tables of annuities, on compound intereft, as in the following Problems:

## PROBLEM VI.

### *Having the Rate of Intereft, and the Annuity, to find the prefent Value of the Reverfion.*

Rule. Find the prefent value of the annuity by Problem I. then, by the tables, find the prefent value of the annuity for the years before the reverfion takes place. Subtract this value from the former value, and the remainder is the prefent value of the reverfion.

Example. What is the value of an eftate, or an annuity of 130*l. per ann.* to continue 20 years? What is the value of the fame, after the expiration of 20 years, to continue for ever? and what is the value of the whole, at 6 *per cent. compound intereft per ann.*?

| | |
|---|---|
| ,06)130,000(2166.6666 | Value of the whole. |
| 1491.0896 | Value of the poffeffion. |
| 675.5770 | Value of the reverfion. |

PROBLEM

# TABLE II.

SHOWING

*The Probabilities from Ten Years Obſervation on the Bills of Mortality in London.*

By Mr. SIMPSON.

| Age. | Perſons living. | Decrement of Life. | Age. | Perſons living. | Decrement of Life. | Age. | Perſons living. | Decrement of Life. |
|---|---|---|---|---|---|---|---|---|
| 0 | 1000 | 320 | 27 | 321 | 6 | 54 | 135 | 6 |
| 1 | 680 | 133 | 28 | 315 | 7 | 55 | 129 | 6 |
| 2 | 547 | 51 | 29 | 308 | 7 | 56 | 123 | 6 |
| 3 | 496 | 27 | 30 | 301 | 7 | 57 | 117 | 5 |
| 4 | 469 | 17 | 31 | 294 | 7 | 58 | 112 | 5 |
| 5 | 452 | 12 | 32 | 287 | 7 | 59 | 107 | 5 |
| 6 | 440 | 10 | 33 | 280 | 7 | 60 | 102 | 5 |
| 7 | 430 | 8 | 34 | 273 | 7 | 61 | 97 | 5 |
| 8 | 422 | 7 | 35 | 266 | 7 | 62 | 92 | 5 |
| 9 | 415 | 5 | 36 | 259 | 7 | 63 | 87 | 5 |
| 10 | 410 | 5 | 37 | 252 | 7 | 64 | 82 | 5 |
| 11 | 405 | 5 | 38 | 245 | 8 | 65 | 77 | 5 |
| 12 | 400 | 5 | 39 | 237 | 8 | 66 | 72 | 5 |
| 13 | 395 | 5 | 40 | 229 | 7 | 67 | 67 | 5 |
| 14 | 390 | 5 | 41 | 222 | 8 | 68 | 62 | 4 |
| 15 | 385 | 5 | 42 | 214 | 8 | 69 | 58 | 4 |
| 16 | 380 | 5 | 43 | 206 | 7 | 70 | 54 | 4 |
| 17 | 375 | 5 | 44 | 199 | 7 | 71 | 50 | 4 |
| 18 | 370 | 5 | 45 | 192 | 7 | 72 | 46 | 4 |
| 19 | 365 | 5 | 46 | 185 | 7 | 73 | 42 | 3 |
| 20 | 360 | 5 | 47 | 178 | 7 | 74 | 39 | 3 |
| 21 | 355 | 5 | 48 | 171 | 6 | 75 | 36 | 3 |
| 22 | 350 | 5 | 49 | 165 | 6 | 76 | 33 | 3 |
| 23 | 345 | 6 | 50 | 159 | 6 | 77 | 30 | 3 |
| 24 | 339 | 6 | 51 | 153 | 6 | 78 | 27 | 2 |
| 25 | 333 | 6 | 52 | 147 | 6 | 79 | 25 | 2 |
| 26 | 327 | 6 | 53 | 141 | 6 | 80 | 23 | 2 |

## TABLE III.

*The Probabilities of Life, calculated from Forty-six Years Observation on the Bills of Mortality at Northampton: viz. from* 1735 *to* 1780.

| Age. | Perfons living. | Decrement of Life. | Age. | Perfons living. | Decrement of Life. | Age. | Perfons living. | Decrement of Life. |
|---|---|---|---|---|---|---|---|---|
| 0 | 11650 | 1894 | 34 | 4085 | 75 | 69 | 1312 | 80 |
| ½ | 9756 | 1106 | 35 | 4010 | 75 | 70 | 1232 | 80 |
| 1 | 8650 | 1367 | 36 | 3935 | 75 | 71 | 1152 | 80 |
| 2 | 7283 | 502 | 37 | 3860 | 75 | 72 | 1072 | 80 |
| 3 | 6781 | 335 | 38 | 3785 | 75 | 73 | 992 | 80 |
| 4 | 6446 | 197 | 39 | 3710 | 75 | 74 | 912 | 80 |
| 5 | 6249 | 184 | 40 | 3635 | 76 | 75 | 832 | 80 |
| 6 | 6065 | 140 | 41 | 3559 | 77 | 76 | 752 | 77 |
| 7 | 5925 | 110 | 42 | 3482 | 78 | 77 | 675 | 73 |
| 8 | 5815 | 80 | 43 | 3404 | 78 | 78 | 602 | 68 |
| 9 | 5735 | 60 | 44 | 3326 | 78 | 79 | 534 | 65 |
| 10 | 5675 | 52 | 45 | 3248 | 78 | 80 | 469 | 63 |
| 11 | 5623 | 50 | 46 | 3170 | 78 | 81 | 406 | 60 |
| 12 | 5573 | 50 | 47 | 3092 | 78 | 82 | 346 | 57 |
| 13 | 5523 | 50 | 48 | 3014 | 78 | 83 | 289 | 55 |
| 14 | 5473 | 50 | 49 | 2936 | 79 | 84 | 234 | 48 |
| 15 | 5423 | 50 | 50 | 2857 | 81 | 85 | 186 | 41 |
| 16 | 5373 | 53 | 51 | 2776 | 82 | 86 | 145 | 34 |
| 17 | 5320 | 58 | 52 | 2694 | 82 | 87 | 111 | 28 |
| 18 | 5262 | 63 | 53 | 2612 | 82 | 88 | 83 | 21 |
| 19 | 5199 | 67 | 54 | 2530 | 82 | 89 | 62 | 16 |
| 20 | 5132 | 72 | 55 | 2448 | 82 | 90 | 46 | 12 |
| 21 | 5060 | 75 | 56 | 2366 | 82 | 91 | 34 | 10 |
| 22 | 4985 | 75 | 57 | 2284 | 82 | 92 | 24 | 8 |
| 23 | 4910 | 75 | 58 | 2202 | 82 | 93 | 16 | 7 |
| 24 | 4835 | 75 | 59 | 2120 | 82 | 94 | 9 | 5 |
| 25 | 4760 | 75 | 60 | 2038 | 82 | 95 | 4 | 3 |
| 26 | 4685 | 75 | 61 | 1956 | 82 | 96 | 1 | 1 |
| 27 | 4610 | 75 | 62 | 1874 | 81 | | 299198 | 11650 |
| 28 | 4535 | 75 | 63 | 1793 | 81 | | | |
| 29 | 4460 | 75 | 64 | 1712 | 80 | | | |
| 30 | 4385 | 75 | 75 | 1632 | 80 | | | |
| 31 | 4310 | 75 | 66 | 1552 | 80 | | | |
| 32 | 4235 | 75 | 67 | 1472 | 80 | | | |
| 33 | 4160 | 75 | 68 | 1392 | 80 | | | |

# TABLE IV.

*The Expectation of Life at every Age, according to the Bills of Mortality for both London and Northampton.*

| Age. | Expectation. London. | Expectation. North. | Age. | Expectation. London. | Expectation. North. | Age. | Expectation. London. | Expectation. North. |
|---|---|---|---|---|---|---|---|---|
| 0 | 19.5 | 25.18 | 34 | 22.4 | 26.20 | 68 | 9.9 | 9.50 |
| 1 | 27.5 | 32.74 | 35 | 22.0 | 25.68 | 69 | 9.6 | 9.05 |
| 2 | 32.5 | 37.79 | 36 | 21.6 | 25.16 | 70 | 9.3 | 8.60 |
| 3 | 34.5 | 39.55 | 37 | 21.2 | 24.64 | 71 | 8.9 | 8.17 |
| 4 | 36.1 | 40.58 | 38 | 20.8 | 24.12 | 72 | 8.6 | 7.74 |
| 5 | 36.5 | 40.84 | 39 | 20.4 | 23.60 | 73 | 8.3 | 7.33 |
| 6 | 36.5 | 41.07 | 40 | 20.1 | 23.08 | 74 | 8.0 | 6.92 |
| 7 | 36.3 | 41.03 | 41 | 19.7 | 22.56 | 75 | 7.7 | 6.54 |
| 8 | 36.1 | 40.79 | 42 | 19.3 | 22.04 | 76 | 7.3 | 6.18 |
| 9 | 35.7 | 40.36 | 43 | 19.0 | 21.54 | 77 | 6.9 | 5.83 |
| 10 | 35.3 | 39.78 | 44 | 18.6 | 21.03 | 78 | 6.5 | 5.48 |
| 11 | 34.8 | 39.14 | 45 | 18.3 | 20.52 | 79 | 6.0 | 5.11 |
| 12 | 34.2 | 38.49 | 46 | 17.9 | 20.02 | 80 | 5.5 | 4.75 |
| 13 | 33.6 | 37.83 | 47 | 17.5 | 19.51 | 81 | | 4.41 |
| 14 | 33.0 | 37.17 | 48 | 17.2 | 19.00 | 82 | | 4.09 |
| 15 | 32.4 | 36.51 | 49 | 16.8 | 18.49 | 83 | | 3.80 |
| 16 | 31.8 | 35.85 | 50 | 16.5 | 17.99 | 84 | | 3.58 |
| 17 | 31.2 | 35.20 | 51 | 16.1 | 17.50 | 85 | | 3.37 |
| 18 | 30.6 | 34.58 | 52 | 15.7 | 17.02 | 86 | | 3.19 |
| 19 | 30.0 | 33.99 | 53 | 15.4 | 16.54 | 87 | | 3.01 |
| 20 | 29.4 | 33.43 | 54 | 15.0 | 16.06 | 88 | | 2.86 |
| 21 | 28.8 | 32.90 | 55 | 14.7 | 15.58 | 89 | | 2.66 |
| 22 | 28.2 | 32.39 | 56 | 14.3 | 15.10 | 90 | | 2.41 |
| 23 | 27.7 | 31.88 | 57 | 13.9 | 14.63 | 91 | | 2.09 |
| 24 | 27.1 | 31.36 | 58 | 13.6 | 14.15 | 92 | | 1.75 |
| 25 | 26.6 | 30.85 | 59 | 13.2 | 13.68 | 93 | | 1.37 |
| 26 | 26.1 | 30.33 | 60 | 12.9 | 13.21 | 94 | | 1.05 |
| 27 | 25.6 | 29.82 | 61 | 12.5 | 12.75 | 95 | | 0.75 |
| 28 | 25.1 | 29.30 | 62 | 12.1 | 12.28 | 96 | | 0.50 |
| 29 | 24.6 | 28.79 | 63 | 11.7 | 11.81 | | | |
| 30 | 24.1 | 28.27 | 64 | 11.3 | 11.35 | | | |
| 31 | 23.6 | 27.76 | 65 | 11.0 | 10.88 | | | |
| 32 | 23.2 | 27.24 | 66 | 10.6 | 10.42 | | | |
| 33 | 22.8 | 26.72 | 67 | 10.3 | 9.96 | | | |

# TABLE V.

*Showing the Number of Years Purchaſe an Annuity on a ſingle Life is worth according to the Probabilities of Life, at London and Northampton, from the Age of* 6 *to* 75 *Years; at* 3, 4, *and* 5 *per Cent.*

| Age. | 3 per cent. | | 4 per cent. | | 5 per cent. | | Age. | 3 per cent. | | 4 per cent. | | 5 per cent. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Lon. | North. | Lon. | North | Lon. | North | | Lon. | North | Lon. | North. | Lon. | North |
| 6 | 18.8 | 20.73 | 16.2 | 17.48 | 14.1 | 15.04 | 41 | 13.0 | 14.62 | 11.4 | 13.02 | 10.2 | 11.70 |
| 7 | 18.9 | 20.85 | 16.3 | 17.61 | 14.2 | 15.17 | 42 | 12.8 | 14.39 | 11.2 | 12.84 | 10.1 | 11.55 |
| 8 | 19.0 | 20.89 | 16.4 | 17.66 | 14.3 | 15.23 | 43 | 12.6 | 14.16 | 11.1 | 12.66 | 10.0 | 11.41 |
| 9 | 19.0 | 20.81 | 16.4 | 17.63 | 14.3 | 15.21 | 44 | 12.5 | 13.93 | 11.0 | 12.47 | 9.9 | 11.26 |
| 10 | 19.0 | 20.66 | 16.4 | 17.52 | 14.3 | 15.14 | 45 | 12.3 | 13.69 | 10.8 | 12.28 | 9.8 | 11.11 |
| 11 | 19.0 | 20.48 | 16.4 | 17.39 | 14.3 | 15.04 | 46 | 12.1 | 13.45 | 10.7 | 12.09 | 9.7 | 10.95 |
| 12 | 18.9 | 20.28 | 16.3 | 17.25 | 14.2 | 14.94 | 47 | 11.9 | 13.20 | 10.5 | 11.89 | 9.5 | 10.78 |
| 13 | 18.7 | 20.08 | 16.2 | 17.10 | 14.1 | 14.83 | 48 | 11.8 | 12.95 | 10.4 | 11.68 | 9.4 | 10.62 |
| 14 | 18.5 | 19.87 | 16.0 | 16.95 | 14.0 | 14.71 | 49 | 11.6 | 12.69 | 10.2 | 11.47 | 9.3 | 10.44 |
| 15 | 18.3 | 19.66 | 15.8 | 16.79 | 13.9 | 14.59 | 50 | 11.4 | 12.44 | 10.1 | 11.26 | 9.2 | 10.27 |
| 16 | 18.1 | 19.44 | 15.6 | 16.62 | 13.7 | 14.46 | 51 | 11.2 | 12.18 | 9.9 | 11.06 | 9.0 | 10.10 |
| 17 | 17.9 | 19.22 | 15.4 | 16.46 | 13.5 | 14.33 | 52 | 11.0 | 11.93 | 9.8 | 10.85 | 8.9 | 9.93 |
| 18 | 17.6 | 19.01 | 15.2 | 16.31 | 13.4 | 14.22 | 53 | 10.7 | 11.67 | 9.6 | 10.64 | 8.8 | 9.75 |
| 19 | 17.4 | 18.82 | 15.0 | 16.17 | 13.2 | 14.11 | 54 | 10.5 | 11.41 | 9.4 | 10.42 | 8.6 | 9.57 |
| 20 | 17.2 | 18.64 | 14.8 | 16.03 | 13.0 | 14.01 | 55 | 10.3 | 11.15 | 9.3 | 10.20 | 8.5 | 9.38 |
| 21 | 17.0 | 18.47 | 14.7 | 15.91 | 12.9 | 13.92 | 56 | 10.1 | 10.83 | 9.1 | 9.98 | 8.4 | 9.19 |
| 22 | 16.8 | 18.31 | 14.5 | 15.80 | 12.7 | 13.83 | 57 | 9.9 | 10.61 | 8.9 | 9.75 | 8.2 | 9.00 |
| 23 | 16.5 | 18.15 | 14.3 | 15.68 | 12.6 | 13.75 | 58 | 9.6 | 10.34 | 8.7 | 9.52 | 8.1 | 8.80 |
| 24 | 16.3 | 17.98 | 14.1 | 15.56 | 12.4 | 13.66 | 59 | 9.4 | 10.06 | 8.6 | 9.28 | 8.0 | 8.60 |
| 25 | 16.1 | 17.81 | 14.0 | 15.44 | 12.3 | 13.57 | 60 | 9.2 | 9.78 | 8.4 | 9.04 | 7.9 | 8.39 |
| 26 | 15.9 | 17.64 | 13.8 | 15.31 | 12.1 | 13.47 | 61 | 8.9 | 9.49 | 8.2 | 8.80 | 7.7 | 8.18 |
| 27 | 15.6 | 17.47 | 13.6 | 15.18 | 12.0 | 13.38 | 62 | 8.7 | 9.21 | 8.1 | 8.55 | 7.6 | 7.97 |
| 28 | 15.4 | 17.29 | 13.4 | 15.05 | 11.8 | 13.28 | 63 | 8.5 | 8.91 | 7.9 | 8.29 | 7.4 | 7.74 |
| 29 | 15.2 | 17.11 | 13.2 | 14.92 | 11.7 | 13.18 | 64 | 8.3 | 8.61 | 7.7 | 8.03 | 7.3 | 7.51 |
| 30 | 15.0 | 16.92 | 13.1 | 14.78 | 11.6 | 13.07 | 65 | 8.0 | 8.30 | 7.5 | 7.76 | 7.1 | 7.28 |
| 31 | 14.8 | 16.73 | 12.9 | 14.64 | 11.4 | 12.97 | 66 | 7.8 | 7.99 | 7.3 | 7.49 | 6.9 | 7.03 |
| 32 | 14.6 | 16.54 | 12.7 | 14.50 | 11.3 | 12.85 | 67 | 7.6 | 7.68 | 7.1 | 7.21 | 6.7 | 6.79 |
| 33 | 14.4 | 16.34 | 12.6 | 14.35 | 11.2 | 12.74 | 68 | 7.4 | 7.37 | 6.9 | 6.93 | 6.6 | 6.54 |
| 34 | 14.2 | 16.14 | 12.4 | 14.20 | 11.0 | 12.62 | 69 | 7.1 | 7.05 | 6.7 | 6.65 | 6.4 | 6.28 |
| 35 | 14.1 | 15.94 | 12.3 | 14.04 | 10.9 | 12.50 | 70 | 6.9 | 6.73 | 6.5 | 6.36 | 6.2 | 6.02 |
| 36 | 13.9 | 15.73 | 12.1 | 13.88 | 10.8 | 12.38 | 71 | 6.7 | 6.42 | 6.3 | 6.08 | 6.0 | 5.76 |
| 37 | 13.7 | 15.52 | 11.9 | 13.72 | 10.6 | 12.25 | 72 | 6.5 | 6.10 | 6.1 | 5.79 | 5.8 | 5.50 |
| 38 | 13.5 | 15.30 | 11.8 | 13.55 | 10.5 | 12.12 | 73 | 6.2 | 5.79 | 5.9 | 5.51 | 5.6 | 5.24 |
| 39 | 13.3 | 15.08 | 11.6 | 13.38 | 10.4 | 11.98 | 74 | 5.9 | 5.49 | 5.6 | 5.23 | 5.4 | 4.99 |
| 40 | 13.2 | 14.85 | 11.5 | 13.20 | 10.3 | 11.84 | 75 | 5.6 | 5.20 | 5.4 | 4.96 | 5.2 | 4.74 |

## THE USE OF THE TABLES.

### *Table* I.

Shows the probability of life, according to Dr. Halley's computation: the firſt column ſhows the ages; the ſecond column, the number of perſons living at thoſe ages; and the third column, the decrement of life, or the number of perſons that died each year: thus, oppoſite age 1, is 1000 in the ſecond column, and 145 in the third column; which ſhows, that of 1000 perſons born in the ſame year, 145 died before the expiration of the year; and of 855, the remainder of the perſons living, 57 died the ſecond year, and ſo on.

But the calculations according to this table differ from thoſe made from the tables of the probabilities of life in London, partly owing to the different ſituations of theſe two places. Breſlaw being an inland town not much frequented by ſtrangers or foreigners, and London being a mercantile port, and crowded with traffickers and travellers from all parts of the world; and partly owing to the difference of climate, difference of food, and different manners of life, between the inhabitants of theſe two places, which is always found to occaſion a different proportion in the deaths at the ſame ages. Theſe conſiderations induced Mr. Simpſon to compoſe a table of the probability of life, calculated from the bills of mortality of London, and which table will conſequently much better anſwer the purpoſe of calculating the value of an annuity for a life at London, than the other table of Dr. Halley. This table may be ſeen page 51.

It muſt, however, be obſerved, that Dr. Halley's table is better adapted for the uſe of all Europe in general than any other particular table.

### *Table* III.

Shows the probability of life at all age
obſervation on the bills of mortality

from the year 1735 to 1780 inclufive. This table is juftly reckoned to be the moft correct of any extant, as it is taken from a large manufacturing town, and which confifts generally of the fame perfons: and alfo that the table is conftructed from a larger number of perfons born than any other table, being 11650, which affords an opportunity of obferving the decrement of life to a greater exactnefs.

## *Table* IV.

Shows the expectation of life, or the number of years which any perfon may be fuppofed to have a fair chance of living, according to an equality of chance at every age, according to the bills of mortality for both London and Northampton. Thus, againft age 2 ftands 32.5 in the fecond column, under London; and 37.79 in the third column, under Northampton; which fhows, that a child of the age of two years has an equal chance of living 32.5 years, according to the London tables, or 32 years 6 months; and according to the Northampton tables 37.79 years, or 37 years and upwards of 9 months.

## *Table* V.

Shows the value of an annuity for a fingle life, according to the probabilities of life at London and Northampton, from the age of 6 to 75 years inclufive, at 3, 4, and 5 *per cent.* Thus, fuppofe, it were required to find the number of years purchafe which an annuity is worth to a perfon of the age of 30: here in the fecond column, and oppofite the age 30, ftands 15.0, which fhows that an annuity for a perfon of 30 years of age in London, and at 3 *per cent.* is worth 15 years purchafe; and an annuity for a Northampton life, at the fame rate of intereft, is worth 16.92 years purchafe; and for a London life, at 4 *per cent.* an annuity is worth 13.1 years purchafe; and for a Northampton life, at the fame rate and

intereft,

is worth 14.78 years purchafe: and an annuity at 5 *per cent.* for a London life, is worth 11.6 years purchafe; and for a Northampton life, at the fame rate of intereft, 13.07 years purchafe. This table alfo fhows the value of an annuity of 1*l.* for a fingle life, at all the above-mentioned rates of intereft: thus, an annuity of 1*l. per ann.* for a fingle life, at 30 years of age, according to the London tables, at 3 *per cent.* is worth 15*l.*; and the fame, according to the Northampton tables, is worth 16*l.* and upwards of 18*s.* &c.

This table is efteemed the beft of any extant, and preferable to any other of a different form. But thofe who fell annuities have generally a table of 2 years more value than the lives in this table, for purchafers who are upwards of twenty years of age.

## *Definitions.*

1. The *probability of life* is the chance that any perfon or perfons have of living to any certain time, and is denoted by a fraction, whofe numerator is the chance of living, and denominator that of living and dying. Thus, fuppofe it were required to find the probability of a perfon of the age of 20 attaining to the age of 37, according to Mr. Simpfon's table. Here it muft be obferved, that of 360 perfons living at the age of 20, only 252 furvive to the age of 37; therefore, 108 perfons have died between the two ages. Thus, 252 is the chance of the faid perfon's living to the age of 37, and 360 the chance of the faid perfon's dying before he attains the age of 37, and the probability of life of that perfon is expreffed by the fraction $\frac{252}{360}$ or $\frac{7}{10}$; therefore, the odds in that perfon's favour, or the chance that he fhall live to that age, is 7 to 3.

2. The *probability of dying* is expreffed by a fraction, which is the difference between the former fraction and unity. Thus, the probability that the aforefaid perfon fhall die before the age of 37 is expreffed by $\frac{108}{360}$ or $\frac{3}{10}$, which

 fhows

ſhows that the chance of that perſon's dying before the ſaid age is as 3 to 7.

3. The *extremity of life* is the period beyond which there is no probability of ſurviving. In the Northampton tables this is 96 years.

4. The *complement of life* is the number of years which any perſon's age wants of the full extremity of life; this in the Northampton table, for a life aged 80, is 17 years.

5. The *expectation of life* is the number of years due to the life of a perſon of a certain age, upon an equality of chance. And it is the number of years purchaſe, which an annuity for life is worth in ready money, without allowing any intereſt. And in ſingle lives it is always equal to the ſum of all the probabilities of ſurviving to the extremity of life.

6. The *number of years purchaſe* of annuities, at any rate of intereſt, is that number which, if multiplied by the annuity, is equal to the preſent value thereof, according to ſuch rate of intereſt; therefore, it is the preſent value of an annuity of 1*l.* according to a given rate of intereſt, as ſeen in Table V.

7. The *reverſion* of a life annuity is where two or more lives are in joint poſſeſſion, and the expectation depends upon the probability of one particular life ſurviving the reſt.

## PROBLEM I.

### *To find the Value of an Annuity for the Life of any Perſon, at a given Rate of Intereſt.*

Rule. Seek the age of the perſon in the firſt column of Table V. and againſt it, under the proper intereſt, is the number of years purchaſe for either the London or Northampton life; or, which is the ſame thing, the preſent value of an annuity of 1*l.* during ſuch life. Multiply this by the annuity, and the product is the anſwer.

Example.

Example. Suppofe the given age be 49, the rate of intereft 3 *per cent.* and the annuity 20*l.* Here again 49, and under 3 *per cent.* ftands 11.6 according to the London bills, which, multiplied by 20*l.* gives 232*l.* for the value of a London life. And in the next column ftands 12.69, the value according to the Northampton bills, which, multiplied by 20*l.* as before, produces 253*l.* 16*s.* for the value of a Northampton life.

## SECT. II.

### OF THE VALUE OF AN ANNUITY DURING JOINT LIVES.

### PROBLEM I.

*To find the Value of an Annuity for the joint Continuance of two Lives; that is, one Life failing, the Annuity to ceafe.*

#### CASE I.

*When both Perfons are of the fame Age.*

Rule. Find the value of any one of the lives from Table V. Multiply this value by the intereft of 1*l.* for a year at the given rate; fubtract the product from 2, divide the aforefaid value by this remainder, and the quotient will be the value of 1*l.* annuity, or the number of years purchafe.

Example. What is the value of 100*l.* annuity, for the joint lives of two perfons, aged 40 years each, according to the London tables, reckoning intereft at 5 *per cent.*? Here, by the table, one life for 40 years is,

| | |
|---|---|
| | 10.3 |
| Multiply by | .05 |
| Subtract this product | .515 |
| From | 2.000 |
| Remains | 1.485 |

And 1.485)10.3(6.9 value of 1*l.* annuity, which multiplied by 100 is 690*l.* the value of the annuity sought.

## CASE II.

*When the two Persons are of different Ages.*

RULE. Find the values of the two lives in Table V. Multiply them one into the other, and call the result the first product; then multiply the said first product by the interest of 1*l.* for a year, at the given rate, calling the result the second product: add the values of the two lives together, and from the sum subtract the second product; divide the first product by the remainder, and the quotient will be the value of 1*l.* annuity, or the number of years purchase.

EXAMPLE. What is the value of 50*l.* annuity, for the joint lives of two persons, whereof one is 20, and the other 30 years of age, according to the Northampton tables, interest at 4 *per cent.*?

| | |
|---|---|
| The value of 20 years is | 16.03 |
| And the value of 30 years is | 14.78 |
| First product | 236.9234 |
| | .04 |
| Second product | 9.476936 |
| Sum of the two lives | 30.810000 |
| Remainder | 21.333064 |

And 21.333064)236.9234(11.1 value of 1*l.* annuity.

| | |
|---|---|
| 50 | |
| 555.0 | Value required. |

PROBLEM

## PROBLEM III.

*To find the Value of an Annuity during the Life of the longeſt Survivor of two Lives; that is, as long as either of the two Parties live.*

Rule. From the ſum of the values of the ſingle lives, ſubtract the value of the joint lives, and the remainder will be the value ſought.

Example. What is the value of an annuity of 1*l.* to continue during the longeſt of two lives: the one perſon being 30, and the other 40 years of age; intereſt at 4 *per cent.* the life of 30 years of age valued according to the London bills, and that of 40 years of age according to the Northampton bills?

| | |
|---|---|
| By the table, the value of 30 years is | 13.1 |
| The value of 40 years is . . | 13.20 |
| | 26.30 |
| The value of their joint lives, by Problem II. Caſe 2, is . . . | 8.9 |
| The value ſought . . . | 17.4 |

If the annuity be any other than 1*l.* multiply the above found value, by the given annuity; and if the two perſons be of equal ages, the value of their joint lives muſt be found by Caſe 1, of Problem II.

## PROBLEM IV.

*To find the Value of an Annuity for the joint Continuance of three Lives; that is, one Life failing, the Annuity to ceaſe.*

Rule. Multiply the value of the three ſingle lives continually into each other, calling the reſult the product of the three lives; multiply that product by the intereſt of 1*l.* and that product again by 2, calling the reſult the double product; then from the ſum of the ſeveral products of the ſaid lives, taken

taken two and two, ſubtract the double product; divide the product of the three lives by the remainder, and the quotient will be the value of the three joint lives.

EXAMPLE. What is the value of an annuity of 1*l.* during the joint lives of three perſons, whereof A is 10 years of age, B 20, and C 30, at 4 *per cent.* according to the London tables?

Here, by the table, the value of A's life is 16.4, that of B's 14.8, and C's 13.1, which three multiplied together is 3179.63; this multiplied by .04, the intereſt of 1*l.* gives 127.18528, which, multiplied again by 2, gives 254.370 for the double product. Then

| | |
|---|---|
| The product of A and B is | 242.72 |
| And the product of A and C is | 214.84 |
| The product of B and C is | 193.88 |
| The ſum of all taken two and two | 651.44 |
| Double product to ſubtract | 254.37 |
| Remainder | 397.07 |

And 397.07)3179.632(8.007 Value ſought.

## PROBLEM V.

### *To find the Value of the Annuity for the longeſt Life of three or more Perſons.*

RULE. Find an age anſwerable to the value of the longeſt life of any two lives, which ſubſtitute in lieu of the two, and then find the value of the longeſt life of that life and the other, and that will be the value of the longeſt life among three lives. If there are four or more lives, ſubſtitute the age correſponding to this value in lieu of the three lives, and find the value of the longeſt life of this age, and the other remaining age, and it will be the value of the longeſt of four lives: proceed in the ſame manner for five or more lives, having regard to the rate of intereſt.

The examples in the foregoing Problems will be found ſufficient to inſtruct the learner how to perform all the following

following Problems. I ſhall, therefore, give a few Problems with their rules, leaving their operation for the exerciſe of the learner.

---

## SECT. III.

### THE VALUE OF CONTINGENT REMAINDERS AND REVERSIONS.

### PROBLEM I.

*To find the Value of the Reverſion of an aſſigned Life after a given Term.*

Rule. Subtract the value of the annuity for the given term of years, from the value of the propoſed life, on the contingency of its ceaſing, upon the extinction of the aforeſaid life, and the remainder will be the anſwer.

### PROBLEM II.

*To find the Value of the Reverſion of an Annuity for the Remainder of a given Term of Years after an aſſigned Life.*

Rule. From the value of an annuity, certain for the given term of years, ſubtract the value of the annuity for the ſaid term, on the contingency of its ceaſing, upon the failing of the propoſed life; and the remainder will be the value of the reverſion.

PROBLEM

## PROBLEM III.

*To find the Value of the Reversion of one Life after another.*

RULE. Subtract the value of the two joint lives from the value of the life in expectation, and the remainder will be the value of the reversion.

## PROBLEM IV.

*To find the Value of the Reversion of two Lives after one.*

RULE. Subtract the value of the life in possession from the value of the longest of the three lives, and the remainder will be the value of the reversion.

## PROBLEM V.

*To find the Value of the Reversion of one Life after two Lives.*

RULE. If the two lives are joint lives, subtract the value of the three joint lives from the value of the life in expectation, and the remainder will be the answer. But if the reversion takes place after the extinction of either life, subtract the value of the two lives in possession from the value of the three lives, and the remainder will be the value of the reversion.

## PROBLEM VI.

*To find the present Value of any Number of Lives in Succession.*

RULE. Multiply the value of each life by the interest of 1*l.* for one year, and subtract each product from unity or 1; multiply all the remainders continually together, and subtract this product from unity; then the remainder multiplied

plied by the perpetuity *, will be the value of all the successive lives.

## PROBLEM VII.

A given sum of money is to be received as a legacy on the decease of D, who is at a given age; what is the value thereof in present money?

RULE. Subtract the value of the life of D from the perpetuity; then say, as the perpetuity is to the remainder, so is the proposed sum to the present value.

## PROBLEM VIII.

*To find the Value of a Sum of Money to be received at the Decease of B, in case A is then deceased also.*

RULE. Subtract the value of the oldest life from the value of an annuity for as many years as are expressed by the complement of B's age; then say, as the complement of the younger life is to the remainder, so is the proposed sum to its present value.

# ANNUITIES UPON TONTINES.

## PROBLEM IX.

*To find the Value of an Annuity for either Person of two, who have a joint Annuity, which at the Decease of either one, is to become the sole Property of the Survivor.*

* The perpetuity, or value of an annuity to continue for ever, is found by dividing 100*l.* by the rate of interest per cent. or by dividing 1*l.* by the interest of 1*l.* for a year; and the quotient is the perpetuity, or value of an annuity of 1*l.* to continue for ever. The perpetuity is also equal to the number of years purchase, which a perpetual annuity is worth, without allowing any interest.

Rule. From the value of the life of either of the two perſons, ſubtract half the value of the two joint lives, and the remainder will be the value of the other perſon's life.

## PROBLEM X.

What is the preſent value of an annuity to be poſſeſſed by D and his heirs, as ſoon as any two of the three lives, A, B, and C, become extinct; D and his heirs holding the ſame during the life of the ſurvivor of the three lives, A, B, C?

Rule. Add thrice the value of the three joint lives, A, B, and C, to the ſum of the value of the three ſingle lives, deducting therefrom twice the ſum of the value of each two joint lives: viz.—of A and B, A and C, and B, C; and the remainder will be the anſwer.

## PROBLEM XI.

What is the value of the right of any one of the three following perſons, viz.—A, B, and C, who enjoy an annuity equally among them, which, upon the deceaſe of any one, is to become the property of the two ſurvivors, during their joint lives, and on the deceaſe of the next perſon to become the property of the laſt ſurvivor during his life?

Rule. Subtract half the ſum of the values of the joint lives, A and B, and the joint lives of A and C, from the value of the life of A; then to the remainder add one third of the value of the three joint lives, and the ſum will be the value of the right of A. The value of the right of either of the other parties may be found by a ſimilar method.

## PROBLEM XII.

What is the value of the two ſucceſſive lives, A and B, A having an annuity for life, and to have the nomination of a ſucceſſor, who is to hold the annuity for his own life, after the deceaſe of A?

Rule.

RULE. Multiply the value of the life of A, by the value of the life put in at his deceafe; divide the product by the perpetuity, and fubtract the quotient from the fum of the faid values, and the remainder will be the anfwer.

## SECT. IV.

## OF ASSURING LIVES.

BY affuring a life, is meant, obtaining fecurity for receiving a certain fum of money, fhould the affured life fail in a certain given time: in confideration of which, a premium is given to the affurer, which is a fufficient compenfation for the lofs he is likely to fuftain, in cafe the life fhould drop. This compenfation, called the *premium*, is varied according to the two following caufes:—Firft, The rate of intereft at which the money is fuppofed to be improved; and, fecondly, the probability of the duration of the life to be affured. If the intereft be high, and the probability of life high alfo, the value of the affurance will be low in proportion: on the contrary, if the intereft be low, and the probability of life alfo low, the value of the affurance will be proportionably high. For example:—Let 100*l.* be fuppofed to be affured on a life, for 1 year; that is, let 100*l.* be payable a year hence, provided a perfon of a given age dies in that time.

 Now,

Now, if the intereſt of the money be 5 *per cent.* and the life *ſure* * of failing, the value of the aſſurance would be the ſame as the preſent value of 100*l.* payable at one year's end, reckoning intereſt at 5 *per cent.* and would be that ſum, which being put out to intereſt now, at 5 *per cent.* would produce the 100*l.* at the end of the year, which ſum is 95*l.* 4*s.* 9*d.*

If it be an even chance, or the odds are equal, whether the life does or does not fail in the year, which is the caſe when one half of a given number of lives fail in a given time, the value of the aſſurance will be half as much as the former value, or 47*l.* 12*s.* 4½*d.*

If the odds againſt the perſon's life failing are two to one, which is the caſe when one third of a given number of lives fail in the time, the value of the aſſurance will alſo be one third of the firſt value (if the intereſt be the ſame), or 31*l.* 14*s.* 11*d.*

If the odds are nineteen to one againſt the life failing, which is the caſe when the twentieth part of the lives fail in the given time, the value of the aſſurance will be a twentieth part of the firſt value, or 4*l.* 15*s.* 2¾*d.*

If the odds are forty-nine to one againſt the life failing, or when only one out of fifty of ſuch lives fails in the given time, the aſſurance will be only a fiftieth part of the firſt value, or 1*l.* 18*s.* 1*d.*

Now the odds of two to one, according to Dr. Halley's table, are, that a life aged 85 years will not drop in a year. The odds of nineteen to one are, that a life aged 64 will not drop in a year. And the odds of forty-nine to one are, that a life aged 39 will not drop in a year. Therefore, the value of the aſſurance of 100*l.* for one year, on a life aged 85, is

* By this word is to be underſtood a certainty, according to the calculations from the tables.

31*l.* 14*s.* 11*d.*;—on a life aged 64, 4*l.* 15*s.* $2\frac{3}{4}$*d.*;—on a life aged 39, 1*l.* 18*s.* 1*d.* at 5 *per cent.* intereſt. But if intereſt be reckoned at 3 *per cent.* theſe three values will be 32*l.* 7*s.*—4*l.* 17*s.*—1*l.* 18*s.* 10*d.* reſpectively.

This calculation ſuppoſes the value of the aſſurance to be paid in one ſingle preſent payment. But the value may be paid in *annual payments*, and be continued till the failure of the life, ſhould that happen within the given term; or, if not, till the determination of the time.

The value of an aſſurance upon a life cannot be diſcovered by any one ignorant of the method of calculating the value of life annuities, delivered in the former part of this chapter. But thoſe who underſtand what has been delivered, may form any calculations upon this ſubject, from the following examples:

Example. What money in hand, and alſo in annual payments during life, ought a perſon of a given age to pay for a given ſum of money, payable at his death to his heir?

Rule. Subtract the value of the given life at the rate *per cent.* as given in Table V. from the perpetuity: multiply the remainder by the product of the ſum to be aſſured, and the rate of intereſt for a year; divide this laſt product by 100*l.* increaſed by its intereſt for a year, and the quotient will be the anſwer, or the money which ought to be given in a ſingle preſent payment: and this payment divided by the value of the life, will quote the ſum that ought to be paid in annual payments during the whole continuance of life.

## CASE I.

### *Where the Premium is to be paid immediately, in a ſingle Payment.*

Question 1. What premium ſhould be given to ſecure 100*l.* at the deceaſe of a perſon aged 45 years, intereſt 3 *per cent.*?

*Operation.*

*Operation.*

| | |
|---|---|
| The perpetuity | 33.333 |
| The value of the life by Table V. at 3 *per cent.* | 12.3 |
| Remainder | 21.033 |
| The given ſum multiplied by its intereſt | 300 |
| | 103)6309.9(61.26116 Anſwer. |

Thus it appears that 61.26116, or 61*l.* 5*s.* $2\frac{1}{2}$*d.* is the premium which ought to be immediately paid to ſecure 100*l.* on the deceaſe of a perſon aged 45 years, at 3 *per cent. per annum*, according to the probability of life for London.

## CASE II.

### *When the Premium is to be paid in fixed annual Payments, during the whole Continuance of Life.*

QUESTION 2. What money ſhould be given in equal annual payments, during the life of a perſon aged 45 years, to ſecure 100*l.* on the deceaſe of the ſaid perſon; intereſt at 3 *per cent. per annum?*

In this caſe, the value of the aſſurance in one preſent payment is to be found as in the foregoing caſe, which value divided by the value of the life, quotes the ſum to be paid annually during the life of the perſon:—Thus, 61.26116 divided by 12.3, quotes 4.98, or 4*l.* 19*s.* 7*d.* which is the ſum to be paid annually during life, in order to ſecure the ſum of 100*l.* at the extinction of the ſaid life.

If

If the foregoing queſtions be repeated, reckoning the intereſt at $3\frac{1}{2}$ *per cent. per annum*, the premium will be leſs, viz. in one preſent payment at $3\frac{1}{2}$ *per cent.* it will be 57*l.* 11*s.* and the annual payments 4*l.* 19*s.* $7\frac{1}{2}$*d.*

Thus it appears upon what very eaſy terms a large ſum of money might be ſecured at the deceaſe of any perſon, if the premium be paid by annual payments. Hence the great advantage of inſtitutions for the aſſurance of lives, provided they be properly conducted, and managed by perſons ſufficiently ſkilled in numbers to avoid errors in making their calculations, which are moſt detrimental to ſocieties of this nature, and from which there are hardly any of theſe inſtitutions exempt.

Aſſurances of this nature might be extended conſiderably more than they are at preſent; and rendered not only ſubſervient to the parochial poor, but alſo of infinite advantage to the nation at large, particularly to the revenue in a financial reſpect, were the ſubject to meet the approbation of the legiſlature; and perhaps more pecuniary aſſiſtance might be derived from eſtabliſhments of this nature, under proper modifications, than from any other mode of funding, and creating permanent debts, as I ſhall prove in another treatiſe.

When an eſtate, or a perpetual annuity, is to be aſſured for the duration of another life, after the failure of the aſſured life, inſtead of aſſuring a groſs ſum, the value of a ſingle payment will be the value of the life ſubtracted from the perpetuity, and the remainder multiplied by the annuity, or by the rent of the eſtate. And the value in annual payments to begin immediately, will be the ſingle payment divided by the value of the life, increaſed by unity. Therefore, an aſſurance of an eſtate or annuity, after any given life or lives, is worth as much more than the aſſurance of a correſponding ſum, as 100*l.* increaſed by its intereſt for a year, is greater

greater than 100*l.* Thus the preſent values, in ſingle and annual payments, of the aſſurance of an eſtate of 5*l. per annum* for ever, and of 100*l.* in money, are to one another as 105*l.* is to 100*l.* The reaſon of the difference is, that the algebraical calculations, by which theſe values are determined, ſuppoſe the groſs ſum and the firſt yearly payment of the annuity are to be received at the ſame time, after the expiration of the life or lives.

The examples here given will be found ſufficient to inſtruct any perſon in the method of finding the value of annuities, in all caſes of reverſions; as alſo in the principles of aſſurances upon lives.

CHAP.

# CHAP. XI.

# OF LOGARITHMS.

## SECT. I.

### OF THE ORIGIN AND NATURE OF LOGARITHMS.

LOGARITHMS are certain artificial numbers, which are the ratios of other natural numbers; and are the indices of the ratio of numbers to one another; or, a ſeries of numbers in an arithmetical proportion, anſwering to as many others in a geometrical proportion, and in ſuch a manner, that o in the arithmeticals is the index of 1 in the geometricals. Logarithms were invented for the eaſe of arithmetical calculations, where the numbers, or operations, are large.

The nature of logarithms depends upon theſe axioms: if a ſeries of quantities increaſe, or decreaſe, according to the ſame *ratio*, it is called a *geometrical progreſſion*, as the numbers 1, 2, 4, 8, 16, 32, which are multiplied by 2: if the ſeries or quantities increaſe, or decreaſe, according to the ſame *difference*, it is called an *arithmetical progreſſion*, as the numbers 3, 6, 9, 12, 15, 18, &c. which increaſe by 3, which is therefore called their common difference. Now, if underneath the numbers proceeding in a geometrical progreſſion, be placed as many other numbers, proceeding in an arithmetical progreſſion, theſe laſt are called the logarithms of the firſt; as in the following:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Terms - | 1. | 2. | 4. | 8. | 16. | 32. | 64. | 128. | 256. | 512. |
| Logarithms | 0. | 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. |

In this progreſſion, 0 is the logarithm of 1, the firſt term: 1 the logarithm of the ſecond, which is 2; and 2 the logarithm of the third term, 4, &c.

Theſe indices, or logarithms, may be adapted to any ſeries in a geometrical progreſſion; and, therefore, there may be as many different kinds of indices, or logarithms, as there can be different kinds of geometrical progreſſions; as may be ſeen in the following ſeries:—

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Log. - | 0. | 1. | 2. | 3. | 4. | 5. | &c. |
| | 1. | 2. | 4. | 8. | 16. | 32. | &c. |
| or | $2^0$. | $2^1$. | $2^2$. | $2^3$. | $2^4$. | $2^5$. | &c. |
| | 1. | 3. | 9. | 27. | 81. | 243. | &c. |
| or | $3^0$. | $3^1$. | $3^2$. | $3^3$. | $3^4$. | $3^5$. | &c. |
| | 1. | 10. | 100. | 1000. | 10000. | 100000. | &c. |
| or | $10^0$. | $10^1$. | $10^2$. | $10^3$. | $10^4$. | $10^5$. | &c. |

The Geometrical Progreſſion.

Here the ſame indices, or logarithms, ſerve for any of the ſix under-written geometrical ſeries, from which it appears, that there may be an endleſs variety of ſets of logarithms, adapted to the ſame common numbers, by varying the ſecond term of the geometrical ſeries, as this will change the original ſeries of terms, whoſe indices are the numbers 1. 2. 3. &c. And by interpolation the whole ſyſtem of numbers may be made to enter the geometric ſeries, and receive their proportionable logarithms, whether they be integers or decimals.

The logarithm of any number is the index of ſuch a power of ſome other number, as is equal to the given one involved to the power denoted by the index of the other number. Thus, if N be equal to $r^n$, then the logarithm of N is n, which may be either poſitive or negative, and r any number whatever, according to the different ſyſtems of logarithms. When N is one, then n is 0, whatever the value of r may be; and, conſequently, the logarithm of 1 is always 0 in every ſyſtem of logarithms. But in the common logarithms, r is equal to 10; ſo that the common logarithm of any number is the index of that power of 10, which is equal to the ſaid number: thus the common logarithm of $N=10^n$, is $n$ the index

index of the power of 10.—For example:—1000 being the third power of 10, has 3 for its logarithm: and if 50 be = $10^{1.69897}$, then 1.69897 is the common logarithm of 50; from which it will follow, that the following decimal ſeries of terms will have the following logarithms reſpectively:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| The Geometric Series. | | 1000, | 100, | 10, | 1, | .1, | .01, | .001 |
| | or | $10^3$, | $10^2$, | $10^1$, | $10^0$, | $10^{-1}$ | $10^{-2}$ | $10^{-3}$ |
| Logarithms | | 3, | 2, | 1, | 0, | —1, | —2, | —3. |

The logarithm of a number, which is contained between any two terms of the firſt ſeries, is included between the two correſponding terms of the latter ſeries; therefore, that logarithm will have the ſame index, whether poſitive or negative, as the ſmaller of theſe two terms, together with a decimal fraction, which will always be poſitive. Thus, the number 50 falling between the numbers 10 and 100, its logarithm will fall between 1 and 2, being equal to 1.69897 nearly; and the number .05 falling between .1 and .01, its logarithm will fall between —1 and —2, and is equal to —2 +.69897, the index of the leſs term, together with the decimal .69897. The index is ſometimes called the characteriſtic of the logarithms, and is always an integer, either poſitive or negative, or elſe 0, and ſhows what place is occupied by the firſt ſignificant figure of the given number, either above or below the place of units, being in the former caſe poſitive, and in the latter negative.

When the characteriſtic of a logarithm is negative, the ſign — is commonly ſet over it to diſtinguiſh it from the decimal part, which being the logarithm found in the tables, is always poſitive; thus, the logarithm of .05 or —2 +.69897 is written thus, $\bar{2}.69897$. But when it is required to reduce the whole expreſſion to a negative form, it is done by making the characteriſtic leſs by 1, and taking the arithmetical complement of the decimal; that is, beginning at the left hand, ſubtract each figure from 0, except the laſt ſignificant figure, which is ſubtracted from 10; then will the remainder form a

 logarithm,

logarithm, wholly negative: thus, the aforementioned logarithm $\overline{2}.69897$, or $-2+69897$ is expreſſed by $-1.30103$, which is all negative. Sometimes it is convenient to expreſs the logarithm as poſitive, which is done by joining to the tabular decimal, the complement of the index to 10: thus, the above logarithm is expreſſed by 8.69897, which is only increaſing the indices in the ſcale by 10.

From the foregoing definitions of logarithms, conſidered either as the indices of a geometric ſeries, or as the indices of the powers of the ſame root, it appears, that numbers may be multiplied together by the addition of their logarithms: and they may be divided by the ſubtraction of their logarithms. Alſo a number may be raiſed to any power by multiplying the logarithm of the root by the index of the power: and the extraction of roots may be performed by dividing the logarithm of the given number by the index of the root required to be extracted.

Logarithms conſidered in their theory, are of very ancient origin, and were known to moſt of the ancients; but the celebrated John Napier, Baron of Merchiſton, in Scotland, was the firſt who applied the uſe of them to Trigonometry; but the form of their conſtruction was not made known till the opinion of the mathematicians was had concerning them. His ſon, Robert Napier, in the year 1619, publiſhed a new edition of his father's work, with the conſtruction of logarithms. And in the ſame year Mr. John Speldell publiſhed an Improvement of Napier's Logarithms.

Other Tables were ſoon after publiſhed by John Kepler, and ſome others; all which tables were of that kind, called Hyperbolical; becauſe the numbers expreſſed are as between the aſymptote and curve of the hyperbola.

Henry Briggs, Profeſſor of Geometry in Greſham College, ſoon after publiſhed the logarithms of the firſt one thouſand numbers on a new ſcale: viz.—In which the logarithm of the ratio of 10 to 1 is 1; whereas the logarithms of the ſame ratio

ratio in Napier's ſyſtem is 2.302 58, &c. And in 1624 he alſo publiſhed his *Arithmetica Logarithmica*, containing the logarithms of 30,000 natural numbers, to 14 places of figures, beſides the index, which form was recommended to him by Napier, and which is the form in preſent uſe. In 1633, he publiſhed, to 14 places of figures, his *Trigonometria Britannica*, which contained the natural and logarithmic ſigns, tangents, ſecants, &c.

From the time of Robert Napier, to the preſent period, ſeveral mathematicians publiſhed logarithmic tables, with various improvements; the principal of which were Gunter, Wingate, Henrion, Miller, and Norwood, Cavalerius, Vlacq, and Rowe, Frobenius, Newton, Caramuel, Sherwin, Gardiner, and Dodſon.

In addition to what has been ſaid, it may be obſerved, that the indices or characteriſtic of logarithms correſpond to the denominative part of the natural number, as the other member of the logarithm does the numerative part of the number; that is, the index ſhows the denomination of, or place of the left-hand figure of the number, and conſequently of all the reſt. Thus, o affixed to a logarithm, denotes the firſt figure of the number to which the logarithm anſwers to be nothing diſtant from the place of units. The index 1 ſhows the firſt figure of the number to be diſtant 1 place from the place of units; that is, in the place of tens, and conſequently the number itſelf to be either 10, or ſome number between 10 and 100; and the ſame may be obſerved of all the other indices. Therefore, all numbers that have the ſame denominative, but not the ſame numerative parts, as all numbers from 1 to 10, from 10 to 100, &c. will have logarithms whoſe indices are the ſame, but the other members different. And again, all numbers, which have the ſame numerative, but not the ſame denominative parts, will have different indices; but the reſt of the logarithms will be the ſame. If a number be purely decimal, to its logarithm is affixed a negative index, ſhowing the diſtance of its firſt ſignificative

ficative figure from the place of units. Thus the logarithm of the decimal .256 is $\overline{1}.408240$; that of the decimal .0256 is $\overline{2}.408240$. Instead of these negative indices, some use their complements to 10, set down with a point on each side, thus, .9. and .8. that is, such a figure is made the index, as, when subtracted from 9, leaves a remainder expressing the number of cyphers prefixed to the decimal, as before observed.

## SECT. II.

### OF THE ARITHMETIC OF LOGARITHMS.

FROM what has been said, the learner may have acquired a tolerable idea of the nature of logarithms. The construction of the logarithmic canon requires more room than is consistent with the present treatise. The table of logarithms is generally printed in a table consisting of nine columns, besides the column of natural numbers and that of the differences, except that part of the table which contains the logarithms of numbers from 1 to 100.

### *To find the Logarithm of a Number, or the Number corresponding to a given Logarithm.*

If the given number be under 100, the logarithm is found at once in the table, standing opposite the number: if the given number be above 100, and under 1000, its logarithm will be found opposite to it in the column under 0: if the number be between 1000 and 10000, the first three figures of the number are to be found in the column marked No, and the fourth figure at the top of one of the other columns, and in the column under the said fourth figure, and even with the first three figures, will be found the logarithm required, changing the index 2 into 3.

If

If the given number be greater than any in the common canon, but lefs than 10000000, cut off four figures on the left of the given number, and find the logarithms thereof as before directed; then multiply the common difference which ftands againft the logarithm in the column marked *Difference*, by the remaining figure or figures of the given number, and from the product cut off as many figures from the right-hand fide as you multiplied by, and add the remainder to the logarithm of the firft four figures of the given number, fitting it with a proper index. Thus, if it were required to find the logarithm of the number 92375: the logarithm of the firft four figures 9237 is 3.965531, then 47 multiplied by 5=235, therefore, cut off 5 and add 23, and the required logarithm (proportioning its index) is 4.965554.

*To find the Number correfponding to any Logarithm.*

Seek the logarithm in the table, beginning in the column under 0 (neglecting the index); and if the logarithm, or the two or three firft figures thereof, cannot be found in this column, feek them in the other columns; and having found it on the next lefs logarithm thereto, lineally againft it in the column of numbers are the three firft figures of the number fought, to which joining the figure over the column of the logarithm, you have the required number; when the number confifts of four places of figures, i. e. when the index of the logarithm is 3; but when the index is 1, the two laft figures of the number are decimal, and only the two firft numbers integral: when the index is 0, the firft number only is an integral, &c.

Thus, to find the number correfponding to a logarithm is the converfe of finding the logarithm of a number.

*To multiply or divide Numbers by Logarithms.*

Rule. To multiply two numbers together, add their two logarithms together, and their fum will be the logarithm of the product, adding to the affirmative indices what is carried from the decimal parts; and if the indices be one affirmative,

and

and the other negative, the difference is to be taken for the index to the logarithm of the product.

The reason of this rule is evident, for as unity is to one of the factors, so is the other factor to the product. Thus, the logarithm of the product is a fourth equi-different term to the logarithm of unity and those of the factors; but the logarithm of unity being 0, the sum of the logarithms of the factors must be the logarithm of the factum, or product.

Therefore, as the factors of a square are equal to each other, that is, a square is the factum or product of its root multiplied into itself; the logarithm of a square will be double the logarithm of the root.

Therefore, unity is to the exponent of the power, as the logarithm of a root to the logarithm of the power.

From hence it appears that the logarithm of the cube is triple the logarithm of the root; the logarithm of the biquadrate, quadruple that of the root; the logarithm of the fifth power quintuple that of the root; that of the sixth power sextuple, &c.

To perform division by logarithms, subtract the logarithm of the divisor from the logarithm of the dividend, and the remainder will be the logarithm of the quotient, first changing the sign of the logarithm of the index of the divisor, and if they be of different signs, the difference is to be taken, but if of the same kind, their sum.

For as the divisor is to the dividend, so is unity to the quotient; therefore the logarithm of the quotient is a fourth equi-different number to the logarithms of the divisor, the dividend, and the logarithm of unity. The logarithm of unity, therefore, being 0, the difference of the logarithm of the divisor, and that of the dividend, is the logarithm of the quotient. For the difference between 7 and 5, or the remainder, is 2, which is the logarithm of the quotient 4, which quotient is obtained by dividing 128 by 32. Also, subtracting 3 from 8, the remainder 5 is the logarithm of the quotient 32, which is obtained by dividing 256 by 8.

*Examples*

## *Examples in Multiplication.*

EXAMPLE 1.

| | *Num.* | *Log.* |
|---|---|---|
| Multiply | 68 | 1.832509 |
| By | 12 | 1.079181 |
| Products | 816 | 2.911690 |

EXAMPLE 2.

| | *Num.* | *Log.* |
|---|---|---|
| Multiply | 9 | 0.954242 |
| By | 9 | 0.954242 |
| | 81 | 1.908484 |

EXAMPLE 3. Multiply 3.902, and 597.16, and .0314728, continually together.

EXAMPLE 4. And alſo multiply 35.86, and 2.1046, and 0.8372, and 0.0294, continually together.

| *Num.* | *Log.* |
|---|---|
| 3.902 | 0.5912873 |
| 597.16 | 2.7760907 |
| .0314728 | $\bar{2}$.4979353 |
| 73.33533 | 1.8653133 |

| *Num.* | *Log.* |
|---|---|
| 35.86 | 0.5546103 |
| 2.1046 | 0.3231696 |
| 0.8372 | $\bar{1}$.9228292 |
| 0.0294 | $\bar{2}$.4683473 |
| .1857618 | $\bar{1}$.2689564 |

In the laſt example the 2 carried from the decimals cancels the $\bar{2}$, and there remains $\bar{1}$ to be ſet down; and in the third example, the $\bar{2}$ cancels the 2, and the 1 carried from the decimals is ſet down.

## *Examples in Diviſion.*

EXAMPLE 1.

| | *Num.* | *Log.* |
|---|---|---|
| Div. | 24163— | 4.3831509 |
| By | 4567— | 3.6596310 |
| Quo. | 5.290782— | 0.7235199 |

EXAMPLE 2.

| | *Num.* | *Log.* |
|---|---|---|
| Div. | 37.149— | 1.5699471 |
| By | 523.76— | 2.7191323 |
| Quo. | .07092752— | $\bar{2}$.8508148 |

In the laſt example, the 1 carried from the decimals is added to the 2, which, with its ſign, changed, becomes $\bar{3}$; conſequently the remainder, or index of the quotient, is $\bar{2}$.

| Example 3. | *Num.* | *Log.* | Example 4. | *Num.* | *Log.* |
|---|---|---|---|---|---|
| Div. | .06314— | $\bar{2}.8003046$ | Div. | .7438— | $\bar{1}.8714561$ |
| By | .007241— | $\bar{3}.8597985$ | By | 12.9476— | $1.1121893$ |
| Quo. | 8.719792— | $0.9405061$ | Quo. | 05744694— | $\bar{2}.7592669$ |

In the third example, there is 1 carried from the decimals and added to the $\bar{3}$, which makes it $\bar{2}$; and which taken from the other $\bar{2}$ leaves 0. In the fourth example, the 1 is taken from the $\bar{1}$, which leaves a remainder $\bar{2}$.

### *To find the Logarithm of a Fraction.*

Rule. Subtract the logarithm of the numerator from that of the denominator, and to the remainder prefix the negative ſign. Thus, ſuppoſe it was required to find the logarithm of the fraction $\frac{3}{7}$.

$$
\begin{array}{ll}
0.845098 & \text{Logarithm of } 7. \\
0.477121 & \text{Logarithm of } 3. \\
\hline
-0.367977 = & \text{Logarithm of } \tfrac{3}{7}.
\end{array}
$$

The reaſon of this rule is evident: for a fraction being the quotient of the numerator divided by the denominator, its logarithm muſt be the difference of the logarithms of theſe two: ſo that the numerator being ſubtracted from the denominator, the difference becomes negative. And the logarithms of proper fractions muſt always be negative, if the logarithm of unity be 0; becauſe a proper fraction is leſs than an unit.

Or the logarithm of the denominator, though greater than that of the numerator, may be ſubtracted from that of the numerator,

numerator, regard being had to the fign of the index, which alone in this cafe is negative. Thus,

$$\begin{array}{ll} 0.477121 & \text{Logarithm of } 3. \\ 0.845098 & \text{Logarithm of } 7. \\ \hline \bar{1}.632023 = & \text{Logarithm of } \tfrac{3}{7}. \end{array}$$

This produces the fame effect in any operation as the logarithm before found, viz.—0.367977, this being to be fubtracted, and the other added.

Or the fraction may be reduced to a decimal, and its logarithm found; which logarithm differs from that of a whole number only in the index, which is to be negative. For an improper fraction, fubtract the logarithm of the denominator from that of the numerator, and the remainder is the logarithm of the fraction, as in the fraction $\frac{9}{5}$.

$$\begin{array}{ll} 0.9542425 & \text{Logarithm of } 9. \\ 0.6989700 & \text{Logarithm of } 5. \\ \hline 0.2552725 = & \text{Logarithm of } \tfrac{9}{5}. \end{array}$$

In the fame manner the logarithm of any mixed number may be found, by reducing the mixed number into an improper fraction.

Or laftly, an improper fraction may be reduced to a mixed number, and its logarithm muft be found as if it were a whole number; and its index taken according to the integral part.

In addition, fubtraction, &c. of logarithms, with negative and affirmative indices, the fame rules are to be obferved as thofe given in algebra, for like and unlike figns.

In addition of logarithms of this nature, all the figures, except the index, are reckoned pofitive; and, therefore, the figure to be carried to the index from the other part of the logarithms, takes away fo much from the negative index. Thus, if 1.863326 be added to $\bar{3}.698972$, the fum is $\bar{1}.562298$.

 And

And in ſubtraction, if either one or both of the logarithms have negative indices, you muſt change the ſign of the index of the ſubtrahend, after you have carried to it what may ariſe from the decimal part, and then add the indices together; thus, if 1.863326 be ſubtracted from $\bar{1}.562298$, the remainder will be $\bar{3}.698972$. In multiplication, what is carried from the product of the other parts of the logarithms muſt be ſubtracted from the product of the indices; thus, if $\bar{2}.477121$ be multiplied by 5, the product will be $\bar{8}.385605$. In diviſion, if the diviſor will exactly meaſure the index, proceed as in common arithmetic. Thus, $\bar{4}.924782$ divided by 2, quotes $\bar{2}.462391$; but if the diviſor will not exactly meaſure the index, add units to the index till you can exactly divide it, and carry theſe units to the next firſt number. Thus, if $\bar{8}.385605$ be divided by 5, it quotes $\bar{2}.477121$.

### *Involution and Evolution.*

Involution is performed by multiplying the logarithm of the number given by the propoſed index of the power; and if the index of the logarithm be negative, the product will be negative; but what is carried from the decimal part of the logarithm will be affirmative; therefore the difference of this carried number and the product of the index will be the index of the product.

EXAMPLE 1. To find the cube or third power of 3.07146.
EXAMPLE 2. To find the fourth power of .09163.

| | *Num.* | *Log.* | | *Num.* | *Log.* |
|---|---|---|---|---|---|
| Root | 3.07146— | 0.4873449 | Root | .09163— | $\bar{2}.9620377$ |
| | | 3 | | | 4 |
| Pow. | 28.97575— | 1.4620347 | Pow. | .0000704938— | $\bar{5}.8481508$ |

Evolution is performed by dividing the logarithm of the power or given number by its power, and the quotient will be

be the logarithm of the root; and when the index of the logarithm is negative, and the divifor is not exactly contained in it, without a remainder, increafe the index of the logarithm by fuch a number as will make it exactly divifible; and carry the units borrowed, as fo many tens, to the decimals; then divide the refults by the index of the root.

Example 1. To find the cube root of 12345.

Example 2. To find the cube root of .00048.

| | *Num.* | *Log.* | | *Num.* | *Log.* |
|---|---|---|---|---|---|
| Pow. | 12345— | 3)4.0914911 | Pow. | .00048— | 3)$\bar{4}$.6812412 |
| Root | 32.11162— | 1.3638304 | Root | .07829735— | $\bar{2}$.8937471 |

In the laft example, 2 is to be added to the $\bar{4}$, to make it become $\bar{6}$, in which the divifor 3 is contained $\bar{2}$ times; and the 2 borrowed is carried to the other figure 681, &c.; and the logarithm of the root is $\bar{2}$.8937471.

## SECT. III.

### OF THE RULE OF PROPORTION BY LOGARITHMS.

*To perform the Rule of Proportion by Logarithms.*

Rule. Add the logarithm of the fecond and third terms together, and from the fum fubtract the logarithm of the firft; and the remainder is the logarithm of the fourth number.

Example. Find a fourth proportional to the numbers 4, 68, and 3.

1.832509

| | |
|---|---|
| 1.832509 | Logarithm of 68. |
| 0.477121 | Logarithm of 3. |
| 2.309630 | Sum. |
| 0.602060 | Logarithm of 4. |
| 1.707570 | Logarithm of 51, the anſwer. |

This rule is founded on the ſame reaſon as the rule of proportion in common arithmetic; for adding the logarithms of the ſecond and third numbers together, and ſubtracting the logarithm of the firſt from the ſum, is the ſame thing as multiplying the ſecond and third numbers together, and dividing the product by the firſt.

Or the operation may be performed by the following rule, viz. Againſt the firſt term write the arithmetical complement of its logarithm, and againſt the ſecond and third terms write the logarithms themſelves; and the ſum of theſe three logarithms, abating 10 in the index, will be the logarithm of the fourth term; thus, in reſolving the aforeſaid queſtion, the operation will ſtand thus:

| | |
|---|---|
| 9.397940 | Arithmetical complement of log. of 4. |
| 1.832509 | Logarithm of 68. |
| 0.477121 | Logarithm of 3. |
| 1.707570 | Logarithm of 51. Anſwer. |

The reſolution of problems of this nature is of eminent ſervice in trigonometry, as will be ſeen hereafter.

CHAP.

# CHAP. XII.

# OF TRIGONOMETRY.

### *Definitions.*

1. TRIGONOMETRY is the art of finding all the fides and angles of a triangle, from having any three of thefe, one of which, at leaft, muft be a fide. Or, to find the ratio of the fides, when the angles are given; and converfely the ratio of the angles, when the fides are given. And it is founded on the mutual proportion which fubfifts between the fides and angles of triangles, which proportions are known by finding the relations between the radius of the circle and certain lines drawn in and about the circle, called *chords*, *fines*, *tangents*, *fecants*, &c.

For this purpofe, the circumference of a circle is divided into 360 parts, called degrees; and every degree fubdivided into 60 other parts, called minutes, and every minute into 60 feconds, and every fecond into 60 thirds, &c.; and any angle is faid to confift of fo many degrees, minutes, and feconds, as are contained in the arc that meafures the angle, or that is intercepted between the legs or fides of the angle; the point of the faid angle being at the centre of the circle.

2. The

2. The *complement* of an arc is the difference between the arc and a quadrant.

3. The *fupplement* of an arc is the difference between the arc and a femicircle.

4. The *right fine* of an arc, commonly called the *fine*, is a perpendicular falling from one end of the arc, to the radius, drawn through the other end of the fame arc, as D E (fig. 1, plate 9) is the fine of the arc D B, and it is always equal to half the fubtenfe of double the arc. Thus, D E is equal to half of D O, which is the fubtenfe or chord of the arc D O; and the arc D O is double the arc D B. Hence the fign of an arc of 30 degrees is equal to one half the radius, becaufe the chord of 60 degrees is equal to the radius.

5. The *fine complement* of an arc is that part of the radius intercepted between the centre and the right fine, as C E, and is alfo the fine of the complement of the arc to a quadrant; for C E is equal to F D, which is the fine of the arc D H.

6. The *cofine* of an arc is the fame as the fine complement.

7. The *verfed fine* is that part of the radius intercepted between the right fine and the circumference of the circle, as E B.

8. The *tangent* of an arc is a perpendicular drawn from the extremity of the radius to the fecant, as B G, which is the tangent of the arc D B.

9. The *fecant* of an arc is a line drawn from the centre of the circle, through one end of the arc, till it meets the tangent; as C G.

10. The *cofecant* and *cotangent* of an arc is the fecant and tangent of that arc, which is the complement of the former arc. And the chord of an arc, and the chord of its complement to a circle, is the fame; fo likewife the fine, tangent, and fecant of an arc are the fame as the fine, tangent, and fecant of its fupplement or complement to a femicircle. Thus, the fine E D, the tangent B G, and fecant C G, is the

the ſine, tangent, and ſecant of the arc, D A, which is the ſupplement of the former arc D B.

11. The *ſinus totus* is the greateſt ſine, being the ſine of an arc of 90 degrees, or one quarter of a circle, and is equal to the radius of the circle.

Thus, the ſines always increaſe from B, at which place they are nothing, till they come to the radius C H, which is the greateſt, being the ſine totus. From hence they decreaſe all the way along the ſecond quadrant from H to A, and at length vaniſh at the point A; whereby we ſee that the ſine of the ſemicircle B H A, is nothing. After this, the ſines are negative, as they proceed along the next ſemicircle A O B, being drawn on the oppoſite ſide, or downwards, from the diameter A B.

As D E is the coſine of D H: the ſine, coſine, and radius of any arc form a right-angled triangle; as, C D E, or C D F; of which, the radius C D is the hypothenuſe: and therefore the ſquare of the radius is equal to the ſum of the ſquares of the ſine and coſine of any arc.

The ſines, coſines, tangents, &c. of every degree and minute, in a quadrant, are calculated to a radius of 1, and ranged in tables for uſe. But to ſhorten the operation in calculations in trigonometry, we uſe the logarithms of them, inſtead of the natural numbers, which are called the artificial ſines, tangents, &c. and theſe numbers ſo ranged in tables, form the trigonometrical canon; and contain every ſpecies of right-angled triangles; ſo that no triangle can be propoſed, but one ſimilar to it can be found there by compariſon, with which the propoſed one may be computed by analogy of proportion. Laſtly, ſometimes the proportion is not expreſſed in numbers: but the ſeveral ſines, tangents, &c. are actually laid down upon lines of ſcales; from whence the line of ſines, of tangents, &c. on the plane ſcale, the conſtruction and uſe of which follow:—

The *plane ſcale* is a mathematical inſtrument of moſt extenſive uſe, commonly two feet in length. The lines uſually

 drawn

drawn upon it, are the following:—1. Lines of equal parts. 2. Of Chords,—3. Rhumbs.—4. Sines.—5. Tangents.—6. Secants.—7. Semi-tangents.—8. Longitude.—And, 9. Latitude. (Fig. 2.)

1. The lines of equal parts are of two kinds: viz.—simply divided, and diagonally divided. The firft of thefe are formed by drawing three parallel lines, and dividing them into any number of equal parts, by fhort lines drawn acrofs them; and in like manner fubdividing the firft divifion into ten other equal fmaller parts, by which numbers or dimenfions of two figures may be taken off. Upon fome rules feveral of thefe fcales of equal parts are ranged parallel to each other, with figures fet to them, to fhow into how many equal parts they divide the inch; as 20, 25, 30, &c. 2. The diagonal divifions are formed by drawing eleven long parallel lines, equidiftant from each other, which are divided into equal parts, and croffed by other fhort lines, as the former; then the firft of thefe equal parts have the two outermoft of the eleven parallel lines divided into ten equal parts, and the points of divifion connected by diagonal lines, as fhown in Menfuration. The whole fcale is thus divided into dimenfions of three places of figures.

The other lines upon the fcale are commonly ufed in Trigonometry, Navigation, Aftronomy, Dialling, &c. &c. and are all conftructed from the divifions of a circle, as follows:—

2. Defcribe a circle* with any convenient radius, and divide it in four equal parts, by two diameters, drawn at right angles to each other, (*fig.* 2.) Continue one diameter C D towards F, and draw the tangent line E A, parallel thereto; then draw the chords D A and D B.

3. To conftruct the line of chords, divide the quadrant A D into 90 equal parts: then on A, as a centre, with the

* Only half the circle is drawn in the figure for want of room; but in general a complete circle is formed.

compaffes,

compaſſes, transfer theſe diviſions to the chord line A D, which mark with the correſponding numbers, and it will become the line of chords, which may be transferred to the ruler.

4. For the line of rhumbs, divide the quadrant B D into eight equal parts: then with the compaſſes, from the centre B, transfer the diviſions to the line B D, which will be the line of rhumbs.

5. For the line of ſines, through each of the diviſions of the arc A D draw right lines parallel to the radius D C, which will divide the radius A C into the ſines, or verſed ſines; numbering it from C to A for the ſines, and from A to C for the verſed ſines.

6. For the line of tangents, lay a ruler on C, and the ſeveral diviſions of the arc A D; and it will interſect the line E A, which will become a line of tangents, transferring the numbers from the arc A D to that line.

7. For the line of ſecants transfer the diviſions from the tangent line to the line F D with the compaſſes, and from C as the centre, marking the diviſions with the correſponding numbers on the tangent line.

8. For a line of ſemi-tangents, lay a ruler on B and the ſeveral diviſions of the arc A D, which will interſect the radius C D in the ſeveral diviſions of the ſemi-tangents, which are to be numbered according to the arc A D.

9. For the line of longitude, divide the radius A C into 60 equal parts, through each of theſe draw lines parallel to the radius C D; the points where theſe lines interſect the arc A D are to be transferred with the compaſſes from A as a centre to the chord A D, and numbered thereon, which will give the line of longitude.

10. For the line of latitude, the ſemicircle A D B muſt be completed to a circle, then a ruler laid on the point D, and on the ſeveral diviſions of the line of ſines, A C will interſect the next quadrant of the circle, in as many points; when from the oppoſite part of the circle to D, as the centre,

the interfections of the arc are to be transferred to its chord, and numbered according to the numbers on the line of fines.

The chief ufes of the lines of fines, tangents, fecants, and femi-tangents, are to find the poles, and centres of the feveral circles, reprefented in a projection of the fphere.

I have been more particular in defcribing the conftruction of this fcale, as it is an inftrument in moft general ufe in mathematics; and by the foregoing directions the learner may conftruct any lines on the fcale himfelf, where there happens not to be a mathematical inftrument maker nigh at hand, and place them on a rule, as feen *fig.* 3.

## SECT. I.

### OF PLANE TRIGONOMETRY.

THE three methods of refolving triangles, or cafes in trigonometry, are:—1. By geometrical conftruction. 2. Arithmetical computation. And, 3. Inftrumental operation. In the firft method, the triangle is conftructed, by drawing, and laying down the feveral parts, viz.—the fides from a fcale of equal parts, and the angles from a fcale of chords, or other inftrument: then the unknown parts are meafured by the fame fcales; and thus they become known.

In the fecond method, the terms of the proportion are ftated according to rule; which terms confift partly of the numbers of the given fides, and partly of the fines, &c. of angles taken from the tables; the proportion is then refolved like all other proportions, in which a fourth term is to be found from three given terms, viz. by the Rule of Three.

In the third method of refolving the triangle, by inftrumental operation, recourfe muft be had to the logarithmic lines, on one fide of the two foot fcales; extending the compaffes from the firft term to the fecond or third, which happens to be of the fame kind with it; then that extent will

reach

reach from the other term to the fourth term. In this operation for the ſides of triangles, is uſed the line of numbers, and for the angles the line of ſines or tangents, according as the proportion reſpects ſines or tangents.

In every caſe in plane trigonometry, there muſt be given three parts, one of which, at leaſt, muſt be a ſide. And every triangle that can be propoſed, will fall under one of the three following caſes:

## CASE I.

*When two of the three given Parts are a Side, and its oppoſite Angle.*

## CASE II.

*When there are given two Sides, and their contained Angle.*

## CASE III.

*When the three Sides are given.*

RULE. For the firſt caſe, viz.—That the ſides are proportional to the ſines of their oppoſite angles: that is, as the one ſide given, is to the ſine of its oppoſite angle, ſo is another ſide given to the ſine of its oppoſite angle. Or, as the ſine of a given angle is to its oppoſite ſide, ſo is the ſine of another given angle to its oppoſite ſide. Thus, to find an angle, we muſt begin the proportion with a given ſide, that is, oppoſite to a given angle; and to find a ſide, we muſt begin with an angle oppoſite to a given ſide.

EXAMPLE. In the triangle, B D C, (*fig.* 4) having the ſide B D equal to 106, the ſide B C equal to 65, and the angle B D C, 31 degrees, 49 minutes, to find the angle B C D, and the ſide C D.

### 1. *By geometrical Conſtruction.*

Draw a line B D equal to 106; at D, make an angle of 31° 49′ by drawing D C; take 65 in the compaſſes, and with one

one foot at B, extend the other foot to C, in the line D C, then draw the line B C, and it is done: for the angle C will be 120° 43′; the angle D, 31° 49′; and the angle B, 27° 28′; and the ſide D C, 56.

## 2. *By arithmetical Computation.*

| | *Log.* |
|---|---|
| As the ſide B C, 65 | 1.81291 |
| Is to the ſine angle D, 31° 49′ | 9.72198 |
| So is the ſide B D, 106 | 2.02531 |
| | 11.74729 |
| | 1.81291 |
| To ſine angle C, 120° 43′ | 9.93438 |

To find the ſide D C.

| | *Log.* |
|---|---|
| As the ſine angle D, 31° 49′ | 9.72198 |
| Is to ſide B C, 65 | 1.81291 |
| So is ſine angle B, 27° 28′ | 9.66392 |
| | 11.47683 |
| | 9.72198 |
| To the ſide D C, 56.88 | 1.75485 |

Or it may be wrought as follows :—

| | |
|---|---|
| 180° 0′ | The ſum of three angles |
| 59°17′ | Supplement of angle C |
| 120°43′ | Angle C |
| 31°49′ | Angle D |
| 152°32′ | Their ſum |
| 180° 0′ | |
| 152°32′ | |
| 27°28′ | Angle B. |

Here it muſt be noted, that when the given angle is obtuſe, the angle ſought will be acute; but when the given angle is acute, and oppoſite to a leſs given ſide, then the required angle is doubtful, whether acute or obtuſe; it ought therefore to be determined before the operation be performed.

For

For the above proportion gives 59° 17′ for the required angle; but as it is obtuse, its supplement to 180° must be taken, viz. 120° 43′.

### 3. *By Gunter's Line, or instrumental Operation.*

RULE. Extend the compasses from 65 to 106 on the lines of numbers, and that extent will reach from 31° 49′ to 59° 17′ on the line of sines.

Secondly. The extent from 31° 49′ to 27° 28′ on the line of sines, will reach from 65 to 56.88 on the line of numbers.

## CASE II.

*When the three given Parts are two Sides and their contained Angle.*

RULE. As the sum of the two given sides is to the difference of the sides, so is the tangent of half the sum of the two opposite angles or cotangent of half the given angle to the tangent of half the difference of those angles.

Then the half difference added to the half sum gives the greater of the two unknown angles, and subtracted, leaves the less of the two angles.

Thus, having all the angles, the remaining third side will be found by the former case.

EXAMPLE. Having the side B C, equal to 109, B D equal to 76, (fig. 5,) and the angle C B D, 101° 30′, to find the angle B D C, or B C D, and the side C D.

### 1. *By geometrical Construction.*

Draw the line B C equal to 109 and B D, so as to make an angle with B C, of 101° 30′, and make B D equal to 76; join B C and B D with a right line, and it is done; for the angle D being measured, is found to be equal to 47° 32′, the angle C 30° 58′, and the side D C 144.8.

## 2. *Arithmetically by Logarithms.*

| | | | | | |
|---|---|---|---|---|---|
| Side B C | 109 | | 109 | | 180° 0′ | |
| Side B D | 76 | | 76 | | 101°30′ | |
| Their Sum | 185 | their | 33 | difference | 78°30′ | Sum of the angle D & C. |
| | | | | | 39°15′ | Half the sum. |

Then to find the angles D and C.

| | Log. |
|---|---|
| As the sum of the sides B C and B D = 185 | 2.26717 |
| Is to their difference 33 | 1.51851 |
| So is tan. of ½ sum of the angles C & D 39° 15′ | 9.91224 |
| | 11.43075 |
| | 2.26717 |
| To tangent of ½ the diff. of the angles C & D 8°17′ | 9.16358 |

| | |
|---|---|
| To ½ the sum of the angles D and C | 39°15′ |
| Add half the difference of the angles C and D | 8°17′ |
| Gives the greater angle D | 47°32′ |
| Subtracted, gives the lesser angle C | 30°58′ |

To find D C.

| | Log. |
|---|---|
| As sine angle D 47° 32′ | 9.86786 |
| Is to the side B C 109 | 2.03743 |
| So is sine angle B 101° 30′ | 9.99119 |
| | 12.02862 |
| | 9.86786 |
| To the side D C, required 144.8 | 2.16076 |

## 3. *By Gunter.*

**First.** The extent from 185 to 33 on the line of numbers will reach from 39° 15′ to 8° 17′ on the line of tangents. Secondly, the extent from the angle D 47° 32′ to 78° 30′, (the supple-

ment

ment of angle B,) on the line of fines, will reach from the fide B C 109 to 144.8, the fide D C required on the line of numbers.

## CASE III.

### *When the three Sides are given, to find the three Angles.*

Rule. Let fall a perpendicular from the greateft angle upon the oppofite fide or bafe, dividing it into two fegments, and the whole triangle into fmaller right-angled triangles: then the proportion will be; as the bafe, or fum of the two fegments, is to the fum of the other two fides; fo is the difference of thofe fides, to the difference of the fegments of the bafe.

Then half the difference of the two fegments added to half the bafe, or half the fum of the two fegments, gives the greater fegment; and fubtracted gives the lefs. Thus, in each of the two right-angled triangles, there are given the hypothenufe and the bafe, befides the right angle; therefore, the other angles may be found by the firft cafe.

Example. Having the fides B C equal to 105 (*fig.* 6), B D equal to 85, and C D equal to 50; to find the three angles D, C, and B.

#### 1. *Geometrically by Conftruction.*

1. Draw the line B C, equal to 105; with the compaffes open to 50, and having one foot on the point C, defcribe an arc; then with the compaffes open to 85 and one foot in B, cut the former arc in D, join B D and C D, and it is done; for the angles meafured, B will be found equal to 28° 4′, and C 53° 7′, which being added together, and fubtracted from 180°, leaves 98° 49′, for the angle D.

 2. *Arith-*

### 2. *Arithmetically by Logarithms.*

The two ſhorteſt ſides of the triangle B D and C D added together, is 135, and their difference 35. The ſegments of the baſe B C are found in the following manner:

| | Log. |
|---|---|
| As the ſide B C equal to 105 | 2.02119 |
| Is to the ſum of the ſides B D and D C 135 | 2.13033 |
| So is their difference 35 | 1.54407 |
| To the difference of the ſegments of B C 45 | 1.65321 |

Thus, having the ſum and difference of the ſegments of the baſe, it is only neceſſary to add half their ſum $52\frac{1}{2}$ to half the difference $22\frac{1}{2}$, and it will give the greater ſegment, which is 75; and which ſubtracted from 105 leaves 30, the leſſer ſegment.

*To find the Angle B D A.*

| | Log. |
|---|---|
| As the hypothenuſe B D 85 | 1.92942 |
| Is to the radius | 10.00000 |
| So is the greater ſegment 75 | 1.87506 |
| To the ſum of the angle B D A | 9.94564 |

Therefore, the angle B D A is equal to 61° 56′.

*To find the Angle A D C.*

| | Log. |
|---|---|
| As the hypothenuſe D C 50 | 1.69897 |
| Is to the radius | 10.00000 |
| So is the leſs ſegment 30 | 1.47712 |
| To the ſine of A D C | 9.77815 |

Therefore, the angle A D C is equal to 36° 53′, and the whole angle B D C equal to 98° 49′.

To

To find the angle B, it is only neceſſary to ſubtract the angle B D A, or 61° 56′, from 90°, and the remainder 28° 4′, is the angle B, and the angle C is equal to 53° 7′.

### 3. *By Gunter.*

1. The extent from 105 to 135 will reach from 35 to 45 on the line of numbers. Secondly, The extent from 85 to 75 on the line of numbers, will reach from the radius to 61° 56′, or the angle B D A, on the line of ſines. Thirdly, The extent from 50 to 30 on the line of numbers, will reach from the radius to 36° 53′, the angle A D C on the line of ſines.

The three foregoing caſes of plane triangles, contain all the varieties of both right and oblique triangles. But there are ſome other theorems, ſuited to ſome particular forms of triangles, which are often more expeditious in uſe, than the foregoing general ones; particularly the following theorem, for right-angled triangles, being a caſe which frequently occurs.

## CASE IV.

*When in a right-angled Triangle there are given the Angles, and one Leg, to find the other Leg, or Hypothenuſe.*

Rule. As the radius is to the given leg A B (*fig.* 7), ſo is the tangent of the adjacent angle A, to the oppoſite leg B C; and ſo is the ſecant of the ſame angle A, to the hypothenuſe A C.

Example. In the triangle A B C, having the leg A B equal to 162, and the angle A equal to 53° 7′ 48″, and conſequently the angle C 36° 52′ 12″; to find the ſides B C and A C.

### 1. *Geometrically.*

Draw A B equal to 162, and erect the indefinite perpendicular B C; make the angle A 53° 7′ 48″; then the ſide A C will cut B C in the point C, and form the triangle A B C, which, by meaſuring, A C is found equal to 270, and B C to 216.

### 2. *Arithmetically.*

| | *Log.* |
|---|---|
| As radius | 10.0000000 |
| Is to A B 162 | 2.2095150 |
| So is the tangent A 53° 7′ 48″ | 10.1249372 |
| | 12.3344522 |
| | 10.0000000 |
| To B C 216 | 2.3344522 |
| So is the ſecant A 53° 7′ 48″ | 10.2218477 |
| To A C 270 | 2.4313627 |

### 3. *By Gunter.*

Extend the compaſſes from 45° at the end of the tangents, (the radius,) to the tangent of $53^\circ\frac{1}{8}$; and that extent will reach from 162 to 216 on the line of numbers for B C. Then extend the compaſſes from 36° 52′ to 90 on the ſines; and that extent will reach from 162 to 270 on the line of numbers for A C.

There is alſo another method of frequent uſe in trigonometry, called, *making every ſide radius*, which is as follows:

Let A B C (*fig.* 8) be a given triangle; make the hypothenuſe A C radius firſt; that is, with the extent of A C as a radius, and on A and C, as two centres, deſcribe the two arcs C D and A E; then each leg A B, B C, will repreſent the ſine of its oppoſite angle: viz. The leg B C will be the ſine of the arc C D, or of the angle A; and the leg A B the ſine of the arc A E, or of the angle C.

Again,

Again, making either leg radius, the other leg will represent the tangent of its oppofite angle, and the hypothenufe the fecant of the fame angle: thus, with the radius A B and centre A, defcribe the arc B K; and B C will reprefent the tangent of that arc, or of the angle A, and the hypothenufe A C the fecant of the fame; or, with the radius B C and centre C, defcribe the arc B G; then the other leg A B is the tangent of that arc B G, or of the angle C, and the hypothenufe C A is the fecant of the fame.

Then the proportions are obvious; for the fides in this figure bear the fame proportions to each other as the parts they reprefent.

## SECT. II.

### OF SPHERICAL TRIGONOMETRY.

SPHERICAL Trigonometry teaches the refolution of a *fpherical triangle*, having three given parts: and, like plane trigonometry, may be either right-angled, or oblique-angled; but before the learner can proceed to the analogies of a fpherical triangle, it is neceffary to be acquainted with the fix following theorems:—

#### *Theorem* I.

In all right-angled fpherical triangles the fine of the hypothenufe is to the radius, as the fine of a leg is to the fine of its oppofite angle. And as the fine of a leg is to the radius, fo is the tangent of the other leg to the tangent of its oppofite angle.

Demonftrated thus. Let E D A F G (*fig.* 9) reprefent the eighth part of a fphere, where the quadrantal planes E D F G, and E D B C, are both perpendicular to the quadrantal

tal plane A D F B, and the quadrantal plane A D G C is perpendicular to the plane E D F G; and the ſpherical triangle A B C is right-angled at B, and therefore C A is the hypothenuſe, and B A, B C, are the legs.

Draw the tangents H F and O B to the arches G F and C B, and the ſines G M, C I, on the radii D F and D B; alſo draw B L the ſine of the arc A B, and C K the ſine of A C: then join I K and O L. Now H F, O B, G M, and C I, are all perpendicular to the plane A D F B. And H D G K, and O L, lie all in the ſame plane A D G C; alſo F D, I K, B L, lie all in the ſame plane A D G C. Therefore, the right-angled triangles H F D, C I K, and O D L, having the equa-angles H D F, C K I, O L B, are ſimilar. And as C K is to D G, ſo is C I to G M; that is, as the ſine of the hypothenuſe is to the radius, ſo is the ſine of a leg to the ſine of its oppoſite angle. For G M is the ſine of the arc G F, which meaſures the angle C A B. Alſo as L B is to D F, ſo is B O to F H; that is, as the ſine of a leg is to the radius, ſo is the tangent of the other leg to the tangent of its oppoſite angle. Q. E. D.

From this it follows, that the ſines of the angles of any oblique ſpherical triangle, as A C D (*fig.* 10), are to one another directly as the ſines of the oppoſite ſides; therefore, in every right-angled ſpherical triangle, having the ſame perpendicular, the ſines of the baſes will be to each other inverſely as the tangents of the angles at the baſes.

### *Theorem* II.

In every right-angled ſpherical triangle, as A B C (*fig.* 11), the proportion is, as radius is to the coſine of one leg, ſo is the coſine of the other leg to the coſine of the hypothenuſe.

Therefore, if two right-angled ſpherical triangles A B C, C B D (*fig.* 10), have the ſame perpendicular B C, the coſines of their hypothenuſes will be to each other directly as the coſines of their baſes.

*Theorem*

### *Theorem* III.

In any ſpherical triangle, the proportion is, As radius is to the ſine of either angle, ſo is the coſine of the adjacent leg to the coſine of the oppoſite angle.

Therefore, in right-angled ſpherical triangles having the ſame perpendicular, the coſines of the angles, at the baſe, will be to each other, directly, as the ſines of the vertical angles.

### *Theorem* IV.

In any right-angled ſpherical triangles, the proportion is, As radius is to the coſine of the hypothenuſe, ſo is the tangent of either angle to the cotangent of the other angle.

Thus, As the ſum of the ſines of two unequal arches is to their difference, ſo is the tangent of half the ſum of thoſe arches, to the tangent of half their difference; and, As the ſum of the coſines is to their difference, ſo is the cotangent of half the ſum of the arches to the tangent of half the difference of the ſame arches.

### *Theorem* V.

In any ſpherical triangles A B C (*fig.* 12 and 13), the proportion is, As the cotangent of half the ſum of half the difference, ſo is the cotangent of half the baſe, to the tangent of the diſtance (D E) of the perpendicular, from the middle of the baſe.

### *Theorem* VI.

In any ſpherical triangle A B C (*fig.* 12), As the cotangent of half the ſum of the angles at the baſe, is to the tangent of half their difference, ſo is the tangent of half the vertical angle, to the tangent of the angle which the perpendicular C D makes with the line C F, biſecting the vertical angle,

*The Solution of the Cafes of right-angled fpherical Triangles, (Fig. 11.)*

| Cafe. | Given. | Sought. | Solution. |
|---|---|---|---|
| 1 | The hypo. AC, and one angle A. | The oppofite leg B C. | As radius : fine hypothenufe A C :: fine A : fine BC. (by Theor. 1.) |
| 2 | The hypo. AC, and one angle A. | The adjacent leg A B. | As radius : cofine of A :: tangent A C : tangent A B. (by Theor. 1.) |
| 3 | The hypothenufe A C, and one angle A. | The other angle C. | As radius : cofine of A C :: tangent A : cotangent C. (by Theor. 4.) |
| 4 | The hypo. AC, and one leg A B. | The other leg B C. | As cofine A B : radius :: cofine A C : cofine B C. (by Theor. 2.) |
| 5 | The hypo. AC, and one leg A B. | The oppofite angle C. | As fine A C : radius :: fine A B : fine C. (by Theor. 1.) |
| 6 | The hypo. AC, and one leg A B. | The adjacent angle A. | As tangent A C : tangent A B :: radius : cofine A. (by Theor. 1.) |
| 7 | One leg AB, and the adjacent angle A. | The other leg B C. | As radius : fine A B :: tangent A : tangent B C. (by Theor. 4.) |
| 8 | One leg AB, and the adjacent angle A. | The oppofite angle C. | As radius : fine A :: cofine A B : cofine C. (by Theor. 3.) |
| 9 | One leg AB, and the adjacent angle A. | The hypothenufe A C. | As cofine A : radius :: tangent A B : tangent A C. (by Theor. 1.) |
| 10 | One leg BC, and the oppofite angle A. | The other leg A B. | As tangent A : tangent B C :: radius : fine A B. (by Theor. 4.) |
| 11 | One leg BC, and the oppofite angle A. | The adjacent angle C. | As cofine B C : radius :: cofine of A : fine C. (by Theor. 3.) |
| 12 | One leg BC, and the oppofite angle A. | The hypothenufe A C. | As fine A : fine B C :: radius : fine A C. (by Theor. 1.) |
| 13 | Both legs A B and B C. | The hypothenufe A C. | As radius : cofine A B :: cofine B C : cofine A C. (by Theor. 2.) |
| 14 | Both legs A B and B C. | Any angle, as A. | As fine A B : radius :: tangent B C : tangent A. (by Theor. 4.) |
| 15 | Both angles A and C. | Any leg, as AB. | As fine A : cofine C :: radius : cofine A B. (by Theor. 3.) |
| 16 | Both angles A and C. | The hypothenufe A C. | As tangent A : cotangent C :: radius : cofine A C. (by Theor. 4.) |

NOTE. The 10th, 11th, and 12th cafes are ambiguous, as it cannot be determined by the data, whether A B C, and A C, be greater or lefs than 90 degrees each.

The

## *The Solution of the Cases of oblique spherical Triangles, (Fig. 12 and 13.)*

| | Given. | Sought. | Solution. |
|---|---|---|---|
| 1 | Two sides AC, BC, and the angle A opposite to one of them. | The angle B, opposite to the other. | As sine BC : sine A :: sine AC : sine B. *Note.* When BC is less than AC, it cannot be determined whether B be acute or obtuse. |
| 2 | Two sides AC, BC, and the angle A opposite to one of them. | The angle ACB. | Let fall the perpendicular CD upon AB, produced (if necessary); then radius : cosine AC :: tangent A : cotangent ACD. |
| 3 | Two sides AC, BC, and the angle A opposite to one of them. | The other side AB. | As radius : cosine A :: tangent AC : tangent AD. (by Theor. 1.) This and the last case are both ambiguous, when the first is so. |
| 4 | Two sides AC, AB, and the included angle A. | The other side BC. | As radius : cosine A :: tangent AC : tangent AB. (by Theor. 1.) Whence AD is also known. |
| 5 | Two sides AC, AB, and the included angle A. | Either of the other angles, as B. | As radius : cosine A :: tangent AC : tangent AD. (by Theor. 1.) Whence BD is known; then as sine BD : sine AD :: tangent A : tangent B. (by Theor. 4.) |
| 6 | Two angles A, ACB, and the side AC betwixt them | The other angle B. | As radius : cosine AB :: tangent A : cotangent ACD. (by Theor. 4.) Whence BCD is also known; then as sine ACD : sine BCD :: cosine A : cosine B. (by Theor. 3.) |
| 7 | Two angles A, ACB, and the side AC betwixt them | Either of the other sides, as BC. | As radius : cosine AC :: tangent A : cotangent ACD. (by Theor. 4.) Whence BCD is also known; then as cosine BCD : cosine ACD :: tangent AC : tangent BC. (by Theor. 1.) |
| 8 | Two angles A, B, and the side AC opposite to one of them. | The side BC, opposite to the other. | As sine B : sine AC :: sine A : sine BC. |
| 9 | Two angles A, B, and the side AC opposite to one of them. | The side AB betwixt them. | As radius : cosine A :: tangent AC : tangent AD. (by Theor. 1.) and as tangent B : tangent A :: sine AD : sine BD, whence AB is also known. |
| 10 | Two angles A, B, and the side AC opposite to one of them. | The other angle ACB. | As radius : cosine AC :: tangent A : cotangent ACD. (by Theor. 4.) and as cosine A : cosine B :: sine ACD : sine BCD. (by Theor. 3.) |
| 11 | All the three sides AB, AC, and BC. | Any angle, as A. | As tangent $\frac{1}{2}$AB tangent $\frac{AC+BC}{2}$ :: tangent $\frac{AC-BC}{2}$: tangent DE the distance of the perpendicular from the middle of the base, whence AD is known. Then as tangent AC : tangent AD :: rad. : cosine A. (by Theor. 1.) |
| 12 | All the three angles A, B, and ACB. | Any side, as AC. | As cotangent $\frac{ABC+A}{2}$: tangent $\frac{ABC-A}{2}$ :: tangent $\frac{ACB}{2}$: tangent of the angle between the perpendicular and a line bisecting the vertical angles, whence ACD is also known. Then as tangent A : to tangent ACD :: radius : cosine AC. |

The following propofitions concerning fpherical triangles, will render them more intelligible.

1. A fpherical triangle is either equilateral, ifofcelar, or fcalene, according as it has the three angles all equal, or two of them equal, or all three unequal; and *vice verfa*.

2. The greateft fide is always oppofite the greateft angle, and the fmalleft fide oppofite the fmalleft angle.

3. Any two fides, taken together, are greater than the third.

4. If the three angles of a fpherical triangle be all acute, or all right, or all obtufe, angles, the three fides will be accordingly all lefs than 90 degrees, or equal to 90 degrees, or greater than 90 degrees; and *vice verfa*.

5. If from the three angles A, B, C, (*fig.* 14) of a fpherical triangle, A, B, C, as poles, there be defcribed upon the furface of the fphere, three arches of a great circle, D E, D F, and F E, forming by their interfections another fpherical triangle D E F; each fide of the new triangle will be the fupplement of the angle at its pole; and each angle of the fame triangle will be the fupplement of the fide oppofite to it in the triangle A B C.

6. In any triangle A B C, or A b C, right angled at A, (*fig.* 15,) the angles at the hypothenufe are always of the fame kind as their oppofite fides. And the hypothenufe is greater or lefs than a quadrant, according as the fides, including the right angle, are of different kinds, or of the fame kind; that is to fay, according as thefe fame fides are either both obtufe, or both acute, or as one is obtufe, and the other acute, and *vice verfa*. Firft, the fides, including the right angle, are always of the fame kind as their oppofite angles. Secondly, the fides, including the right angle, will be of the fame or different kinds, according as the hypothenufe is lefs or more than 90 degrees; but one of them at leaft will be of 90 degrees, if the hypothenufe be fo.

CHAP.

# CHAP. XIII.

# OF GEOGRAPHY.

## SECT. I.

### GEOGRAPHICAL DEFINITIONS.

1. Geography is the knowledge of the earth, or a defcription of the terreftrial globe, particularly of the furface and known habitable parts thereof, with all its different divifions.

2. The earth is a globular body, furrounded with an atmofphere of air, by which all terreftrial bodies are confined to its furface, being attracted thereto by the laws of gravity.

That the earth is of a globular form, has been demonftrated by a number of experiments, particularly by obfervations of the eclipfes of the moon; in which it appears, that the fhadow of the earth always appears circular, whichever way it is projected. Alfo, by the obfervation of fhips at fea, which, after their departure from any coaft, gradually difappear to an obferver on land, from the bottom upwards; that is, the firft part which difappears from the fight, is the keel, or lower part of the fhip; then thofe parts which are higher up, and fo on; the top of the maft being the part

that is laſt ſeen: this is owing to the convexity of the waters, which have the ſame globular figure as the earth *.

3. The earth alſo has a diurnal motion on its own axis, performing one revolution in 24 hours; thereby occaſioning the changes of the day and night, as will be ſeen in Aſtronomy.

4. The circumference of the globe is ſuppoſed to be divided into 360 parts, called degrees, and each degree ſubdivided into 60 minutes, and each minute into 60 ſeconds, &c. Every degree contains 60 geographic miles; conſequently, the circumference of the globe is 21,600 ſuch miles; and the diameter 6900 miles. But as 60 geographical miles are above 69 Britiſh meaſure, the circuit of the globe is therefore 24,840 Engliſh miles, and the diameter almoſt a third, or 7900 in round numbers.

5. The globe of the earth conſiſts of land and water: the proportion of the land to the water is not accurately known, but it is generally believed to be near one third.

6. The waters are divided into three oceans, (beſides the ſmaller ſeas:) viz. The Atlantic, Pacific, and the Indian ocean. 1. The Atlantic or Weſtern ocean divides Europe from America, and is 3000 miles wide. 2. The Pacific ocean divides America from Aſia, and New Holland, and is 10,000 miles wide. 3. The Indian ocean lies between the Eaſt Indies and Africa, and is 3000 miles wide. The parts or

* The philoſophers of the laſt age differed greatly in their deſcriptions of the true ſpherical figure of the earth, of which there were two general opinions. The one maintained that the earth was a prolate ſpheroid: of this number was Caſſini. The other party maintained it to be an oblate ſpheroid: of this Sir Iſaac Newton was the chief, who inſiſted that the polar was ſhorter than the equatorial diameter by 36 miles. A party of philoſophers from France was ſent by the king of that country, to meaſure a degree on the polar circle, and alſo on the equator; the reſult of their experiment turned out exactly in favour of Sir Iſaac Newton's theory. But this inequality of the ſhape of the earth, as well as the inequalities occaſioned by the mountains, &c. make no *ſenſible* difference in the form of the earth.

branches of thefe oceans, called feas, as the Mediterranean fea, &c. receive their names generally from the countries they border upon.

7. A *bay*, or *gulf*, is a part of the fea, almoft furrounded by land; as the gulf of Mexico, the bay of Bifcay, &c.

8. A *ftrait* is a narrow paffage out of one fea into another; as the ftrait of Gibraltar.

9. A *lake* is a water furrounded by land; as the lake of Geneva.

10. *Rivers* are ftreams of water, iffuing from fprings in high grounds, and falling into the fea, or other rivers; and are wider near their mouths than towards their heads or fprings: they are defcribed in maps by black lines.

11. The land is divided into two great continents: viz. The eaftern and weftern continents, befides iflands. The eaftern continent is fubdivided into three parts: viz. Europe, which is the north-weft part; Afia, the north-eaft; and Africa, the fouth. The weftern continent confifts of America only, divided into North and South America.

12. An *ifland* is a piece of land entirely furrounded by water.

13. A *peninfula* is a country, or piece of land, furrounded by water, except on one fide, where it joins to fome other land.

14. An *ifthmus* is a narrow neck of land which joins a peninfula to fome other country; as the ifthmus of Suez, which joins Africa to Afia; the ifthmus of Darien, which joins North and South America.

15. A *cape*, called fometimes a promontory, or head-land, is a point of land extending fome way into the fea.

16. The furface of the earth is fuppofed to be divided by feveral imaginary circles, for the better determining the fituation and boundaries of the feveral countries and parts of the world, of which the moft confiderable circles are the following:—1. The *equator*, called alfo the equinoctial line, which divides the globe of the earth into two equal parts, or hemifpheres,

hemifpheres, the one north and the other fouth. This circle is every where equally diftant from the two poles; and upon this circle the degrees of longitude are marked. 2. The two *tropical circles*: viz. The *tropic of Cancer*, or the northern tropic, which encompaffes the globe at the diftance of 23½ degrees from the equator; and the *tropic of Capricorn*, or fouthern tropic, which encompaffes the globe at the fame diftance on the fouth fide of the equator. The fpace between thefe two tropics is called the *Torrid Zone*. 3. The two polar circles: viz. the *arctic circle*, which furrounds the north pole; and the *antarctic circle*, which furrounds the fouth pole; each at the diftance of 23½ degrees from its refpective pole. The fpace included between the tropic of Cancer and the arctic circle is called the northern *Temperate Zone*, and that fpace between the arctic circle and the north pole, is called the north *Frigid Zone*; and the correfponding fpaces on the fouthern hemifphere have fimilar names, as the fouthern *Temperate Zone*, and the fouthern *Frigid Zone*. 4. The *meridional lines*, which are lines drawn at right angles to the equator, and coinciding at the poles. Thefe lines run directly north and fouth: and when the fun appears full fouth of any place, he is then faid to be on the meridian of that place; and it is then twelve o'clock at noon at that place. The latitude of places is always numbered on thefe lines.

17. The *longitude* is the diftance of one country from another, and is either eaft or weft, and meafured on the equator. The longitude of a place is always meafured from the capital of the country where the author or traveller is; thus, when a perfon in England mentions the longitude of any other place, it is always underftood that the longitude is reckoned from London; that is, the degrees of longitude are meafured on the equator, and from that part of the equator, where the meridian paffing through London, interfects the equator, at that part of the equator which is cut by the meridian of the other place meafured to.

18.

18. The *latitude* of a place is the diſtance of that place from the equator, meaſured on the meridional line; and is either north or ſouth.

19. The inhabitants of the earth are diſtinguiſhed from each other by their relative ſituations; of which there are reckoned three ſorts, *Periœci*, *Antiœci*, and *Antipodes*:— 1. The *Periœci* are thoſe people who live at the ſame diſtance from the equator, but under oppoſite meridians: the length of their days and ſeaſons is the ſame; but when it is mid-day with one, it is midnight with the other. 2. The *Antiœci* live under the ſame meridian, but oppoſite parallels, and live equally diſtant from the equator; the one being in the ſouth latitude, and the other in the north. Theſe have the ſun at the ſame hour at noon; but the longeſt day of the one is the ſhorteſt day of the other, and their ſeaſons of the year are different; for when it is ſummer with one, it is winter with the other. 3. The *Antipodes* are ſituated directly on oppoſite ſides of the globe to each other, the feet of the one being directly oppoſite to the feet of the other. Theſe lie under oppoſite meridians, and oppoſite parallels: it is noon-day with the one, when it is midnight with the other; the longeſt day with the one, is the ſhorteſt day with the other; and when it is ſummer with the one, it is winter with the other.

20. The inhabitants of the earth are ſometimes diſtinguiſhed from each other (in geography) by the direction of their ſhadows at noon-day; and are called *Amphiſcii*, *Aſcii*, *Heteroſcii*, or *Periſcii*. 1. The *Amphiſcii* are thoſe ſituated in the torrid zone, and have their ſhadows, one part of the year, directed towards the north at noon-day, and at another part of the year, towards the ſouth, at noon-day, according to the part of the ecliptic the ſun is in; conſequently, the ſun is vertical to theſe people twice a year. They are then called: —2. *Aſcii*, ſhowing no ſhadow at noon-day. 3. The *Heteroſcii*, are thoſe who inhabit the temperate zones, and whoſe ſhadows at mid-day always fall one way: viz. The ſhadows of

of thofe in the northern temperate zone, falling always towards the north, at noon-day; and thofe in the fouthern zone, always fouth at noon-day. 4. The *Periſcii* are thofe who inhabit either of the frigid zones. Thefe have their fhadows moved entirely round them every 24 hours, when the fun is in their hemifpheres, and fo far declined towards their pole, as not to fet for feveral days.

21. The *horizon* is properly a double circle; one of the horizons being called the *fenfible*, and the other the *rational horizon*. The former comprehends only that fpace which we can fee around us, upon any part of the earth; and is very different, according to the difference of our fituation. The other, called *rational*, is parallel to the former, but paffing through the centre of the earth, and fuppofed to be continued as far as the celeftial fphere itfelf; whereas the former is fuppofed to pafs over the furface of the earth, where the fpectator ftands: but in geography, when the horizon is mentioned, the rational horizon is always underftood. By reafon of the round figure of the earth, every different part has a different horizon. The poles of the horizon, that is, the points directly above the head, and oppofite the feet of the obferver, are called the *zenith* and *nadir*.

22. The *zenith* is that pole of the horizon directly over the obferver's head.

23. The *nadir* is the oppofite pole of the horizon, or that directly under the obferver's feet.

A TABLE

## A TABLE,

SHOWING

*The Number of Miles in a Degree of Longitude, in every Degree of Latitude, from the Equator.*

| Degrees of Latitude. | Miles. | Degrees of Latitude. | Miles. | Degrees of Latitude. | Miles. |
|---|---|---|---|---|---|
| 1 | 59.96 | 31 | 51.43 | 61 | 29.04 |
| 2 | 59.94 | 32 | 50.88 | 62 | 28.17 |
| 3 | 59.92 | 33 | 50.32 | 63 | 27.24 |
| 4 | 59.86 | 34 | 49.74 | 64 | 26.30 |
| 5 | 59.77 | 35 | 49.15 | 65 | 25.36 |
| 6 | 59.67 | 36 | 48.54 | 66 | 24.41 |
| 7 | 59.56 | 37 | 47.92 | 67 | 23.45 |
| 8 | 59.40 | 38 | 47.28 | 68 | 22.48 |
| 9 | 59.20 | 39 | 46.62 | 69 | 21.51 |
| 10 | 59.08 | 40 | 46.00 | 70 | 20.52 |
| 11 | 58.89 | 41 | 45.28 | 71 | 19.54 |
| 12 | 58.68 | 42 | 44.95 | 72 | 18.55 |
| 13 | 58.46 | 43 | 43.88 | 73 | 17.54 |
| 14 | 58.22 | 44 | 43.16 | 74 | 16.53 |
| 15 | 58.00 | 45 | 42.43 | 75 | 15.52 |
| 16 | 57.60 | 46 | 41.68 | 76 | 14.51 |
| 17 | 57.30 | 47 | 41.00 | 77 | 13.50 |
| 18 | 57.04 | 48 | 40.15 | 78 | 12.48 |
| 19 | 56.73 | 49 | 39.36 | 79 | 11.45 |
| 20 | 56.38 | 50 | 38.57 | 80 | 10.42 |
| 21 | 56.00 | 51 | 37.73 | 81 | 9.38 |
| 22 | 55.63 | 52 | 37.00 | 82 | 8.35 |
| 23 | 55.23 | 53 | 36.18 | 83 | 7.32 |
| 24 | 54.81 | 54 | 35.26 | 84 | 6.28 |
| 25 | 54.38 | 55 | 34.41 | 85 | 5.23 |
| 26 | 54.00 | 56 | 33.55 | 86 | 4.18 |
| 27 | 53.44 | 57 | 32.67 | 87 | 3.14 |
| 28 | 53.00 | 58 | 31.70 | 88 | 2.09 |
| 29 | 52.48 | 59 | 30.90 | 89 | 1.05 |
| 30 | 51.96 | 60 | 30.00 | 90 | 0.00 |

## *Defcription and Ufe of the Globes and Armillary Sphere.*

By means of maps, the true fituations of the different places of the earth, with regard to one another, and every other particular relative to them, may be eafily known; confequently the hour of the day, feafon of the year, &c. for any particular place may be difcovered. But thefe problems, to be refolved by maps, would be tedious and complex; therefore, thofe machines called the celeftial and terreftrial globes, and the armillary fphere, have been invented, by which many calculations are faved, and every problem in geography may be folved mechanically, in the moft eafy and expeditious manner.

If a map of the world be accurately delineated on a fpherical ball, the furface thereof will reprefent the furface of the earth; for the higheft hills bear no greater proportion to the bulk of the whole earth than fo many grains of fand do to a common mathematical globe of twelve or eighteen inches diameter; the diameter of the earth being near 8000 miles, and no hill upon its furface is above three miles in perpendicular height.

The armillary fphere is a large hollow fphere of glafs, having as many bright ftuds fixed on its infide, as there are vifible ftars in the heavens, and of the fame magnitude, and at the fame angular diftances from each other. This fphere is a true reprefentation of the heavens, to an eye fuppofed to be placed in the centre; for to an obferver placed any where within the furface of an indefinite fphere, all objects will appear equally diftant, though fome be much nearer than others: and if a fmall globe, having a map of the earth upon it, be placed on an axis in the centre of this fphere, and the fphere be made to turn round its axis, it will reprefent the *apparent* motion of the heavens round the earth: but if the globe be turned round its axis, while the fphere remains fixed, it will reprefent the *true* motion of the earth.

If

If there be drawn a great circle upon this ſphere, equally diſtant from its poles, and having the plane of the circle perpendicular to the axis of the ſphere, it will repreſent the celeſtial equinox, which divides the heavens into two equal parts or hemiſpheres; and the two axes of the ſphere will repreſent the two poles of the heavens.

If there be another great circle drawn upon the ſphere, cutting the equinoctial at an angle of 23½ degrees, in two oppoſite points, this circle will repreſent the ecliptic, or circle of the ſun's apparent annual motion; one half of which is on the north ſide, and the other half on the ſouth ſide of the equinoctial.

If there be made a large ſtud to move eaſtward in this ecliptic, and with ſuch a motion as to go quite round it in the time that the ſphere is turned round weſtward upon its axis 366 times; this ſtud will repreſent the ſun changing its place every day in the ecliptic, a 365th part; and going round weſtward in the ſame direction as the ſtars, but with a motion ſo much ſlower than that of the ſtars, that they will make 366 revolutions in the time that the ſun makes only 365, about the axis of the ſphere.

If the terreſtrial globe in this machine be about one inch in diameter, and the diameter of the ſtarry ſphere about five or ſix feet, a ſmall inſect, placed upon the globe, would ſee only a very ſmall portion of its ſurface; but it would ſee one half of the ſurface of the ſtarry ſphere, the convexity of the globe hiding the other half from its view. If the ſphere be ſet in motion as before directed, and the globe alſo revolving on its own axis, the inſect will ſee all the phenomena obſerved by the inhabitants of this world, in the diurnal rotation of the earth round its axis.

The exterior parts of this machine are ſeveral braſs rings, which repreſent the principal circles in the heavens: viz. 1. The *equinoctial*: 2. the *ecliptic*, divided into the ſigns and degrees, and alſo into the months and the days of the

Q 2 year,

year, to ſhow in what point of the ecliptic the ſun is on any given day in the year: 3. the *two tropics*: 4. the *arctic* and *antarctic circles*: 5. the *equinoctial colure*, which is a great circle paſſing through the north and ſouth poles of the heavens, and through the equinoctial circle at the points where the equinoctial is cut by the ecliptic: 6. the *ſolſtitial colure*, which is a great circle paſſing through the poles of the heavens, and at right angles to the equinoctial colure. Hence the ſolſtitial colure paſſes through the equinoctial at the points where the equinoctial is at the greateſt diſtance from the ecliptic. Theſe points in the equinoctial are called the *ſolſtitial points*.

In the north pole of the ecliptic is a nut, to which is fixed one end of a quadrantal wire, having at the other end a ſmall ſun, which is carried round the ecliptic, by turning the nut; and in the ſouth pole of the ecliptic, another quadrantal wire is fixed, with a ſmall moon upon it, which may be moved round by the hand. There is alſo a particular contrivance, for cauſing the moon to move in her own orbit.

On the axis of the ſmall globe is fixed a flat celeſtial meridian, which may be ſet directly over the meridian of any place on the globe; and then turned round with the globe, ſo as to keep over the ſame meridian. This globe has alſo a moveable horizon, which turns upon two wires, which proceed from it on the eaſt and weſt points of the globe, and entering the globe at the oppoſite points in the equator, which is a moveable braſs ring, let into the globe in a groove. The whole fabric is ſupported on a pedeſtal, and may be elevated or depreſſed to any number of degrees, from 0 to 90.

### *Deſcription of the Terreſtrial Globe.*

On the terreſtrial globe are drawn all the principal circles before mentioned, as the equator, ecliptic, tropics, polar circles, and meridians. The ecliptic is divided into twelve ſigns, and each ſign into thirty degrees. Each tropic is 23½ degrees

degrees from the equator; and each polar circle 23½ degrees from its refpective pole. There are alfo circles drawn parallel to the equator, at every 10 degrees diftance from it, on each fide towards the poles; thefe circles are called *parallels of latitude*. There are, alfo, feveral other circles, drawn perpendicularly to the equator, and interfecting each other at the poles; thefe circles are called *meridians*, and fometimes circles of *longitude*, or *hour circles*; and on large globes they are drawn through every tenth degree of the equator; but on globes of lefs than 12 inches diameter they are drawn through every fifteenth degree.

The globe is hung in a brafs ring, called the *brazen meridian*, turning upon a wire in each pole, funk into one fide of the meridian ring. This meridian is divided into 360 degrees; one half of thefe degrees are numbered from the equator to the poles, to fhow the latitude of places; the other half are numbered from the poles to the equator, to fhow how to elevate either of the poles above the horizon. This ring divides the globe into two equal parts, called the *eaftern* and *weftern hemifpheres*; as the equator divides it into the *northern* and *fouthern hemifpheres*.

The brazen meridian is let into two notches, made in a broad flat ring, called the *wooden horizon*; the upper furface of which divides the globe into two equal parts, called the *upper* and *lower hemifpheres*. This horizon correfponds to the true rational horizon; and upon it are feveral concentric circles, which contain the months of the year, the figns and degrees anfwering to the fun's place for each month and day, the thirty-two points of the compafs, and the circles of amplitude and azimuth, with fome other circles.

There is a fmall horary circle, fixed to the north part of the brazen meridian, and having the wire in the north pole of the globe in its centre; on which wire is an index, which goes over all the twenty-four hours of the circle, as the globe is turned round its axis. Sometimes there are two horary circles, one at each pole.

There

meridian to the degree of the given latitude; and under that degree of latitude will be the place required.

## PROBLEM III.

*To find the Difference of Longitude, or Difference of Latitude, between any two given Places.*

Bring each of the two given places to the brazen meridian, and mark their latitudes; then, if both places are on the ſame ſide of the equator, the leſſer latitude ſubtracted from the greater will give the difference; but if they are on different ſides of the equator, both latitudes muſt be added together. And the difference of longitude is found by bringing each place to the meridian, and reckoning on the equator the difference of degrees between the meridians of the two places, if it be leſs than 180 degrees, or half a circle; but if the difference be greater, it muſt be ſubtracted from 360, and the remainder is the difference of longitude.

## PROBLEM IV.

*To find all thoſe Places that have the ſame Latitude and Longitude with any given Place.*

Bring the given place to the brazen meridian; then all thoſe places which lie under the ſaid meridian will have the ſame longitude. Then turn the globe round on its axis, and all thoſe places which paſs under the ſame degree of latitude in the brazen meridian, that the given place does, have the ſame latitude.

## PROBLEM V.

*To find the Diſtance between any two Places on the Globe.*

Lay the graduated edge of the braſs quadrant of altitude, over both the places, and count the number of degrees intercepted

tercepted between them on the quadrant, which will be the diſtance in degrees; and which multiplied by 60, will give the diſtance in geographical miles; but multiplied by 69½ gives the diſtance in Engliſh miles. Or, the diſtance between the two places may be taken with a pair of compaſſes, and that extent applied to the equator, will ſhow the number of degrees diſtant.

## PROBLEM VI.

*The Hour of the Day at any Place being given, to find what is the Hour at any other Place.*

Bring the given place to the brazen meridian, and ſet the index to the given hour; then turn the globe, until the place where the hour is required, comes to the meridian; and the index will point to the hour at that place.

## PROBLEM VII.

*To find the Sun's Place in the Ecliptic, and his Declination, for any given Day in the Year.*

Look on the wooden horizon for the given day, and againſt it there is placed the degree of the ſign in which the ſun is on that day at noon. Find the ſame degree of this ſign in the ecliptic line upon the globe, and having brought it to the brazen meridian, obſerve what degree of the meridian ſtands over it; and that is the ſun's declination, reckoned from the equator.

## PROBLEM VIII.

*To find all thoſe Places in the north Frigid Zone where the Sun begins to ſhine conſtantly, without ſetting, on any given Day: which muſt be between the 21ſt of March and [illegible] of September.* (See Fig. 3, *Plate* [illegible]

Ha[illegible] [illegible]en day to the br[illegible] Problem

VII.) count as many degrees on the meridian from the north pole as are equal to the ſun's declination, and mark that degree from the pole on the meridian; then turning the globe round on its axis, obſerve what places in the north frigid zone paſs directly under that mark; for they are the places required.

The ſame problem may be reſolved for places within the ſouth frigid zone, for the other half of the year.

## PROBLEM IX.

*To find the Place over which the Sun is vertical at any Hour of a given Day.*

Find the ſun's declination for the given day, (by Problem VII.) which mark on the brazen meridian; then bring the place where you are, (ſuppoſe London,) to the brazen meridian, and ſet the index to the given hour; then turn the globe on its axis until the index point to 12 at noon; and the place on the globe which is directly under the point of the ſun's declination, marked upon the meridian, has the ſun that moment in the zenith, or directly vertical.

*Note.* The hour 12 at noon on the hour circle is the uppermoſt 12.

## PROBLEM X.

*Having the Day and Hour of a Lunar Eclipſe, to find all thoſe Places of the Earth to which it will be viſible.*

When the moon is eclipſed, ſhe is always at the full, and, conſequently, oppoſite to the ſun; therefore, whatever part of the earth the ſun is vertical to, the moon muſt be vertical to the antipodes of that part; conſequently, the ſun will be viſible then to one half the earth, and the moon to the other half.

Therefore, find the place to which the ſun is vertical at the given hour, (by Problem IX.) elevate the pole to the latitude

latitude of that place, and bring the place to the upper part of the brazen meridian: then as the ſun will be viſible to all thoſe parts of the globe which are above the horizon, the moon will be viſible to all thoſe parts below it, at the middle of the eclipſe.

## PROBLEM XI.

### *To rectify the Globe for the Latitude, the Zenith, and the Sun's Place.*

Find the latitude of the place, (by Problem I.) and if the place be in north latitude, raiſe the north pole as many degrees above the horizon, counting upon the meridian from the north pole to the horizon; but if the place be in ſouth latitude, raiſe the ſouth pole as many degrees: then turn the globe till the place comes under its latitude on the brazen meridian, and faſten the quadrant of altitude to the meridian, ſo that the chamfered edge of its nut may be joined to the zenith. Then bring the ſun's place in the ecliptic for the given day, to the graduated ſide of the brazen meridian, and ſet the hour index to 12 at noon, and the globe will be rectified.

## PROBLEM XII.

### *Having the Latitude of any Place between the two Polar Circles, to find the Time of the Sun's Riſing and Setting, or the Length of the Day and Night, for any given Day in the Year.*

Rectify the globe for the latitude, and the ſun's place, by the foregoing problem; then bring the ſun's place in the ecliptic to the eaſtern ſide of the horizon, and the hour index will ſhow the time of ſun-riſing; then turn the globe on its axis, until the ſun's place comes to the weſtern ſide of the horizon, and the index will then ſhow the time of ſun-ſetting.

The hour of ſun-ſetting being doubled, gives the length of the day; and the hour of the ſun-riſing doubled, gives the length of the night.

## PROBLEM XIII.

*Having the Latitude of a Place, and the Day of the Month, to find when the Morning Twilight begins, and Evening Twilight ends.*

Rectify the globe, and bring the ſun's place in the ecliptic to the eaſtern ſide of the horizon; then mark that point of the ecliptic which is in the weſtern ſide of the horizon, which is the point oppoſite to the ſun's place; and lay the quadrant of altitude over the ſaid point, and turn the globe eaſtward, keeping the quadrant at the ſame mark, until the ſaid point on the ecliptic is 18° high on the quadrant, and the index will point out the time when the morning twilight begins; for the ſun's place will be 18° below the eaſtern ſide of the horizon. Then, to find the time when the evening twilight ends, bring the ſun's place to the weſtern ſide of the horizon, and the point oppoſite to it, which was marked, will be riſing in the eaſt; bring the quadrant over that point, and keeping it thereon, turn the globe weſtward, until the ſaid point be 18° above the horizon on the quadrant, and the index will ſhow the time when the evening twilight ends; as the ſun's place will be then 18° below the weſtern ſide of the horizon.

When the ſun does not go 18° below the horizon of any place, the twilight continues the whole night in that place: and between 49° of latitude, and the polar circles, the twilight continues for ſeveral nights together, in the ſummer ſeaſon: and the nearer the place is to the polar circle, the greater is the number of theſe nights.

PROBLEM

## PROBLEM XIV.

*To find what Day of the Year the Sun begins to ſhine conſtantly without ſetting, on any given Place in the Frigid Zone, and how long it continues to do ſo.*

Rectify the globe for the latitude of the place, and turn it about till ſome point of the ecliptic, between *Aries* and *Cancer*, (if the given place be in the north frigid zone,) coincides with the north point of the horizon, where the brazen meridian cuts it. Then find on the wooden horizon, what day of the year the ſun is in that point of the ecliptic; for that is the day on which the ſun begins to ſhine conſtantly on the given place, without ſetting. Then turn the globe until ſome point of the ecliptic, between *Cancer* and *Libra*, coincide with the north point of the horizon, where the brazen meridian cuts it; and find on the wooden horizon on what day the ſun is in that point of the ecliptic; which is the day the ſun leaves off conſtantly ſhining on the ſaid place, and riſes and ſets to it, as to other places on the globe. The number of natural days, or complete revolutions of the ſun about the earth, between the two days above found, is the time that the ſun keeps conſtantly above the horizon without ſetting; for all that portion of the ecliptic which lies between the two points which interſect the horizon in the very north, never ſets below it; and there is juſt as much in the oppoſite part of the ecliptic that never riſes; therefore, the ſun will keep as long conſtantly below the horizon of every place upon the globe in winter, as he is above it in ſummer.

## PROBLEM XV.

*Having the Latitude, the Sun's Place, and his Altitude, to find the Hour of the Day, and the Sun's Azimuth, or Number of Degrees that he is diſtant from the Meridian.*

Having rectified the globe, and brought the ſun's place to the given height upon the quadrant of altitude, which muſt be

be on the eaftern fide of the horizon, if the time be in the forenoon; and on the weftern fide if it be afternoon; then the index will fhow the hour of the day; and the number of degrees in the horizon intercepted between the quadrant of altitude and the fouth point, will be the fun's true azimuth at that time.

## PROBLEM XVI.

### *To find what Hour of the Day it is, in any Part of the World.*

Rectify the globe for the latitude of the place; and having fet the index to the hour of the day, turn the globe, and bring the places of which the hour is required, fucceffively to the brazen meridian, and the index will point to the feveral hours. For example: if the place be London, and the hour 12 at noon, the globe being rectified for London, and London brought to the meridian, and the index fet to the hour 12, turn the globe, till Naples comes to the brazen meridian, and the index then will point to the hour 1; Naples being 15° eaftward of London. Then continue to turn the globe 15° further, and Peterfburgh, Conftantinople, and Grand Cairo, will come under the brazen meridian, or very near it, then the index will point to the hour 2; thefe three cities having the noon-day fun about two hours before us in London. And turning the globe 15° further, the index will point to the hour 3; and all the places under the meridian will have the fun vertical to them. And thus for every 15° of longitude eaftward, the inhabitants of thofe places have the fun an hour fooner. On the contrary, all the inhabitants fituated to the weftward of London, have the fun later in the fame proportion; that is, an hour later for every 15° of weftern longitude.

Moft of the foregoing Problems may be refolved by a map as well as a globe, though the operation may be fomewhat

more

more tedious, particularly by Plate 10, where the two hemiſpheres of the world repreſent the ſurface of a terreſtrial globe in plano.

## WINDS.

Winds are generally divided into two parts, according to the different parts of the earth on which they blow; being the tropical winds, or thoſe which blow between the two tropics, and thoſe which blow without the tropics.

The tropical winds generally extend to 30° on each ſide the equator, and are of three kinds: 1. The *general trade-winds.* 2. The *monſoons.* 3. The *ſea and land breezes.*

1. The trade-winds blow from north-eaſt on the north ſide of the equator, and from the ſouth-eaſt on the ſouth ſide of the equator, and near the equator almoſt due eaſt; but under the equator, and from 2 to 5° on each ſide of the equator, the winds are variable; and ſometimes it is calm for a month together.

2. The monſoons are periodical winds, which blow ſix months in one direction, and the other ſix months in the oppoſite direction. At the change or ſhifting of the monſoons, are generally violent ſtorms of wind, thunder, lightning, and rain, which always happen about the equinoxes. The monſoons extend about two hundred leagues from land, and are chiefly in the Indian ſeas.

3. The ſea and land breezes are alſo periodical winds, which blow from the land in the night, and beſt part of the morning; and from the ſea from ten in the forenoon till ſix in the evening. Theſe do not extend above two or three leagues from ſhore.

Near the coaſt of Guinea, in Africa, the wind blows almoſt conſtantly from the weſt.

On the coaſt of Peru, in South America, the wind blows conſtantly from the ſouth-weſt.

Between the third and tenth degree of fouth latitude, the fouth-eaft trade-wind continues from April to October; but during the reft of the year, the wind blows from the north-weft. Between Sumatra and New Holland this wind blows from the fouth from March to September; but from September to April blows in the oppofite direction. Between Africa and Madagafcar its direction is influenced by the coaft; for it blows from the north-eaft from October to April; and from the fouth-weft the reft of the year.

In the Indian ocean, to the northward of the third degree of fouth latitude, the north-eaft wind blows from October to April, and the oppofite wind the reft of the year. This wind blows nearly from the fouth in the fummer months, from the ifle of Borneo, along the coaft of Malacca, as far as China; and in the winter months it blows from the north by eaft.

In temperate zones the winds are very irregular, and no certain rule can be formed of their changes. But when winds are violent and continue long, it is generally found that they extend over a large tract of country; particularly if they blow from the north or eaft. By the multiplication and comparifons of meteorological tables, the following theorems have been deduced.

In Virginia, the prevailing winds are between the fouth-weft, weft, north, and north-weft; but the moft frequent is the fouth-weft. At Ipfwich, in New England, the prevailing winds are the fame, but the moft frequent is the north-weft. At Cambridge, in the fame province, the moft frequent is the fouth-eaft. The predominant winds of New York are the north and weft. And in Nova Scotia, north-weft. And at Hudfon's Bay, weft.

It appears from thefe obfervations, that the wefterly winds are the moft frequent over the whole eaftern coafts of North America; but in the fouthern provinces the fouth-weft wind is predominant; and the north-weft wind becomes gradually more frequent as we approach the frigid zone.

In

In Egypt, from May to September, inclufive, the wind blows almoft conftantly from the north, varying in a few points from eaft to weft, in the months of June and July.

In the Mediterranean fea the wind blows nearly nine months of the year from the north; and at the equinoxes there is always an eafterly wind in that fea. But in the ftraits of Gibraltar the winds are either from eaft or weft.

In Italy the prevailing winds differ confiderably, according to the fituation of the places: at Rome and Padua they are northerly; at Milan eafterly.

The prevailing wind in Spain and Portugal is the weft; particularly on the weftern coafts of thefe countries; but at Madrid it is north-eaft.

In France, along the whole fouth coaft of that country, the wind blows moft frequently from the north, or north-eaft and north-weft. On the weftern coaft of the Netherlands, as far north as Rotterdam, the prevailing wind is the fouth-weft.

From the regifter, kept for the fpace of ten years, by order of the Royal Society at London, the average of the winds at that place, blow in the following order:—

| Winds. | Days. | Winds. | Days. |
|---|---|---|---|
| South-weft | 112 | South-eaft | 32 |
| North-eaft | 58 | Eaft | 26 |
| North-weft | 50 | South | 18 |
| Weft | 53 | North | 16 |

It appears from this regifter, that the fouth-weft wind blows at an average more frequently than any other winds, during every month of the year, but particularly in July and Auguft; and the north-eaft blows moft conftantly during the months January, March, April, May, and June, and moft feldom during February, July, September, and December; and the north-weft blows more frequently from November to March; and more feldom in September and October than in any other months.

### *Tides.*

The tide is that riſe and fall of the water obſerved on all maritime coaſts.

It is obſervable, that on the ſhores of the ocean, and in all bays, creeks, harbours, &c. which have a free communication with the ocean, the waters riſe up above a certain mean rate twice a day, and as often ſink below: this is what is called the *flood* and *ebb*; or an high and low water. The whole interval between high and low water is called a *tide*: the water is ſaid to *flow* and to *ebb*; and the riſing is called the *flood tide*, and the falling the *ebb tide*.

This riſe and fall of the waters is very variable in quantity. Thus, at Plymouth it is ſometimes twenty-one feet between the greateſt and leaſt depth of the water in one day, or between high and low water; and ſometimes it is only twelve feet.

The greateſt flow of tide in any place is called a *ſpring tide*, and the leaſt flow is called a *neap tide*; and the different heights of the tide gradually increaſe every day from a neap to a ſpring tide; and then gradually decreaſe from a ſpring to a neap tide.

The whole time between the ſpring and neap tide is about fifteen days; and two of theſe intervals will make an exact lunation, or change of the moon. For the ſpring tide is obſerved to happen at a certain interval of time (generally between two and three days) after the new or full moon; and a neap tide at a certain interval after the half moon. Thus, the high water happens at new and full moon, when the moon has a certain determined poſition with reſpect to the meridian of the place of obſervation, preceding or following the moon's ſouthing a certain interval of time; which is always conſtant with reſpect to that place, but very different in different places.

2 The

The interval between two ſucceeding high waters is very variable. It is leaſt of all about new and full moon, and greateſt when the moon is at her quadratures. As two high waters happen every day, we may call the double of their interval, a tide day. Now, this tide day is ſhorteſt about new and full moon, being then about 24 hours and 37 minutes; but longeſt at the moon's quadratures, being then 25 hours and 27 minutes.

The tides, being in ſimilar circumſtances, are greateſt when the moon is at her leaſt diſtance from the earth; and leaſt, when ſhe is at her greateſt diſtance from the earth.

The ſame may be remarked with reſpect to the ſun's diſtance. Thus, the greateſt tides are obſerved during the winter months in Europe, or when the ſun is at his leaſt diſtance.

The tides in every part of the ocean increaſe as the moon, by changing her declination, approaches the zenith of that place.

The tides which happen while the moon is above the horizon are greater than thoſe of the ſame day, when the moon is below the horizon.

Theſe are all the regular phenomena of the tides. They are of the utmoſt importance to all commercial nations, and have therefore been much attended to by all navigators and aſtronomers.

## SECT. III.

### THE GRAND DIVISIONS OF THE EARTH.

The principal divifions of the earth, as before mentioned, are, into land and water.

The land is divided into two great continents, befides iflands: viz. The eaftern and weftern continents. The eaftern continent is fubdivided into the following parts: viz. Europe on the north-weft, Afia on the north-eaft, and Africa on the fouth, being joined to Afia by the Ifthmus of Suez, which is 60 miles over. The weftern continent confifts of North and South America, joined by the Ifthmus of Darien, between 60 and 70 miles in breadth.

Europe is again fubdivided into the following principal parts, and is fituated between the tenth degree weft longitude and the fixth-fifth degree eaft longitude, and between the thirty-fixth and feventy-fecond degree of north latitude. It is bounded on the north by the Frozen Ocean, on the eaft by Afia, on the fouth by the Mediterranean Sea, which divides it from Africa; and on the weft by the Atlantic Ocean, which feparates it from America; being 3000 miles in length, from Cape St. Vincent in the weft to the mouth of the river Oby in the north-eaft; and 2500 miles broad from north to fouth, from North Cape in Norway to Cape Cayha in the Morea, the moft fouthern point of Europe. It contains the following ftates and kingdoms:

A TABLE

## A TABLE OF EUROPE.

| *Nations.* | *Length.* | *Breadth* | *Chief Cities.* | *North Latitude.* | | | *Longitude from Greenwich.* | | | *Difference of Time from Greenwich.* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | D. | M. | S. | D. | M. | S. | H. | M. | S. |
| England | 360 | 300 | London | 51 | 31 | 0 | 0 | 5 | 37W | 0 | 0 | 22½aft |
| Scotland | 300 | 150 | Edinburgh | 55 | 57 | 57 | 3 | 12 | 15W | 0 | 12 | 49 aft. |
| Ireland | 285 | 160 | Dublin | 53 | 21 | 12 | 6 | 6 | 30W | 0 | 24 | 26 aft. |
| Norway | 1000 | 300 | Bergen | 60 | 11 | 0 | 5 | 45 | 0E | 0 | 24 | 0bef. |
| Denmark | 240 | 180 | Copenhagen | 55 | 40 | 45 | 12 | 35 | 15E | 0 | 50 | 21bef. |
| Sweden | 800 | 500 | Stockholm | 59 | 20 | 35 | 18 | 3 | 55E | 1 | 12 | 16bef. |
| Ruſſia | 1500 | 1100 | Peterſburgh | 59 | 56 | 0 | 30 | 19 | 15E | 2 | 1 | 17bef. |
| Poland | 700 | 680 | Warſaw | 52 | 14 | 0 | 21 | 0 | 30E | 1 | 24 | 2bef. |
| Pruſſia | 609 | 350 | Berlin | 52 | 32 | 30 | 13 | 26 | 15E | 0 | 53 | 45bef. |
| Germany | 600 | 500 | Vienna | 48 | 12 | 40 | 16 | 22 | 30E | 1 | 5 | 30bef. |
| Bohemia | 300 | 250 | Prague | 50 | 4 | 30 | 14 | 45 | 0E | 0 | 59 | 0bef. |
| Holland | 150 | 100 | Amſterdam | 52 | 22 | 45 | 4 | 45 | 30E | 0 | 19 | 2bef. |
| Flanders | 200 | 200 | Bruſſels | 50 | 51 | 0 | 4 | 21 | 45E | 0 | 17 | 27bef. |
| France | 600 | 500 | Paris | 48 | 50 | 14 | 2 | 20 | 0E | 0 | 9 | 20bef. |
| Spain | 700 | 500 | Madrid | 40 | 25 | 0 | 3 | 25 | 45W | 0 | 13 | 43 aft. |
| Portugal | 300 | 100 | Liſbon | 38 | 42 | 25 | 9 | 9 | 59W | 0 | 36 | 40 aft. |
| Swiſſerland | 260 | 100 | Bern | 40 | 0 | 0 | 7 | 40 | 0E | 0 | 28 | 0bef. |
| Italy | 750 | 400 | Rome | 41 | 53 | 54 | 12 | 29 | 15E | 0 | 49 | 50bef. |
| Hungary | 300 | 200 | Buda | 47 | 20 | 0 | 19 | 22 | 0E | 1 | 17 | 0bef. |
| Turkey in Europe | 1400 | 730 | Conſtantinople | 41 | 0 | 24 | 28 | 53 | 49E | 1 | 55 | 35bef. |

Beſides the foregoing ſtates, Europe contains ſeveral iſlands, of which the following are the principal.

Iſlands.

| | Islands. | Chief Towns. | Subject to |
|---|---|---|---|
| In the Northern Ocean | Iceland | Skalholt | Denmark |
| In the Baltic Sea | Zealand, Funen, Alfen, Falfter, Langland, Laland, Femeren, Mona, Bornholm | – | Denmark |
| | Gothland, Aland, Rugen | – | Sweden |
| | Ofel Dagho | – | Ruffia |
| | Ufedom, Wollin | – | Pruffia |
| Mediterranean Sea | Ivica | Ivica | Spain |
| | Majorca | Majorca | Spain |
| | Minorca | PortMahon | Spain |
| | Corfica | Baftia | France |
| | Sardinia | Cagliari | K. Sardinia |
| | Sicily | Palermo, | K. of 2 Sici. |
| Gulf of Venice | Lufiena, Corfu, Cephalonia, Zant, Leucadia | – | Venice |
| Archipelago & Levant Seas | Candia, Rhodes, Negropont, Lemnos, Tenedos, Scyros, Mytelene, Scio, Samos, Patmos, Paros, Cerigo, Santorin, &c. being part of Ancient and Modern Greece | – | Turkey |

Afia is fituated between 25 and 180 degrees of eaft longitude, and between the equator and 80 degrees north latitude; being about 4740 miles in length, from the Dardanelles on the weft to the eaftern fhore of Tartary; and about 4380 in breadth, from the moft fouthern part of Malacca to the moft northern cape of Nova Zembla. It is bounded by the Frozen Ocean on the north, on the weft it is feparated from Africa by the Red Sea, and from Europe by the Levant or Mediterranean, the Archipelago, the Hellefpont, the Sea of Marmora, the Bofphorus, the Black Sea, the river Don, and a line drawn from it to the river Tobel, and

and from thence to the river Oby, which falls into the Frozen Ocean; on the eaſt it is bounded by the Pacific Ocean or South Sea, which ſeparates it from America; and on the ſouth, by the Indian Ocean: thus, it is almoſt ſurrounded by the ſea. The principal diviſions are as follow:

## A TABLE OF ASIA.

| *Nations.* | *Length.* | *Breadth.* | *Chief Cities.* | *North Latitude.* | | | *Eaſt Longitude from Greenwich.* | | | *Difference of Time from Greenwich.* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | D. | M. | S. | D. | M. | S. | H. | M. | S. |
| Tartary. Independent | *Boundaries* | | Samarcand | 39 | 50 | 0 | 69 | 0 | 0 | 4 | 36 | 0bef. |
| Tartary. Mogulean | *of theſe* | | Tibet | 37 | 0 | 0 | 8 | 0 | 0 | 5 | 40 | 0bef. |
| Tartary. Chineſe | *countries* | | Chynian | 48 | 0 | 0 | 12 | 0 | 0 | 8 | 4 | 0bef. |
| Tartary. Ruſſian | *variable.* | | Tobolſki | 58 | 12 | 18 | 68 | 12 | 45 | 4 | 38 | 41bef. |
| Perſia | 1300 | 1100 | Iſpahan | 32 | 25 | 0 | 52 | 40 | 0 | 3 | 31 | 20bef. |
| India | 2000 | 1000 | Siam or Pegu | 14 | 18 | 0 | 100 | 50 | 0 | 6 | 43 | 20bef. |
| Empire of Mogul | 2000 | 1500 | Delhi | 28 | 20 | 0 | 79 | 25 | 0 | 5 | 16 | 0bef. |
| China | 1440 | 1260 | Pekin | 39 | 54 | 30 | 116 | 14 | 15 | 7 | 45 | 37bef. |
| Georgia | 210 | 140 | Teflis | 43 | 0 | 0 | 46 | 15 | 0 | 3 | 10 | 0bef. |
| Turcomania | 360 | 300 | Erzerum | 39 | 56 | 35 | 48 | 35 | 45 | 3 | 14 | 23bef. |
| Diarbec or Meſopotamia | 560 | 310 | Bagdat | 33 | 20 | 0 | 43 | 46 | 30 | 2 | 55 | 6bef. |
| Natolia | 750 | 388 | Burſa or Smyrna | 38 | 28 | 7 | 35 | 20 | 0 | 2 | 21 | 20bef. |
| Paleſtine | 210 | 90 | Jeruſalem | 31 | 55 | 8 | 27 | 19 | 45 | 1 | 49 | 15bef. |
| Syria | 270 | 160 | Aleppo | 35 | 45 | 23 | 37 | 20 | 0 | 2 | 29 | 20bef. |
| Part of Arabia | 1300 | 1200 | Mecca. | 21 | 45 | 0 | 40 | 55 | 0 | 2 | 52 | 0bef. |

*Asiatic Islands in the Indian and Pacific Ocean.*

| *Islands.* | *Towns.* | *Belonging to* |
|---|---|---|
| New Holland | Sydney Cove | English |
| The Japanese Isles | Jeddo and Meaco | Dutch |
| The Ladrones | Guam | Spain |
| Formosa | Tai-ouan-fou | China |
| Anian | Kiontcheow | China |
| The Philippines | Manilla | Spain |
| The Molucca or Clove Isles | Victoria Fort | Dutch |
| The Banda or Nutmeg Isles | Lantor | Dutch |
| Amboyna | Amboyna | Dutch |
| Celebes | Macasser | Dutch |
| Gilolo | Gilolo | Dutch |
| The Sunda Isles { Borneo, | Borneo Caytongee | Several Nations |
| Sumatra, | Achen & Bencoolen | English & Dutch |
| Java, &c. | Batavia and Bantam | Dutch |
| The Andaman and Nicobar Isles | Andaman, Nicobar | Several Nations |
| Ceylon | Candia | English |
| The Maldives | Caridon | English |
| Bombay | Bombay | English |
| The Kurile Isles in the Sea of Kamschatka, discovered by the Russians | – | Russia |
| New Guinea, New Britain, New Ireland, New Hebrides, New Caledonia, New Zealand, and the Friendly, Sandwich, and Society Islands, are uncolonized. | | |

Africa, the third grand division of the globe, is generally represented as bearing some resemblance to the form of a pyramid, whose vertex or point is the Cape of Good Hope, and its base the shores of the Mediterranean Sea. It is a peninsula of great extent, joined to Asia by the Isthmus of Suez: its greatest length from north to south, from Cape Bona in the Mediterranean, to the Cape of Good Hope, is 4600 miles; and the breadth, from Cape Verd to Cape Guardafui, is 3500 miles. It is bounded on the north by the Mediterranean Sea, which separates it from Europe; on the east by the Isthmus

Iſthmus of Suez, the Red Sea, and the Indian Ocean, which divides it from Aſia; on the ſouth by the Southern Ocean; and on the weſt by the great Atlantic Ocean, which ſeparates it from America.

Very few travellers have penetrated into the interior part of this quarter of the world; conſequently we ſtill remain ignorant of the bounds, and even of the names of many of the inland parts; but, according to the beſt accounts, it is divided according to the following table:

| *Nations.* | *Length.* | *Breadth* | *Chief Cities.* | *Latitude.* | *Longitude from Greenwich.* | *Difference of Time from Greenwich.* |
|---|---|---|---|---|---|---|
| | | | | D. M. S. | D. M. S. | H. M. |
| Barbary. Morocco, Tafilet, &c. | 500 | 480 | Fez | 33 40 0N | 6 0 0W | 0 24 aft. |
| Barbary. Algiers | 600 | 400 | Algiers | 36 49 0N | 2 12 0E | 0 9 bef. |
| Barbary. Tunis | 400 | 250 | Tunis | 36 40 0N | 10 0 0E | 0 39 bef. |
| Barbary. Tripoli | 400 | 300 | Tripoli | 32 50 0N | 21 30 0E | 1 26 bef. |
| Barbary. Barca | 700 | 240 | Polemeta | 32 53 0N | 13 5 0E | 0 52 bef. |
| Egypt | 600 | 250 | Grand Cairo | 30 2 0N | 31 18 0E | 2 5 bef. |
| Biledulgerid | 2500 | 350 | Dara | – | 8 0 0W | 0 32 aft. |
| Zaara | 2400 | 660 | Tegeſa | 21 40 0N | 6 0 0W | 0 24 aft. |
| Negroland | 2200 | 840 | Madinga | – | – | 0 38 aft. |
| Guinea | 1800 | 360 | Benin | 7 40 0N | 5 4 0E | 0 20 bef. |
| Upper Ethiop. Nubia | 940 | 600 | Nubia | 17 0 0N | 33 0 0E | 2 12 bef. |
| Upper Ethiop. Abyſſinia | 900 | 800 | Gondar | 13 10 0N | 35 0 0E | 2 20 bef. |
| Upper Ethiop. Abex | 540 | 130 | Doncala | 15 6 0N | 39 0 0E | 2 36 bef. |

The middle parts, called Lower Ethiopa, are very little known to Europeans, but are computed at one million two hundred thouſand miles.

| | | | | | | |
|---|---|---|---|---|---|---|
| Loango | 510 | 300 | Loango | 5 0 0N | 11 0 0E | 0 44 bef. |
| Congo | 640 | 420 | St. Salvador | 5 0 0S | 15 0 0E | 1 1 bef. |
| Angola | 460 | 250 | Loando | 8 30 0S | 14 30 0E | 0 58 bef. |
| Benguela | 430 | 180 | Benguela | 11 0 0S | 14 30 0E | 0 58 bef. |
| Mataman | 450 | 240 | No towns | | | |
| Ajan | 900 | 300 | Brava | 1 0 0N | 45 0 0E | 3 0 bef. |
| Zanguebar | 1400 | 350 | Mozambique | 15 0 0S | 40 0 0E | 2 40 bef. |
| Monomotapa | 960 | 660 | Monomotape | – | – | 1 18 bef. |
| Monemugi | 900 | 660 | Chicova | – | – | 1 44 bef. |
| Sofola | 480 | 300 | Sofola | 20 0 0S | 36 40 0E | 2 26 bef. |
| Terra de Natal | 600 | 330 | No town | | | |
| Caffraria, or Hottentots | 708 | 660 | Cape of G. Hope | 33 55 0S | 18 23 0E | 1 13 bef. |

| *Islands.* | *Towns.* | *Belong to* |
|---|---|---|
| Babel Mandel | Babel Mandel | |
| Zocotra in the Indian Ocean | Calanfia | |
| The Comora Ifles, ditto | Joanna | |
| Madagafcar, ditto | St. Auftin | |
| Mauritius, ditto | Mauritius | French. |
| Bourbon, ditto | Bourbon | Ditto. |
| St. Helena, in the Atlan. Ocean | St. Helena | Englifh. |
| Afcenfion Ifle, Ditto | — | Uninhabited. |
| St. Thomas | St. Thomas | |
| St. Matthew, ditto | — | Uninhabited. |
| Anaboa, Princefs Iflands / Fernandogo, ditto | Anaboa | Portuguefe. |
| Cape Verd Iflands, ditto | St. Domingo | Portuguefe. |
| Goree, ditto | Fort St. Michael | French. |
| Canaries, ditto | Palma, St. Chriftopher | Spain. |
| Madeiras, ditto | Santa Cruz | Portuguefe. |
| The Azores, or Weftern Ifles, which are at an equal diftance from Europe, Africa, and America, ditto | Angra, St. Michael | Portuguefe. |

America, the great weftern continent, called the New World, runs north and fouth through every habitable climate upon the earth; extending from the eightieth degree of north latitude to the fifty-fixth degree of fouth latitude; and its breadth, where it is known, extends from the thirty-fifth to the one hundred and thirty-fixth degree of weft longitude from London; being near 9000 miles in length, and 3690 in breadth. Extending into both the hemifpheres, it has confequently two fummers and two winters, and has all the variety of climates to be met with on the face of the earth. On the eaft it is bounded by the great Atlantic Ocean, which divides it from the eaftern continent. On the weft it has the Pacific Ocean, or great South Sea, which feparates it from Afia. It is compofed of two parts, called North and South America, joined together by a narrow neck of land, called the

the Iſthmus of Darien, in the kingdom of Mexico, 1500 miles long, and at one part being only 60 miles in breadth; ſo that to effect a communication between the two oceans is by no means difficult. In the great gulf, which is formed between the iſthmus and the northern and ſouthern continents, lie a multitude of iſlands, denominated the Weſt Indies, in contradiſtinction to the iſlands of Aſia, beyond the Cape of Good Hope, which are called the Eaſt Indies. The grand diviſions of North America are as follows:

## *North America.*

| *Countries.* | *Length.* | *Breadth.* | *Chief Towns.* | *Latitude.* | *Longitude from Greenwich* | *Difference of Time from Greenwich.* |
|---|---|---|---|---|---|---|
| New Britain | *Not known.* | | | D. M. S. | D. M. S. | H. M. S. |
| Canada | 800 | 200 | Quebec | 46 55 0 N | 69 53 0 W | 4 39 30 aft. |
| Nova Scotia | 350 | 250 | Halifax | 44 45 0 N | 64 30 0 W | 4 18 0 aft. |
| United States | 1390 | 700 | Philadelphia | 39 57 0 N | 75 8 0 W | 5 0 0 aft. |
| Eaſt Florida / Weſt Florida | 500 | 440 | St. Auguſtine | 30 8 0 N | 81 10 0 W | 5 25 0 aft. |
| | | | Penſacola | 30 32 0 N | 87 20 0 W | 5 49 0 aft. |
| Louiſiana | *Not known.* | | New Orleans | 30 0 0 N | 87 5 0 W | 5 48 0 aft. |
| N. Mexico and California | 2000 | 1600 | Santa Fee / St. Juan | 35 32 0 N | 105 0 0 W | 7 0 0 aft. |
| Mexico, or New Spain | 2000 | 600 | Mexico | 19 54 0 N | 100 5 0 W | 6 40 0 aft. |

## *South America.*

| *Countries.* | *Length.* | *Breadth.* | *Chief Towns.* | *Latitude.* | *Longitude from Greenwich* | *Difference of Time from Greenwich.* |
|---|---|---|---|---|---|---|
| Terra Firma | 1400 | 700 | Panama | 8 48 0 N | 80 28 0 W | 5 22 0 aft. |
| Peru | 1800 | 500 | Lima | 12 1 0 S | 76 49 0 W | 5 7 0 aft. |
| Amazonia | 1200 | 960 | *Little known.* | | | |
| Guiana | 780 | 480 | Surinam | 6 0 0 N | 55 30 0 W | 3 42 0 aft. |
| | | | Cayenne | 4 56 0 N | 52 15 0 W | 3 29 0 aft. |
| Brazil | 2500 | 700 | St. Sebaſtian | 22 59 0 S | 44 16 0 W | 2 57 0 aft. |
| | | | St. Salvadore | 12 0 0 S | 38 0 0 W | 2 32 0 aft. |
| Paraguay, or La Plata | 1500 | 1000 | Aſſumption | 34 10 0 S | 60 40 0 W | 4 3 0 aft. |
| | | | and Buenos Ayres | 34 35 0 S | 58 31 0 W | 3 54 0 aft. |
| Chili | 1200 | 500 | St. Jago | 33 40 0 S | 77 0 0 W | 5 8 0 aft. |
| Terra Magellanica, or Patagonia | 700 | 300 | Uncoloniſed by Europeans. | | | |

Grand

## *Grand Divisions of South America.*

| Nations. | Length. | Breadth | Chief Towns. | Distances & Bearing from London. | Belongs to |
|---|---|---|---|---|---|
| Terra Firma | 1400 | 700 | Panama | 4650 S. W. | Spain. |
| Peru | 1800 | 600 | Lima | 5520 S. W. | Ditto. |
| Amazonia | 1200 | 960 | | | |
| Guiana | 780 | 680 | Surinam<br>Cayenne | 3840 S. W. | Dutch.<br>French. |
| Brazil | 2500 | 700 | St. Sebastian | 6000 S. W. | Portugal. |
| La Plata | 1500 | 1000 | Buenos Ayres | 6040 S. W. | Spain. |
| Chili | 1200 | 500 | St. Jago | 6600 S. W. | Ditto. |
| Terra Magellanica, or Patagonia | 1400 | 460 | | | |

The

*The principal Iſlands in North America belonging to the Europeans.*

| | *Iſlands.* | *Length.* | *Breadth* | *Chief Towns.* | *Belongs to* |
|---|---|---|---|---|---|
| In the Gulf of St. Lawrence. | Newfoundland | 350 | 200 | Placentia | England |
| | Cape Breton | 110 | 80 | Louiſburg | Ditto |
| | St. John | 60 | 30 | Charlotte | Ditto |
| In the Atlantic. | The Bermuda Iſles | 400 *in number* | – | St. George | Ditto |
| | The Bahama Iſles | – | – | Naſſau | Ditto |
| The Weſt India Iſlands, lying in the Atlantic, between N. and S. America. | Jamaica | 140 | 60 | Kingſton | Ditto |
| | Barbadoes | 21 | 14 | Bridgetown | Ditto |
| | St. Chriſtopher's | 20 | 7 | Baſſe Terre | Ditto |
| | Antigua | 20 | 20 | St. John's | Ditto |
| | Nevis | 6 | 3 | Charleſtown | Ditto |
| | Montſerrat | 5 | 4 | Plymouth | Ditto |
| | Barbuda | 20 | 12 | — | Ditto |
| | Anguilla | 30 | 10 | — | Ditto |
| | Dominica | 28 | 13 | — | Ditto |
| | St. Vincent | 24 | 18 | Kingſton | Ditto |
| | Granada | 30 | 15 | St. George | Ditto |
| | Cuba | 700 | 90 | Havannah | Spain |
| | Hiſpaniola | 450 | 150 | St. Domingo | France |
| | Porto Rico | 100 | 49 | Porto Rico | Spain |
| | Trinidad | 90 | 60 | St. Joſeph | England |
| | Margarita | 40 | 23 | — | Spain |
| | Martinico | 60 | 30 | St. Peter | France |
| | Guadaloupe | 45 | 38 | Baſſe Terre | Ditto |
| | St. Lucia | 23 | 12 | — | England |
| | Tobago | 32 | 9 | — | Ditto |
| | St. Bartholomew | – | – | — | Sweden |
| | Deſeada | – | – | — | France |
| | Marigalante | – | – | — | Ditto |
| | St. Euſtatia | 7 | 4 | The Bay | Dutch |
| | Curaſſou | 30 | 10 | — | Ditto |
| | St. Thomas | 5 | 3 | — | Denmark |
| | St. Croix | 30 | 10 | Baſſe End | Ditto |



I ſhall

I ſhall here ſubjoin a table of the ſuperficial content of the ſeveral parts of the globe in ſquare miles, accounting 60 miles to a degree on the equator.

| | *Square Miles.* | *Iſlands.* | *Square Miles.* | *Iſlands.* | S. M. |
|---|---|---|---|---|---|
| The Globe | 199,512,595 | Cuba | 38,400 | Funen | 768 |
| Seas and unknown Parts | 160,522,026 | Java | 38,250 | Yvica | 625 |
| | | Hiſpaniola | 36,000 | Minorca | 520 |
| The habitable World | 38,990,567 | Newfoundland | 35,500 | Rhodes | 480 |
| | | Ceylon | 27,730 | Cephalonia | 420 |
| Europe | 4,456,065 | Ireland | 27,457 | Amboyna | 400 |
| Aſia | 10,768,822 | Formoſa | 17,000 | (Orkney) | |
| Africa | 9,654,807 | Anian | 11,900 | Pomona | 324 |
| America | 14,110,874 | Gilolo | 10,400 | Scio | 300 |
| Perſian Emp. under Darius | 1,650,800 | Sicily | 9,400 | Martinico | 260 |
| | | Timor | 7,800 | Lemnos | 220 |
| Roman Emp. in its greateſt height | | Sardinia | 6,600 | Corfu | 154 |
| | 1,610,000 | Cyprus | 6,300 | Providence | 168 |
| | | Jamaica | 6,000 | Man | 160 |
| Ruſſian | 4,161,685 | Flores | 6,000 | Bornholm | 160 |
| Chineſe | 1,749,000 | Ceram | 5,400 | Wight | 150 |
| Great Mogul | 1,116,000 | Breton | 4,000 | Malta | 150 |
| Turkiſh | 950,057 | Socotra | 3,600 | Barbadoes | 140 |
| Preſent Perſia | 800,000 | Candia | 3,220 | Zant | 120 |
| ISLANDS. | — | Porto Rico | 3,200 | Antigua | 100 |
| Borneo | 228,000 | Corſica | 2,520 | St. Chriſtopher | |
| Madagaſcar | 168,000 | Zealand | 1,935 | | 80 |
| Sumatra | 129,000 | Majorca | 1,406 | St. Helena | 80 |
| Japan | 118,000 | St. Jago | 1,400 | Guernſey | 50 |
| Great Britain | 72,926 | Negropont | 1,300 | Jerſey | 43 |
| Celebes | 68,400 | Teneriff | 1,272 | Bermudas | 40 |
| Manilla | 58,500 | Gothland | 1,000 | Rhodes | 36 |
| Iceland | 46,000 | Madeira | 950 | | |
| Terra del Fuego | | St. Michael | 920 | | |
| | 42,075 | Skye | 900 | | |
| Mindanao | 39,200 | Lewis | 880 | | |

There are alſo ſeveral other conſiderable iſlands, chiefly in the South Seas, the exact dimenſions of which are not certainly known, but they may be ranged according to their magnitude in the following order: New Holland being nearly equal in ſize to the whole continent of Europe.

New

| | |
|---|---|
| New Holland. | Otaheite, or King George's Iſland. |
| New Guinea. | Friendly Iſlands. |
| New Zealand. | Marqueſas. |
| New Caledonia. | Eaſter, or Davis's Iſland. |
| New Hebrides | |

## SECT. IV.

### OF THE DIFFERENT GOVERNMENTS OF THE WORLD.

MANKIND were no ſooner united into civil ſocieties, than they diſcovered an inclination to oppreſs each other. That ſyſtem of equality, in which they were left by nature, gave the ſtrongeſt, and the moſt crafty, the advantage over his weaker and undeſigning neighbours. From hence aroſe the neceſſity of forming conjunctions of ſeveral individuals, or families together, who ſhould implicitly follow the dictates or commands of ſome choſen ſuperior, or leader. And, to prevent the altercations, ſtrife, and conſequently bloodſhed, that inevitably followed the nomination of every new leader, or prince, they cauſed the office to be made hereditary. Conſequently, abſolute and hereditary monarchy was the firſt original form of government; as appears from ſacred writ; where Nimrod is repreſented by his courage and dexterity to have acquired a ſuperiority of fame and power above his contemporaries; and he founded, at Babylon, the firſt monarchy whoſe origin is mentioned in hiſtory.

In the year 1496 before Chriſt, the Greeks were the firſt people who, by the advice and public-ſpirited endeavours of Cecrops, and Cranaus his ſucceſſor, formed a regular council. For Amphictyon, one of thoſe diſintereſted cha-

racters who live for the good of the community of which he is a member, endeavored to find an expedient to unite the feveral independent kingdoms of Greece into one body; and thus to put a ftop to thofe fatal confequences of inteftine divifion, and civil difcord, which rendered them a prey to each other, and an eafy conqueft to the invader. He, therefore, engaged the kings, or leaders, of twelve different cities, to unite together for their mutual fecurity and welfare. Two deputies from each city affembled twice a year at Thermopylæ, and formed the Amphictyonic Council. In this affembly the general interefts of the ftates were difcuffed. Amphictyon, in order to render thofe feveral connexions more durable, connected them with religious charge, intrufting the care of the temple at Delphi, with the riches that accrued to this place from thofe who confulted the oracles, to the care alfo of thefe deputies. This affembly was the firft political eftablifhment of a plurality of power, that we have any authentic account of in hiftory; and gave an energy of action to Greece, which enabled them to defend their liberty and independence againft the great force of the Perfians.

This was the firft deviation from abfolute monarchy, recorded in profane hiftory; from that time, various have been the modes and forms of government in different nations; though, if we except fome part of the Roman hiftory, Greece, and a few nations of lefs note, the monarchical form of government was the moft prevailing for the next two thoufand years.

Athens is an inftance of the pernicious effects of divifion in a ftate; and alfo difplays the benefits of unanimity. Thefeus, king of Attica, about the year before Chrift 1234, perceiving the danger to which his country was expofed by this twelvefold divifion, endeavoured to form a conjunction of the ftates; for this purpofe he detached the leaders of the different tribes as much as poffible from the people they governed; he abolifhed the different courts eftablifhed in different parts of

Attica

Attica, and appointed one council hall, common to all the Athenians. He eſtabliſhed a common form of religion, with certain religious ceremonies to be performed at Athens, the more effectually to ſtrengthen civil allegiance; and by inviting ſtrangers from all parts of the world, by the promiſe of privileges and protection, he raiſed the city to the higheſt pitch of fame and popularity. The ſplendour of Athens eclipſed that of all the other ſtates of Greece.

This monarchy ſoon gave place to an overbearing influence. Theſeus had formed his kingdom into three diſtinct claſſes; the nobles, the artizans, and the huſbandmen. And to prevent the increaſing power of the nobles, he granted many immunities and privileges to the two other claſſes. This ſyſtem of politics, in a few years, gave the two inferior claſſes an opportunity of acquiring conſiderable property: and, conſequently, they became important members of the ſtate; and, by their riches and independence, upon the death of Codrus, a prince of great merit, in the year B. C. 1070, they had power and influence enough to aboliſh the regal power, under pretence of finding no one worthy of filling the throne of Codrus, who had devoted himſelf to death for the ſafety of his people. Thus they proclaimed Jupiter king, declaring none elſe was fit to govern Athens. This was the firſt inſtance of a republican form of government in Greece.

From this period, ſo various have been the modes and forms of government, that it is impoſſible to diſtinguiſh them all. Governments are generally divided into three diſtinct forms, each of which has its partizans, viz. the monarchical, ariſtocratical, and democratical.

The monarchical form of government is, where a nation is governed by a king, or monarch; and is divided into two parts, called abſolute, and limited, monarchy. Abſolute monarchy is, where the ſovereign is entirely unreſtrained, having the legiſlative as well as the executive power. A limited

ed monarchy is, where the ſovereign is reſtrained by certain laws, beyond which he cannot paſs.

An ariſtocracy is, where the legiſlative and executive authority is veſted in the hands of a ſelect number of perſons, generally titled nobility; and in whom the office is moſtly hereditary.

A democracy is that government in which the legiſlative and executive authority is veſted in a certain number of individuals, who hold their office by election; and generally elected by the majority of the nation at large.

From the various modifications of theſe different forms of government, all the governments of the earth are formed; ſome approaching nearer to one, and ſome to another form. For there is hardly a government exiſting, that is entirely either an abſolute monarchy, a perfect ariſtocracy, or a complete republic.

## SECT. V.

### OF RELIGION.

Religion is coeval with the origin of mankind: without it the preſent order of the univerſe would be entirely overturned; and mankind, from their natural depravity, be rendered worſe than the moſt voracious of the brute creation.

The diſtinguiſhing religions in antiquity were Judaiſm, and Polytheiſm, or Paganiſm.

But in modern times the prevailing religions may be divided into the four following, viz. the Jewiſh, Chriſtian, Mahometan, and Pagan.

Before

Before treating of the four foregoing ſyſtems, it may be neceſſary to premiſe the following general axiom, viz. That all ſyſtems of religion contribute more or leſs to the welfare of ſociety. From hence we deduce the following theorem; that all religion muſt have ſomewhat in its origin of a divine nature, however it may be transformed, corrupted, or miſapplied, by the ignorance or artifice of its propagators.

In conſidering the Jewiſh code of religion, it does not appear as a complete ſyſtem of religion, adapted to all countries and ages, but ſeems particularly deſigned by the all-wiſe Creator, for the people to whom it was ſent; for the age they lived in, being over-run by idolatry; the circumſtances in which they had lived in Egypt, and the means by which they were to form their new ſettlement in the land of Canaan.

From hence they were enjoined the obſervation of the ſabbath, in honour to that Being who created the heavens and the earth, with all the hoſt of heaven; which hoſt, ſun, moon, ſtars, &c. were worſhipped by the Egyptians as *eternal beings*. To prevent their communication with the neighbouring idolatrous nations, they were proſcribed the uſe of certain animals for food, and permitted others; that, by being forbidden the uſe of thoſe animals for food, ſuch as the hog, &c. which the Gentile nations conſidered as the greateſt luxury, a perpetual bar might be kept up between the Jews and Gentiles. And by being permitted to eat other animals, ſuch as goats, ſheep, oxen, &c. which were worſhipped in Egypt, and from which the Egyptians religiouſly withheld all violence, the Jews would ſoon overcome any religious prejudices they might have acquired from the Egyptian idolatry.—The reſtitution of property, in the year of Jubilee, which would anſwer no purpoſe in another ſtate, was deſigned to preſerve the order of rank, and that diviſion of property, originally eſtabliſhed.

In condeſcenſion to their rude and groſs notions of Deity,

the Creator permitted them, in their wanderings through the Wildernefs, to have a tabernacle, or portable temple, in which he fometimes deigned to difplay fome rays of his glory.

From this general view of the Jewifh religion, it appears happily adapted to promote the welfare of its followers. In comparing it with other religions, it is neceffary to reflect on the peculiar purpofes for which it was eftablifhed; which were principally two; firft, to preferve the Jews a feparate people; and fecondly, to guard them from the idolatry with which they were every where furrounded. The religion of the Jews was not formed, nor defigned, to be propagated through all the earth; that would have been inconfiftent with the purpofes for which it was inftituted: therefore we fee the Jewifh religion, though near four thoufand years old, wants that effential attribute for propagation, to be found in all other religions, viz. a difference of fentiment, and, confequently, a divifion and fubdivifion into different fects.

The Chriftian religion is to be confidered as an improvement of the Jewifh. The effects of the Jewifh religion were indeed beneficial, but were confined almoft to them alone; whereas the effects of the Chriftian religion are extended to all mankind; reprefenting them with true philanthropy as children of the fame God, and heirs of the fame falvation. It levels all diftinctions of rich and poor, native and foreigner, as accidental and infignificant diftinctions with that impartial Being, who rewards or punifhes according to the demerits of his creatures.

The precepts of the Chriftian religion are more happily calculated to promote the happinefs of mankind, than thofe of any other religion. Its whole defign is to infpire mankind with mild, benevolent, and peaceable difpofitions. Its diftinguifhing rule, by which it excels all other religions, is, *to do unto others, as we would they fhould do unto us*; and fuch is its purity, that it does not allow an impure thought. It requires its followers to abandon their vices, however dear; and

to

to join the cautious wifdom of the ferpent with the innocent fimplicity of the dove. And to prevent perfeverance in immorality, it offers a pardon for the paft, provided the offender forfake his vicious practices. The practice and belief of the Gofpel have a peculiar tendency to raife the mind of man above the trifling purfuits of time; and to render its followers incorruptible by wealth, honour, or pleafures. It not only requires the Chriftian to abftain from injuring his neighbour, but even enjoins him to forgive any unmerited injuries which he himfelf fuffers, upon the principle of his being forgiven by his offended Creator. It reprefents the Deity and his attributes in the faireft light, fo as to render our ideas of him confiftent with the correct principles of reafon and philofophy. The rites of this gofpel are few and fimple; eafy to perform, expreffive, and edifying. It inculcates no duties, but what are founded in the principles of human nature, and on the relation on which man ftands to God, as his Creator, Redeemer, and Sanctifier. The affiftance of the Spirit of God is there promifed to thofe who labour to difcharge the duties which it enjoins. It teaches us that worldly afflictions are cafual accidents; incident to both bad and good men: *a doctrine highly encouraging to virtue, confoling in affliction, preventing defpair, and encouraging in difficulty.*

Such are the precepts and fpirit of the Chriftian religion. And even thofe who have refufed to give credit to its hiftory, and follow its doctrines, have acknowledged the excellency of its precepts. Bolingbroke, one of its moft zealous oppofers, fays, that "no religion ever yet appeared in the world, of which the natural tendency was fo much directed to promote the peace and happinefs of mankind, as the Chriftian; and that the Gofpel of Chrift is one continued leffon of the ftricteft morality, of juftice, benevolence, and univerfal charity." Thus we can pronounce, with confidence, that the precepts of a religion, which is fo happily formed to promote all that

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is juſt and beneficial to mankind, cannot but be in the higheſt degree divine. By reviewing the effects which it has produced, we ſhall be more confirmed in our aſſertion.

Chriſtianity has produced the moſt beneficial change in the circumſtances of domeſtic life. It has greatly contributed towards the abolition of ſlavery, and towards the mitigation of the rigours of ſervitude. We meet with no laws in Chriſtian countries ſo inhuman as thoſe practiſed at Rome; where maſters were allowed to remove their ſick or infirm ſlaves to an iſland in the Tiber, there to periſh without any aſſiſtance. The rigours of ſlavery are eaſed and aboliſhed; not by any particular precept of the Goſpel, but by the gentle and humane ſpirit which breathes through the general tenour of the whole ſyſtem. And though it may be objected, that a trade in ſlaves is at preſent carried on by people who preſume to call themſelves Chriſtians, and ſanctioned by the legiſlature of ſome Chriſtian ſtates; yet it muſt be remembered, that the ſpirit of the Chriſtian code condemns the practice; and the true Chriſtian will not engage in it.

Chriſtianity is alſo gradually ſoftening barbarous nations into humanity. The influence of ſelfiſhneſs has been checked and reſtrained. And even war, with all the pernicious improvements, by which mankind has ſought to render it more terrible, has aſſumed much more the ſpirit of mildneſs and peace, than ever entered into it under the influence of Paganiſm.

Theſe are a few of the excellencies of the Chriſtian ſyſtem. Its laſt diſtinction I ſhall mention, is that of its extending its benefits to thoſe nations who have not received its doctrines and precepts. The virtues aſcribed to Julian the apoſtate, are, no doubt, owing to his acquaintance with Chriſtianity; and after the propagation of Chriſtianity through the Roman empire, even while the purity of its doctrines was deſpiſed, it had a remarkable effect on the manners of thoſe unconverted

verted Pagans, who, in their religious doctrines and worſhip, became leſs immoral and abſurd.

Upon the whole, we muſt conclude, that Chriſtianity is infinitely ſuperior to every other religious ſyſtem, both in point of its religious doctrines, and the effects it has produced upon ſociety. It is an univerſal religion; formed to exert its happy influence in all ages, and among all nations; and has a tendency to diſpel the ſhades of barbarity and ignorance; to promote the cultivation of the powers of the human underſtanding; and to encourage every virtuous refinement in manners.

As the Chriſtian religion is deſtined to be of an univerſal nature, and to be diſſeminated into all parts of the world; ſo, in order for its more effectual propagation, its all-wiſe Founder has ordained that it ſhall be divided into different ſects and parties; that the leaders of each being governed by a mutual emulation, might endeavour to propagate their reſpective opinions, and thereby form a grand junction for propagating a religion, the fundamentals of which would be ultimately the ſame.

The two principal ſects into which the Chriſtian religion is divided, are the Proteſtant and Romiſh churches.

The Romiſh church differs from the Proteſtant chiefly in the following particulars: 1. In believing every thing that was defined by the Council of Trent, concerning original ſin and juſtification. 2. In believing tranſubſtantiation, or the converſion of the material bread and wine, given at the ſacrament, into the real body and blood of Jeſus Chriſt. 3. In the belief of a purgatory; and that ſouls are kept priſoners there after their departure from the body; and that they receive help by the prayers of the faithful. 4. That the ſaints reign together with Chriſt, and are to be worſhipped as mediators for man. 5. That the images of Chriſt, the Virgin Mary, and other ſaints, ſhall be retained, and due honour

and veneration be given unto them. 6. That the power of indulgences was left by Chriſt to the church. 7. That the holy church of Rome is the mother and miſtreſs of all churches; and that the biſhop of Rome, or pope, is the ſucceſſor of St. Peter, the prince of the apoſtles, and vicar of Jeſus Chriſt on earth; and that he is infallible and invincible.

Theſe are the chief tenets which diſtinguiſh the church of Rome from the Proteſtant church. The implicit obedience which the followers of this church pay to their leaders, has been a ſource of a very black corruption and error; of which their numerous perſecutions of the Proteſtants are an ample proof. But, on the other hand, it muſt be allowed, that there is no religion ſo zealous of propagating its doctrines. Their miſſionaries have been ſent to all parts of the earth, ſome of whom, by their perſeverance and abſtemiouſneſs, were as great an honour, as others by their profligacy were a diſgrace, to the cauſe in which they were concerned.

The church of Rome is now divided into two ſects: that already deſcribed, which prevails over moſt parts of Italy, Spain, and France, and ſeveral other parts of the continent of Europe; and the Greek church, which differs from the former in not allowing the pope's ſupremacy, nor worſhipping idols, though they have many in their churches, and in not enjoining their prieſts to celibacy.

The Proteſtant religion is divided into numerous ſects and parties: the two principal of which are the Lutherans and the Calviniſts.

The Lutherans maintain, that man is a free agent, perfectly capable of performing good or evil: that according to his actions he ſhall be rewarded or puniſhed hereafter: that he is left at perfect liberty to chooſe the good or evil; and that God has no predilection for any particular perſons: that the ſacrament of the Lord's ſupper is nothing but a mere ordinance.

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The Calvinifts, on the contrary, affert, that man is not a free agent, that he has no power to perform any good action without the Spirit of God affifting him; that God, according to his fore-knowledge, has elected a certain number of individuals to be faved: that he is the former of every good thought; and faves the elect, not from any goodnefs in themfelves, but merely from his own unmerited mercy; confequently, that Chrift did not die for all the world: that the facrament is a fpiritual rite; that the bread and wine is confubftantiated (not tranfubftantiated) in a fpiritual manner into the body and blood of Chrift.

Befides thefe two divifions of the Proteftant religion, one or both of which prevail in moft Proteftant countries on the continent of Europe, there are a great number of inferior fects in England, America, Holland, Germany, and other parts, and fome of them very numerous; as the Quakers, Baptifts, Diffenters, Methodifts, &c. which are too well known to need any defcription. Suffice it to fay, each of them differs from the eftablifhed church, on account of fome trifling errors, which they imgaine they have detected in the national church.

The next divifion of religion that deferves our notice is that of Mahomet, which ftill makes fuch a confpicuous figure in the world, extends over a large tract of country, and is profeffed by very powerful nations. Like the Jewifh religion, it is not merely a fyftem of religious doctrines, and moral precepts; but it forms both the civil legiflature and religious fyftems of the nations by which it is profeffed. It alfo appears to be rather calculated for one particular period, in the progrefs of mankind from barbarity to refinement, than for all ages, and all parts of the world.

In viewing this fyftem of religion, there are many parts of it which feem copied from the Chriftian, the Jewifh, and the Pagan religions. It is difficult to tell which is the greater, the purity of fome parts of this doctrine, or the abfurdity of other parts.

The greateſt abſurdities, or that which tends moſt effectually to promote impurity of manners, are the Prophet's ideas of heaven and hell. Paradiſe, or the place of future rewards, he makes to abound with rivers, trees, fruits, and ſhady groves; wine, without its intoxicating quality, is to be there ſerved out to believers, who, as they enjoy perpetual youth, their powers of enjoyment are to be enlarged and invigorated, according to the delights they are to enjoy. Mahomet celebrates the pearls and diamonds, robes of ſilk, palaces of marble, diſhes of gold, numerous attendants, wines, and dainties, with the whole train of ſenſual luxury, reſerved for the faithful in theſe regions. Seventy-two black-eyed damſels of reſplendent beauty, blooming youth, virgin purity, and exquiſite ſenſibility, will be created for the uſe of the meaneſt believer. A moment of pleaſure will be prolonged to one thouſand years; and the faculties will be increaſed a hundred fold, to render him worthy of his felicity. There are alſo certain more refined enjoyments; as, believers are to ſee the face of God morning and evening—a pleaſure which is to exceed all the other pleaſures of Paradiſe.

In hell, the place of future puniſhments, the wicked are to drink nothing but boiling, ſtinking water; eat nothing but briers and thorns, and the fruit of a tree that grows in the bottom of hell, whoſe branches reſemble the heads of devils, and whoſe fruit ſhall be in their bellies like burning pitch: they are to breathe nothing but hot winds, and dwell for ever in continual burning, fire, and ſmoke.

Thus Mahometiſm appears to be a ſtrange mixture of abſurdities, with a few truths and valuable precepts incongruouſly intermixed. A great part of it is incompatible with virtue, and the progreſs of knowledge and refinement. It ſubſtitutes trifling ſuperſtitious ceremonies in the room of genuine piety and virtue; and preſents ſuch a proſpect of futurity, as renders purity of heart no neceſſary qualification for ſeeing God.

However, Mahometiſm forms in ſome meaſure a regular ſyſtem

ſyſtem of religion, as it has borrowed many of its precepts and doctrines from both Judaiſm and Chriſtianity, which are, however, greatly degraded and corrupted. It has, nevertheleſs, conſiderably contributed towards the ſupport of civil government in thoſe countries in which it is eſtabliſhed.

It is divided into a numerous party of ſects, which, however, differ ſo little from each other, as ſcarcely to deſerve mention in this place.

Paganiſm next deſerves our notice. It was the moſt prevailing religion of antiquity, and may be principally divided into two parts:—1. The Pagan religion of the ancient barbarous nations; and, 2. The Polytheiſm of the more civilized Greeks and Romans.

The Paganiſm of the ancient nations preſents us with a moſt ſhocking picture of ignorance, ſuperſtition, and abſurdity. We there behold the moſt abſurd doctrines concerning a future ſtate. Various nations have imagined, that the ſcenes and objects of the world of ſpirits are only a ſhadowy repreſentation of the things of the preſent world. According to them, not only the ſouls of men inhabit thoſe regions, but all the inferior animals and vegetables, and even inanimate bodies that are killed or deſtroyed here, are ſuppoſed to paſs into that viſionary world, and exiſting there in unſubſtantial forms, execute the ſame functions, or ſerve the ſame purpoſes, as on earth. By this belief they were ſtimulated, on the death of a king or other great perſonage, to provide for his accommodation in the world of ſpirits, by burying with his corpſe, meat and drink for his ſubſiſtence, ſlaves for his attendance, and wives for his enjoyment. His faithful ſubjects vied with each other in their offerings upon this occaſion; one brought a ſervant, another a wife, a third, a ſon or daughter, to accompany their monarch in his future ſtate. Similar practices, on the ſame occaſion, prevailed in New Spain, in the iſland of Java, in the kingdom of Benin, and among the inhabitants of Hindoſtan. A like belief alſo prevailed among the Japaneſe. They not only bribed their

priefts to folicit for them a place in the blifsful manfions of futurity; but looking upon the prefent life with difguft and contempt, when fet in competition with the joys of futurity, they ufed to dafh themfelves from precipices, or cut their throats, in order to get to Paradife as foon as poffible. Various other fuperftitions, fubfifting among rude nations, might here be adduced, as inftances of the perverfion of the religious principles of the human heart, which render them injurious to virtue and happinefs. Innumerable are the ways of torture which have been invented and practifed on themfelves by men ignorantly ftriving to obtain the favour of Heaven. Thefe are fufficient proofs of religious fentiments having been fo ill directed by the influence of imagination, and unenlightened erring nature, aided by the corrupt defigns of artful priefts.

The Polytheifm of the Greeks and Romans, though more favourable to virtue and civilization, than the Pagan notions of antiquity, is yet a very imperfect, not to fay a pernicious, code of religion: the vicious characters of their deities, the abfurd notions they entertained concerning the government of the univerfe and a future retribution, the abfurdities of their religious rites and ceremonies, the frivolous practices with which they were intermixed, muft all together have a great tendency to pervert both the reafoning and moral principles of the human mind: however, it cannot be denied, that this fyftem was friendly to the encouragement of arts; particularly of fuch as depend on the vigorous exertion of a fine imagination, as mufic, poetry, fculpture, architecture, and painting: all thefe arts appear to have been confiderably indebted for that perfection to which they attained, to the fplendid and fanciful fyftem of mythology which was received by thofe people, particularly by the Greeks.

The effect of this religion, to reform the lives of its votaries, was very imperfect. Sacrifices and prayers, temples and feftivals, not purity of heart and integrity of life, were the means prefcribed for obtaining the favour of their deities.

There were alfo other means of gaining admiffion into the Elyfian fields, or the feat of the councils of the Gods; but none of thefe means appear to have been thofe commanded by the Chriftian religion. And whatever might be the effects of the religion of Greece and Rome in general, upon the civil and political eftablifhments, and on the manners of the people, yet it muft be confeffed to have been but ill adapted to imprefs the heart with fuch principles as might in all circumftances direct to a firm, uniform tenour of righteous conduct.

From this view of religion, it appears, that though fome particular forms, fuch as thofe of the Chriftian, have had a greater influence in reforming the manners of their followers; yet as they all have often contributed to form the mind to virtue, it muft be acknowledged, that they have always, and under all their forms, been infinitely more beneficial than hurtful to mankind.

When we view the different fyftems in a comparative light, with refpect to their influence on the welfare of fociety, no one will hefitate to prefer the Polytheifm of the Greeks and Romans to the ruder ideas of the more ancient Pagans; and Mahometifm to the Polytheifm of the Greeks and Romans: Judaifm is, however, greatly preferable to Mahometifm; and Chriftianity to all of them.

SECT.

## SECT. VI.

### OF EUROPE.

EUROPE, though the leaſt quarter of the globe, is by far the moſt eminent in modern hiſtory; and is at preſent the moſt diſtinguiſhed part of the globe for the literature, arts, and ſciences, to which it has given birth and encouraged, and for the learned men it has produced. It is alſo the moſt civilized quarter of the globe. Here are no public marts for buying and ſelling the human ſpecies, as are found in Aſia and Africa. The Chriſtian religion alſo prevails here almoſt univerſally. Its languages are as mixed as its inhabitants, but all derived from the ſix following: viz. the Celtic, Sclavonian, Teutonic, Greek, Latin, and Gothic. It extends about 3000 miles in length, and 2500 in breadth: and is divided into ſeveral kingdoms and ſtates, as ſeen in the Table, page 133.

The BRITISH ISLES, lying on the weſtern part of Europe, conſiſt of Great Britain (which compriſes England, Wales, and Scotland), Ireland, and the Iſles of Man, Jerſey, Guernſey, Alderney, Sark, and Wight. England lies between 50 and 56 degrees north latitude, and between 2 degrees eaſt, and 6 degrees 20 minutes weſt longitude; and is divided into forty counties. Its conſtitution is that of a limited monarchy, conſiſting of king, lords, and commons, with certain prerogatives and privileges annexed to each.

The legiſlative authority, or power of making laws and raiſing

raiſing money, is veſted in theſe three branches of the government; and each branch has a negative voice.

The crown is made hereditary in the Hanover line, by ſeveral acts of Parliament, provided they do not profeſs Popery, marry Papiſts, or ſubvert the conſtitution.

The peers are created by the crown; but their honours are hereditary, and cannot be taken from them any more than their lives or eſtates, unleſs forfeited by the commiſſion of high treaſon; and they can only be tried by the Houſe of Peers, being ſubject to no other juriſdiction. This Houſe is the laſt reſort in all civil caſes, and the higheſt court in the kingdom.

Any bill for making a new law, or altering an old law, may be brought in firſt in the Houſe of Peers; but a bill relating to the revenues, or public taxes, muſt be brought into the Houſe of Commons firſt; and it cannot be *altered* by the Peers, though it may be rejected.

The Houſe of Peers can apprehend and commit any man for a reflection on their judicature.

The Commons are compoſed of 658 members: viz. 80 knights, every county in England ſending two, and elected by the freeholders; 50 citizens, two being ſent from each of the 25 cities in England (London ſending four, and Ely none); 334 burgeſſes, from 167 boroughs, ſending two each; five burgeſſes, from the boroughs of Abingdon, Banbury, Bewdley, Higham Ferrars, and Monmouth; four repreſentatives, from the two univerſities; 16 barons, from the five Cinque Ports, Haſtings, Dover, Sandwich, Romney, and Hythe, and their three dependants, Rye, Winchelſea, and Seaford; 12 knights, from the 12 counties of Wales; 12 burgeſſes, from the 12 boroughs in Wales (Pembroke ſending two, and Merioneth none); 30 knights, from the ſhires of Scotland; 15 burgeſſes, from the Scotch boroughs; and 100 members from Ireland.

WALES is ſituated on the weſt and north-weſt of England, to which it joins; and is divided into 12 counties: it is a principality;

principality; and always confidered as the right of the King's eldeft fon, who is therefore titled Prince of Wales. It was peopled in the year 410, by the ancient inhabitants of England, who fled thither from the perfecution of the Picts and Scots.

IRELAND is fituated between fix and ten degrees of weft longitude, and between 51 and 55 degrees of north latitude. Bounded by the eaft by St. George's Channel, or the Irifh Sea, which divides it from Great Britain. It is divided principally into four provinces: viz. Ulfter, on the north; Leinfter, on the eaft; Munfter, on the fouth; and Connaught, on the weft.

The climate of thefe iflands is in general mild for the latitude, but very changeable, the weather never continuing a month the fame, owing to the exhalations from the furrounding fea, which render the air humid. But the foil is in general fruitful, and has been of late years greatly improved.

Thefe iflands have feveral very good mines of tin, copper, iron, and lead; gold has alfo been found in Scotland, in folid pieces, in the brooks, after a great torrent.

The chief manufacture of England is woollen cloth, which is accounted the ftaple trade of the kingdom; as linen cloth is that of Ireland.

DENMARK, including Norway, is the moft northern kingdom of Europe, and includes Denmark Proper, the territories in Germany, Norway, part of Lapland, and feveral iflands in the Baltic Sea, and in the German Ocean; and extends from 52 degrees of north latitude, to the fartheft habitable part of the Arctic Circle. Denmark Proper is bounded on the north by the Cattegate or Skaggerac; on the fouth by Germany; on the weft by the German Ocean; and on the eaft by the Sound.

The eftablifhed religion is Lutherifm. The king is abfolute, though in general mild in his government. It is di-

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vided into two parts, called North Jutland and South Jutland.

The air of this country is ſharp, but the exhalations from the ſea abate its ſeverity. The ſummers are very ſhort and hot, but the ſoil is in general fruitful, for a northern latitude, except on the tops of mountains. The manufactures of this country are chiefly hardware; and their artiſts and mechanics, in every branch, are generally ſkilful.

NORWAY is bounded on the ſouth by the Cattegate, on the weſt and north by the Northern Ocean, and on the eaſt by the mountains which ſeparate it from Sweden; and is divided into the north, ſouth, and middle diviſions. The air of Norway is generally healthy and dry in the inland parts of the country, but on the ſea-coaſt it is moiſt. In winter it is exceſſively cold, the whole country being covered with ſnow; it is alſo very hot in the ſummer. Their trade conſiſts of copper, timber, iron, marble, mill-ſtones, fiſh, fowls, tallow, tar, oil, alum, vitriol, &c. Their language is the ſame as that uſed in Denmark; and their religion is that of Lutheriſm.

ICELAND is ſituated in the Northern or Atlantic Ocean; being 726 miles in length from eaſt to weſt, and 300 in breadth: extending from ſixty-three to ſixty-eight degrees of north latitude; and from fourteen to twenty-nine degrees of weſt longitude. It has a milder climate than any other country in the ſame latitude. It is a very mountainous country, but well watered, with ſeveral large rivers. In this country there are ſome large ſprings of boiling hot water, the principal of which is Geyſer, near Skalholt. The water iſſues from this ſpring ſeveral times a day, with a violent noiſe, like that of a great torrent, ſometimes riſing to the height of 60 fathoms, and ſeldom leſs than 90 feet.

There are alſo ſeveral burning mountains in this country, of which the moſt remarkable are, Hecla, Kotlegau, and Oraiſe, the eruptions of which have ſometimes done conſiderable damage. The inhabitants live chiefly by fiſhing, and

breeding cattle, attending very little to agriculture. Their commerce is monopolized by a Daniſh company of merchants, and conſiſts chiefly of ſalt meat, butter, tallow, oil, wool, ſkins, furs, and feathers. The revenue ariſing from this country to the king of Denmark amounts to 30,000 crowns *per annum*.

GREENLAND is the moſt northern boundary of the king of Denmark's dominions; and is the fartheſt part of the globe northward which has been diſcovered. Eaſt Greenland extends beyond 76 degrees of north latitude; and between 10 and 11 degrees of eaſt longitude. There are no inhabitants here, except a few convicts tranſported from Ruſſia, and who gain their liberty by procuring ſkins, furs, tuſks of morſe, &c. for the ſovereign of Ruſſia.

Weſt Greenland extends beyond 60 degrees of north latitude; and between 5 and 50 degrees of weſt longitude. There are a few natives who inhabit this country, and many of whom have lately been converted to Chriſtianity, by the Daniſh and Moravian miſſionaries.

SWEDEN extends from 55 degrees 20 minutes to 69 degrees 30 minutes north latitude; and from the 12th to the 30th degree of eaſt longitude. It is bounded on the ſouth by the Baltic, the Sound, and the Cattegate Sea; on the north by Daniſh Lapland; by Ruſſia on the eaſt; and by the mountains of Norway on the weſt: and principally divided into ſeven provinces: viz. 1. Sweden, properly ſo called, lying between Norway and the Gulf of Bothnia; 2. Gothnia, or Gothland; 3. Livonia, on the ſouth of Finland Gulf; 4. Ingria, on the north-eaſt of Livonia; 5. Finland, on the eaſt ſide of the Gulf of Bothnia; 6. Swediſh Lapland, in the northern parts; 7. the iſlands of Gothland, Œland, Aland, Hogland, and Rugen.

*Note.* The provinces of Livonia and Ingria, with Kexholm and Karelia in Finland, and the iſlands of Dagho and Oſel, are under the government of Ruſſia.

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The natural ſoil of this country is in general barren, but has been greatly improved of late years, by the induſtry of the inhabitants, aſſiſted by the affluent part of the nation, ſo that they have now fruitful harveſts. Their manufactures are chiefly in ſilver, copper, and iron; and vaſt quantities of theſe metals, with timber, tar, hemp, flax, hides, furs, fiſh, &c. conſtitute the chief articles of their trade.

Their religion is the ſame as that of Denmark and Norway. Their language is alſo partly the ſame, being only a dialect of the Teutonic language. The government of this country is a limited monarchy.

Russia, the largeſt empire upon the globe, and greater than all the reſt of Europe beſides, extends in length from the Baltic Sea on the weſt, to within a few miles of America on the eaſt, upwards of 6000 miles; and above 2400 miles in breadth from north to ſouth. It is bounded on the weſt by Sweden and the Baltic; on the eaſt by China, and the Pacific Ocean, which ſeparates Aſia from America; on the north by the Frozen Ocean; and on the ſouth by Pruſſia, Poland, Turkey, Perſia, and Tartary. Its meaſured length from the iſle of Dagho to its eaſtern bounds is near 170 degrees. Thus it contains ſeveral different climates. In the ſouthern parts, the longeſt day is ſcarcely ſixteen hours, while in the northern parts it is nearly three months. In the ſouthern provinces it is very hot; and extremely cold in the northern parts. The ſoil beyond the 60th degree of north latitude ſcarcely ever produces corn to any perfection; and beyond the 70th degree ſcarcely any ſpecies of fruit is found; but in the middle provinces the ſoil is fruitful, and produces good paſture for cattle, and excellent grain. The ſouthern provinces being hot, have all the fertility of a warm country, where there is a ſufficient depth of ſoil. There is a great variety of inhabitants in this extenſive country: viz. the Tartars, Kamſchatdales, Samoiedes, Laplanders, &c. There is conſiderable variety in the manners of the natives

of these different countries. In some of the northern parts they live in caverns, not five feet in height; in other parts they lead a wandering life: the natives of some parts practise agriculture; but in others live on the spontaneous productions of the soil.

The religion of some parts is next to Paganism, the natives idolizing inanimate objects, as a sheep's skin; but in other parts they make no public profession of religion. The established religion of Russia is the Greek church.

The European part of Russia, called Muscovy, is divided into the following provinces: viz.

In the northern division: Lapland, Samoieda, Bellamornsckoy, Meseen, Dwina, Syrianes, Permia, Rubeninski, Belacseda.—In the middle division: Pereslaf, Belozero, Wologda, Jereslaf, Tweer, Moscow, Belgorod.—In the eastern division: Bulgar, Kansan, Little Novgorod, Don Cossacks.—In the western division: Great Novgorod, Rus, Finland, Kexholm, Karelia, Ingria.—In the southern division: Livonia, Smolensko, Zernigof, Seefsk, Ukrain. Their articles of commerce and manufacture are the same as those of Sweden and Denmark; they have, moreover, silk, cotton, teas, gold, &c. which they bring from China and India, in caravans, by the way of the Caspian Sea.

The language is derived from the Sclavonian, to which are added many words from the Greek; their alphabet consists of forty-two characters, which are principally Greek. The people of high rank generally speak French and High Dutch, but their priests speak the modern Greek.

POLAND, before its late dismemberment, was bounded on the north by Livonia, Muscovy, and the Baltic Sea; on the east, by Muscovy; on the south, by Hungary, Turkey, and Little Tartary; and on the west, by Germany; extending from 47 degrees 40 minutes, to 56 degrees 30 minutes, north latitude; and from 16 to 34 degrees east longitude. It was divided into the provinces of Great and Little Poland, Polish

Polifh Pruffia, Samogitia, Courland, Lithuania, Mafovia, Podolachia, Polefia, Red Ruffia, Podolia, and Volhinia. The foil of Poland is in general very fruitful, and the air moftly temperate, except in the northern parts, where it is very cold. Their pafture land is fo fruitful, that the height of the grafs often conceals the cattle from the view of a paffenger at two hundred yards diftance. Great numbers of beafts, as horfes, affes, oxen, buffaloes, bears, foxes, wolves, &c. run wild in the forefts. There are feveral mines in the country, of gold, filver, copper, lead, iron, &c.

The greateft curiofities in this country are the falt-mines, of which that of Wielitfka is the largeft in the world, and has been wrought above 600 years. It is 743 feet below the furface of the ground, and 1115 feet in breadth, and 6691 in length; and appears like a fpacious plain, with vaulted roofs, fupported by columns of falt, which have been left ftanding. Many public lights are placed in this mine, for general ufe, which reflect a moft luminous appearance from every part of the mine. Here are alfo great numbers of huts for the accommodation of the miners and their families, many of whom are born and fpend their lives in this place, without ever making their appearance on the furface of the earth. Through the midft of the mine is the great road, which paffes to the mouth of the mine: this road is generally crowded with carriages full of falt. A ftream of frefh water alfo runs through the mine.

The wild men who have been feen of late years in the woods of Poland form another curiofity.

The Poles at prefent feem almoft annihilated, and their country divided among the Auftrians, Pruffians, and Ruffians.

PRUSSIA is bounded on the north by Samogitia; on the fouth by Poland Proper and Mafovia; on the eaft by part of Lithuania; and on the weft by Polifh Pruffia and the Baltic: but if we take it in its full extent, this kingdom confifts

confifts of various territories, different parts of Germany, Poland, Swifferland, and other northern countries.

The principal divifions of this kingdom are, Regal Pruffia, fituated in Poland; and Upper Saxony, containing Brandenburg, Pruffian Pomerania, and Swedifh Pomerania, Magdeburg, and Halberftadt in Lower Saxony; Glatz in Bohemia; Minden, Ravenfburg, Lingen, Cleves, Meures, and Mark, in the dutchy of Weftphalia; Eaft Friefland, Lippe, Gulick, and Tacklenburg, in the circle of Weftphalia; the margraviate of Anfpach, in the circle of Franconia; Gelder in the Netherlands; Neufchatel in Swifferland; and part of Silefia.

Pruffia carries on a confiderable trade, and the balance in favour of Pruffia is reckoned greater than that of any other European ftate; great quantities of glafs, iron works, cloth, camlet, filk, linen, paper, powder, copper, and brafs, are annually exported.

Amber is here found in great quantities, from which the crown of Pruffia receives 26,000 dollars annually; alfo great fums from the bitumen, of which feveral kinds are found in the Baltic Sea.

The religions of Pruffia are thofe of the Lutherans and Calvinifts; but all religions are tolerated. His Pruffian Majefty is abfolute through all his dominions. The Pruffian army, even in times of peace, confifts of 180,000 men, which are reckoned the beft difciplined troops in the world; but in time of war it has been augmented to between 3 and 400,000 men.

GERMANY is bounded on the north by the German Ocean, Denmark, and the Baltic; on the eaft, by Poland, Hungary, and Bohemia; on the fouth, by Swifferland and the Alps; and on the weft, by France and the Low Countries. It extends from 45 degrees 4 minutes, to 54 degrees 40 minutes, north latitude; and from 6 degrees to 19 degrees 45 minutes eaft longitude. Germany is a great empire, having feveral dependant fovereignties under it, under different modifications

of

of government, ſome of which ſcarcely exceed an Engliſh manor in extent. It is divided into nine circles, three of which lie in the north, three in the middle, and three in the ſouth: viz. Upper Saxony, Lower Saxony, Weſtphalia; Upper Rhine, Lower Rhine, Franconia; Auſtria, Bavaria, and Suabia. Theſe circles are ſubdivided into principalities, dutchies, marquiſates, electorates, palatinates, counties, baronies, abbies, biſhoprics, &c.

The climate of Germany is in general healthy and agreeable, except in the moſt northern and ſouthern parts. And the ſoil is particularly fruitful; for though only a ſmall proportion of the country is cultivated, yet proviſions are in general cheaper than in moſt other countries of Europe. They have alſo a greater quantity of domeſtic animals and wild beaſts, as boars, hares, rabbits, foxes, badgers, goats, &c. &c. than other European countries. They alſo abound in moſt of the ſpecies of tame fowl, as well as wild fowl.

There are ſeveral mines in Germany of ſilver, copper, iron, lead, quickſilver, ſulphur, nitre, &c. and coal-pits are found in every part of the empire.

Germany is alſo in great eſteem in all other European countries for its mineral ſprings and baths, the moſt remarkable of which are thoſe of Aix-la-Chapelle, Spa, Pyrmont, Ems, Wiſbaden, Schwalbach, Wildungen, and Brakel, which laſt is encloſed, as the waters are ſo ſtrong as to be capable of intoxication.

The manufactures of Germany conſiſt of velvets, ſilks, cotton and woollen ſtuffs, linen, fuſtian, ribands, lace, tapeſtry, &c. They alſo make beautiful porcelain and lacquered ware, and every kind of hard ware.

The Germans have a conſiderable commerce, owing to their centrical ſituation, and the balance of trade is greatly in their favour. The eſtabliſhed religion is either Romiſh, Lutheran, or Calviniſt, being different in the different parts of the empire; but moſt other religions are tolerated at preſent.

The German language is a dialect of the Teutonic, and is called the High Dutch, being the mother tongue of the whole empire: but every different province has a different dialect.

The government of Germany is in the hands of about 300 civil and ecclesiastical princes, each of whom is absolute in the government of his own state; and the whole of them form a great confederacy, governed by political laws, at the head of which is the Emperor, whose power in the collective body is only executive. The Emperor is elected; but the empire for some centuries has belonged to the House of Austria, as being the most powerful of the German princes. The nine electors of the empire have each a particular office in the Imperial court: they have the sole election of the Emperor, and are as follow:—1. The Archbishop of Mentz, who is high chancellor of the empire, when in Germany.—2. The Archbishop of Treves, who is high chancellor of France and Arelat (a dignity merely nominal).—3. The Archbishop of Cologne.—4. The King of Bohemia, who is cup-bearer.—5. The Elector of Bavaria, who is grand sewer.—6. The Elector of Saxony, who is great marshal of the empire.—7. The Elector of Brandenburg (now King of Prussia), who is great chamberlain.—8. The Elector Palatine.—9. The Elector of Hanover (King of Great Britain), who claims the post of arch-treasurer.

The revenue of the Emperor, as such, is about 5 or 6000 pounds sterling per annum, arising from the fiefs in the Black Forest. The Austrian revenues are immense, amounting to 12,000,000 pounds sterling.

The military force of this country amounts to near half a million of men; the secular princes bringing upwards of 379,000, the ecclesiastical 7450, and the Emperor, as the head of the House of Austria, 90,000.

Hungary, Bohemia, and the provinces of Transylvania, Sclavonia, Croatia, and Morlachia, may be considered as part

of

of the German empire, having been brought under the dominion of the houfe of Auftria. The eftablifhed religion of thefe countries is the church of Rome.—Bohemia lies between 48 and 52 degrees north latitude, and between 12 and 19 degrees eaft longitude. Bounded on the north by Saxony and Brandenburg; on the eaft by Poland and Hungary; on the fouth by Auftria and Bavaria; and on the weft by Bavaria.—Tranfylvania lies between 45 and 48 degrees north latitude, and between 22 and 25 degrees eaft longitude. Bounded on the north by Hungary and Poland; on the fouth by Walachia; on the eaft by Moldavia; and on the weft by Hungary.—Sclavonia lies between 45 and 47 degrees north latitude, and between 16 and 22 degrees eaft longitude. Bounded by the river Drave on the north; by Auftria on the fouth; by the Danube on the eaft; and by the Save on the weft.—Croatia lies between 44 and 47 degrees north latitude, and between 15 and 17 degrees eaft longitude. Bounded on the north by the Save; on the fouth by Morlachia; on the eaft of Bofnia; and on the weft by Carniola.—Morlachia lies between 44 and 46 degrees north latitude, and between 16 and 17 degrees eaft longitude. Bounded on the north by Carniola; on the fouth by Dalmatia; on the eaft by Bofnia; and on the weft by the Gulf of Venice.

Swisserland is bounded on the north by Suabia; on the eaft by the lake of Conftance, Tirol, and Trent; on the fouth by Italy; and on the weft by France: extending from 45 to 48 degrees north latitude, and from 6 to 11 degrees eaft longitude; and is divided into thirteen cantons: viz. Bern, Fribourg, Bafil, Lucern, Soloturn, in the weft divifion; Schaffhaufen, Zurich, Appenzel, in the eaft divifion; and Zug, Swifs, Glaris, Uri, and Underwald, in the middle divifion. Seven of thefe cantons profefs the Romifh religion: Fribourg, Lucern, Soloturn, Zug, Swifs, Uri, and Underwald; the other fix are Proteftants. The climate of this country is very various, on account of the inequality of the furface of the ground, being fituated among the Alps; the higheft

mountains in Europe: ſo that it is common for the inhabitants to be reaping on one ſide of the mountain, while thoſe on the other ſide of the ſame mountain are ſowing. The froſts in winter are very ſevere; and in the ſummer the heat is, in ſome parts, intenſe.

The commerce of Swiſſerland conſiſts of their cattle, horſes, cheeſe, butter, hides, ſkins, and the productions of their own manufacture, the principal of which are ſilks, brocades, linen, lace, woollens, ſtuffs, hats, paper, leather, porcelain, toys, watches, and clocks.

Each canton forms a ſeparate republic; but when any controverſy ariſes, it is referred to the general diet, which ſits at Baden, where each canton has a vote, and ſends two deputies.

The NETHERLANDS lie between 50 and 54 degrees north latitude, and between 2 and 7 degrees eaſt longitude. They are bounded on the north by the German ocean; on the eaſt by Germany; on the weſt by the Britiſh channel; and on the ſouth by France and Lorrain. The Netherlands are divided into ſeventeen provinces; the ſeven northerly ones are called Holland, or the United Provinces, and the other ten are called Flanders, or the Auſtrian and French Netherlands.

The provinces of Holland, are Holland, Zealand, Friezland, Groningen, Overyſſel, Guelderland and Zutphen, and Utrecht.

The air of theſe provinces is very moiſt and foggy; their harbours are generally frozen up four months in the year; and the ſoil is very unfavourable for vegetation; but the induſtry of the inhabitants has greatly improved it, by making canals and ditches to drain the land.

Their commerce is carried on to ſuch an extent, that there is hardly a commodity of traffic on the face of the globe but may be bought here, and almoſt as cheap as in the places where it is produced.

The religion of this country is Calvinifm; but all profeffions and focieties are tolerated, of which there are great numbers.

The government of Holland is a democracy, and has fo continued for upwards of two hundred years; notwithftanding they had a prince under the title of ftadtholder, whofe powers had very little of the regal nature.

The ten other provinces of the Netherlands, called Flanders, have been divided among the Auftrians, French, and Dutch, but are now chiefly claimed by the French, and contain the ten following provinces: viz.—Brabant, Antwerp, Malines, Limburgh, Luxemburgh, Namur, Hainault, Cambrefis, Artois, and Flanders.

The foil in moft of thefe provinces is extremely fruitful, and the air generally healthy, except in Brabant, and fome parts of the fea-coafts.

The commerce of thefe provinces confifts chiefly of their own manufacture, viz.—fine linens, cambricks, laces, and woollen manufactures.

FRANCE extends from 42 to 51 degrees north latitude; and from 5 degrees weft, to 8 degrees eaft longitude. It is bounded on the north by the Netherlands and the Englifh channel; on the eaft by Germany, Swifferland, and Italy; on the fouth by the Mediterranean fea and Pyrenean mountains; and on the weft by the Bay of Bifcay. France was formerly divided into 12 provinces; but at the late revolution it was divided into 84 departments, each department being divided into diftricts, and each diftrict into cantons. The eighty-four departments are as follows:—1. Straits of Calais: 2. North. 3. Lower Seine. 4. Somme. 5. Aifne. 6. Ardennes. 7. Channel. 8. Calvados. 9. Eure. 10. Oife. 11. Marne. 12. Meufe. 13. Mofelle. 14. Lower Rhine. 15. Finifterre. 16. North coaft. 17. Ifle and Vilaine. 18. Mayenne. 19. Orne. 20. Eure and Loire. 21. Seine and Oife. 22. Paris. 23. Seine and Marne. 24. Aube. 25.

Upper Marne. 26. Meurte. 27. Vosges. 28. Upper Rhine. 29. Morbihan. 30. Lower Loire. 31. Mayenne and Loire. 32. Sarthe. 33. Loire and Cher. 34. Loirett. 35. Yonne. 36. Cote d'Or. 37. Upper Soanne. 38. Doubes. 39. Vendée. 40. Two Sevres. 41. Vienne. 42. Indre and Loire. 43. Indre. 44. Cher. 45. Nievre. 46. Soanne and Loire. 47. Jura. 48. Lower Charente. 49. Charente. 50. Upper Vienne. 51. Creuze. 52. Allier. 53. Rhone and Loire. 54. Ain. 55. Gironde. 56. Dordogne. 57. Correze. 58. Puy de Dome. 59. Upper Loire. 60. Isere. 61. Landes. 62. Lot and Garonne. 63. Lot. 64. Cantal. 65. Lozere. 66. Ardeche. 67. Dreme. 68. Upper Alps. 69. Lower Pyrenees. 70. Gers. 71. Upper Garonne. 72. Tarne. 73. Aveiron. 74. Herault. 75. Gard. 76. Lower Alps. 77. Upper Pyrenees. 78. Arriege. 79. Aude. 80. East Pyrenees. 81. Mouths of Rhone. 82. Var. 83. Corsica. 84. Mount Blanc.

The climate of France is reckoned, upon the whole, to be more settled than that of any other country in Europe. In the north the winters are very cold; but in the interior parts the air is very temperate and healthy; and in the south it is so mild, that invalids retire thither from all the northern countries, to avoid the rigour of their own climates.

The commerce of France consists of wines, brandy, vinegar, drugs, oils, fruits, of which they have great variety, silks, cambricks, laces, paper, parchment, hardware, toys, &c. and their trade is very considerable and lucrative both to the East and West Indies; but particularly to the European countries.

The national religion was always Romish. And their monarchs were always limited till the three last sovereigns of France. The executive power is now vested in three consuls, the chief of whom is created consul for life, with the power of nominating a successor.

SPAIN

Spain lies between 36 and 44 degrees north latitude; and between 10 degrees weſt, and 3 degrees eaſt longitude. It is bounded on the north by the Bay of Biſcay and the Pyrenean mountains; on the ſouth by Gibraltar ſtraits; on the eaſt by the Mediterranean ſea; and on the weſt by Portugal and the Atlantic Ocean. It is divided into the following kingdoms or provinces: Galicia, Aſturia, Biſcay, Navarre, Arragon, Catalonia, Valencia, Murcia, Granada, Andaluſia, Old Caſtile, New Caſtile, Leon, and Eſtremadura.

Spain enjoys a dry, clear, temperate air, except during the equinoctial rains; and in the ſouthern provinces during the ſummer months, where it is very hot. The ſoil is as fruitful as the ſoil of any part of Europe; but the natives are very indolent. In many parts the choiceſt fruits grow ſpontaneouſly. They alſo have a great variety of aromatic herbs. Seville is celebrated for its oranges; and Murcia produces mulberry-trees in ſuch abundance, that the ſilk exported from this part amounts to 200,000 pounds per annum.

The chief articles of commerce in Spain are gold and ſilver, which they derive from their ſettlements in South America. The principal manufactures are ſilk, wool, iron, copper, and hardware.

The national religion of Spain is the profeſſion of the church of Rome. The Inquiſition always reigned in this country, till, by a late edict, it was put under ſome reſtrictions.

The conſtitution of Spain is the moſt abſolute monarchy in Europe. And the revenue from Old Spain only, amounts to upwards of 6,000,000 ſterling: what the exact amount of the whole revenue is, is not accurately known.

The military force of Spain is never leſs than 70,000 men in time of peace; and in time of war the king has raiſed near 200,000.

Portugal joins to Spain, and is bounded by it on the north and eaſt; and on the ſouth and weſt by the Atlantic Ocean.

Ocean. It extends from 36 degrees 50 minutes to 43 degrees north latitude, and from 7 to 10 degrees weſt longitude.

The climate of Portugal is more temperate than that of Spain, on account of its vicinity to the ſea. Their commerce conſiſts chiefly of wines, fruits, ſalt, linen, woollen, and ſome coarſe ſilk. Their religion is that of the church of Rome; and the Inquiſition has greater power here than in any other country. The conſtitution is, like that of Spain, an abſolute monarchy.

ITALY, including Sicily, lies between 37 and 47 degrees north latitude, and between 7 and 19 degrees eaſt longitude. On the eaſt, ſouth, and weſt, it is waſhed by the Adriatic and Mediterranean ſeas; and on the north it is ſeparated from the reſt of Europe by the Alps. It contains the following countries: Piedmont, Montferrat, part of Milan, Sardinia Iſle, Naples, Sicily, Milaneſe, Mantua, Tuſcany, the Duke of Parma's territories, Genoeſe territories, Oneglia, the Duke of Modena's territories, Venetian territories, Pope's dominions, Corſica Iſle, Malta Iſle, and ſome other ſmall iſlands. All theſe countries are diſtinct from each other; having different forms of government, different trade, and ſeparate intereſts.

Italy has a fine ſoil, and temperate but warm climate; the ſoil however is greatly neglected, owing to the indolence of the inhabitants.

The religion, univerſally profeſſed throughout Italy, is that of the church of Rome; but people of all other religions generally live unmoleſted in moſt parts of Italy. The commerce and manufactures are various, according to the different ſtates; but wines, fruits, and oil, conſtitute the chief articles. The curioſities to be met with in this extenſive tract of country are almoſt innumerable, it being the ſeat of ſo many nations of antiquity, particularly of ancient Rome: hence, there are innumerable remains of the arts, the

works of ancient artifts: the burning mountains alfo confti-tute one of their greateft natural curiofities. The Italian language is derived from the Latin; with an intermixture of words from the Goths, and other barbarous nations; but every feparate ftate has a different dialect.

To defcribe the form of government of each ftate, would be to enter into too minute a detail, as they are different in every ftate.

Turkey extends into both Europe and Afia.

European Turkey extends from 17 to 40 degrees eaft longitude, and between 37 and 49 degrees north latitude. It is bounded on the north by Ruffia, Poland, and Sclavonia; on the eaft by the Black Sea, the Hellefpont, and the Archipelago; on the fouth by the Mediterranean; and on the weft by the Mediterranean, and Venetian and Auftrian territories.

Turkey in Europe contains fome of the moft genial climates in the world; and is divided into the following provinces: Crim and Little Tartary, Budzaic Tartary, Beffarabia, Moldavia, Wallachia, Bulgaria, Servia, Bofnia, Romania, Macedonia, Janna, Livadia, Epirus, Albania, Dalmatia, Ragufa, Corinth, Argos, Sparta, Olympia, Arcadia, Elis.

The foil of Turkey is extremely fruitful, where the leaft induftry has been employed: and all the fruits common to all the warm climates are produced here in great perfection; and many valuable drugs are natives of this country.

The commerce and manufactures of Turkey are chiefly filks, drugs, dying ftuffs, in their natural ftate; with cottons, carpets, leather, velvets, foap, &c.; but though the Turks are fituated in the moft advantageous part of both Europe and Afia for traffic, yet they fhamefully neglect it.

The religion which the Turks univerfally profefs, is Mahometifm; but they are divided into as many fects as the profeffors of Chriftianity. The high prieft, or Mufti, is an officer

officer of ſuch honour, that whenever he comes into court, the Grand Seignior riſes from his ſeat and meets him. Moſt other religions are tolerated here by paying an annual tax.

The government of Turkey is that of an abſolute monarchy; and in this empire there is no hereditary ſucceſſion by law to any property; yet the rights of individuals are rendered ſecure by being annexed to the church, by which means even Jews and Chriſtians may ſecure their property in lands to the lateſt poſterity. The revenue of Turkey amounts to upwards of twenty-five millions per annum, but does not produce four millions to the emperor's treaſury; the reſt being expended in collecting, &c. The forces of the Turkiſh empire are of two ſorts; the one has certain lands for their maintenance, and the other is paid out of the treaſury. The former amount to 268,000 troopers; the latter, called the horſe-guards, are about 12,000; and the janizaries, or foot-guards, 25,000; beſides 100,000 foot ſoldiers in different parts of the empire.

## SECT. VII.

### OF ASIA.

Asia forms the moſt remarkable quarter of the globe in ancient hiſtory. It was here that the firſt man was created—here the patriarch Noah was preſerved during the flood—and from this quarter the world was repeopled a ſecond time. In Aſia lived all the patriarchs recorded in Scripture—and

and this was the ſcene of all the tranſactions recorded in Holy Writ—and, finally, it was here Jeſus Chriſt appeared, and wrought the ſalvation of mankind—and from hence the Chriſtian religion was propagated.

This quarter of the globe enjoys the moſt ſerene air and fruitful ſoil of all the quarters, and produces the moſt delicious fruits, odoriferous ſhrubs, ſpices, and valuable drugs, gums, &c.

Idolatry and Mahometiſm are almoſt univerſal in this quarter of the globe, except in a few European ſettlements. The languages in uſe in this quarter are chiefly the Arabic, Perſian, Malayan, Chineſe, Japaneſe, Tartarian, Ruſſian, and Turkiſh.

Aſia is bounded on the weſt by the Red Sea, the Mediterranean, the Archipelago, the Black Sea, and Europe; on the north by the Frozen Ocean; on the eaſt and ſouth by the Pacific and Indian Oceans. It is ſituated between the equator and the frigid zone, and between 25 and 180 degrees of eaſt longitude; it is about 4800 miles in length, and 4300 in breadth, and contains the following countries.

TURKEY in Aſia, being the other part of the Turkiſh empire, is about 1000 miles in length from eaſt to weſt, and 800 in breadth from the northern parts to the deſerts of Arabia. It is bounded on the north by the Black Sea and Circaſſia; on the eaſt by Perſia; on the ſouth by Arabia and the Levant ſea; and on the weſt by the Archipelago and the Helleſpont.

This part of Turkey was the principal ſcene of all the tranſactions recorded in ancient writ, ſacred and profane.

TARTARY is an extenſive country taken in its full extent, and ſtretches from Muſcovy on the weſt, to the Pacific Ocean on the eaſt; and from the nations of China, India, Perſia, and Turkey, on the ſouth, to the impenetrable regions of the north. It extends from the thirtieth degree of north latitude to the frozen regions of the north pole; and from 50 to 190 degrees eaſt longitude; and contains Ruſſian, Chineſe, Mo-

gulean, and Independent Tartary, which are its four grand divifions, 4000 miles in length, and 2400 in breadth.

Through fuch an extenfive tract of country the foil and climate muft neceffarily partake of a great variety.

Their manners, language, &c. muft alfo be as various.

CHINA lies on the eaftern borders of the continent of Afia, and is divided from Chinefe Tartary on the north, by a prodigious wall, and, in fome places, by inacceffible mountains; on the eaft it is bounded by the Yellow Sea and Pacific Ocean, which feparates it from America; on the fouth by the Chinefe fea, and the kingdom of Tonquin; and on the weft by Tibet. It extends from 21 to 44 degrees north latitude, and from 94 to 133 degrees eaft longitude.

In fuch an extenfive country there muft no doubt be a variety of climates. The fouthern parts are very hot, and have violent rains, while the northern parts are very cold, and their rivers frozen for fome months during the winter; but the middle parts are temperate and pleafant. The foil alfo partakes of a great variety, though there is no part of this extenfive country but is fruitful, either from nature or art; for fuch is the induftry of the Chinefe, that they fuffer very little, if any land, to lie uncultivated.

The Chinefe have a confiderable trade with every European nation, and with North America, exporting filks, cotton, gold and filver ftuffs, painted gauzes, teas, china-ware, paper, and Indian ink, for which they receive ready money; defpifing the manufactures of every other country but their own.

There are a great number of natural and artificial curiofities in China. Among the latter are reckoned the famous wall which divides China from Tartary, extending over mountains and vallies, of 1500 miles in length, and from 20 to 25 feet in height, and broad enough for fix horfemen to travel abreaft. It has ftood near 1800 years, and is now almoft entire. 2. Their canals are works of great magnitude, infinitely

infinitely exceeding thofe in Europe. 3. The bridge over the river Saffrany, which confifts of a fingle arch, whofe fpan is 400 cubits, and its height 500. 4. The Cientao, or road of pillars, which is a road broad enough for four horfes to travel abreaft, and near four miles in length, defended by an iron railing; and unites the fummits of feveral mountains, in order to avoid the winding of the roads. It refts upon ftrong ftone pillars for the moft part. 5. The bridge of chains, which is a bridge built upon a number of ftrong iron chains, and hangs over a very deep valley, in the neighbourhood of King-Tung. 6. The triumphal arches of China, of which there are above 1100; 200 of them are very magnificent; they were erected in memory of their great princes, legiflators, &c. 7. The tower of Nan-King, called the Porcelain Tower, being wholly covered with the moft beautiful china; upwards of three hundred feet in height, nine ftories high; each ftory decreafing gradually to the top. The whole forms the moft correct and grand piece of architecture to be met with in the Eaft.

Among the natural curiofities may be reckoned their waterfalls and volcanoes.

Their religion is that of Paganifm; the deities are men that have been eminent in arts and fciences. They alfo worfhip inanimate beings, as mountains, woods, and rivers; but they acknowledge only one Supreme Being.

INDIA, or HINDOSTAN, is an extenfive country taken in its full extent. Bounded on the north by Tibet and Ufbeck Tartary; on the fouth by the Indian Ocean; on the eaft by China and the Pacific; and on the weft by Perfia and the Indian Ocean. It extends from 1 degree to 40 degrees north latitude, and from 66 to 109 degrees eaft longitude; and is principally divided into three parts:—1. The peninfula of India beyond the Ganges, on the eaft; 2. the main land, or empire of the Great Mogul, on the north; 3. the peninfula within the Ganges, or on this fide of it, on the weft.

A great part of the fea-coaft of India, as well as confiderable

diſtricts in the interior, belong to the Engliſh Eaſt India Company, where there are many large and rich ſettlements, from which we receive great quantities of Eaſt India commodities.

As the country extends through ſo many degrees of latitude, there is a great difference in the climates of the different parts. In the northern parts the air is very dry and healthy; but in the ſouthern parts near the ſea, in low lands, the air is very hot and moiſt: they divide the year into the dry and wet ſeaſons.

The ſoil, in general, throughout the whole country, is very fruitful, producing all the variety of plants, drugs, and fruits, to be met with in the other tropical climates. There are alſo mines of gold, diamonds, rubies, topazes, and other precious ſtones.

In the European ſettlements the religion is Chriſtianity; but in the northern and inland parts they are either Mahometans or Pagans; and divided into ſeveral kingdoms, each of which is governed by one or more abſolute monarchs.

Persia extends from 25 to 45 degrees north latitude, and from 45 to 67 degrees eaſt longitude. It is bounded on the eaſt by the Mogul's dominions; on the north by Uſbeck Tartary, the Caſpian Sea, and Circaſſia; on the ſouth by the Indian Ocean and Gulf of Perſia; and on the weſt by Arabia and the Turkiſh empire.

The climates of this country are very various. In the northern parts, and near the mountains, which are covered with ſnow, the air is very cold; in the midland parts it is ſerene, pure, and healthy; but towards the ſouthern parts there are ſometimes hot ſuffocating winds, which blow over a ſandy deſert from ſouth and eaſt; a blaſt of which has ſometimes ſtruck the unwary traveller with death in an inſtant. The ſoil is various, being in ſome parts very barren, but where it is well watered it is very fruitful.

The principal commodities of traffic are ſilks, camlets, carpets, leather, embroidery, gold and ſilver threads, mohair, &c.

The national religion of Perfia is that of Mahometifm, and the fect of Ali.

Arabia extends from 35 to 60 degrees eaft longitude, and from 12 degrees 30 minutes to 30 degrees north latitude. It is bounded on the north by Afiatic Turkey; on the fouth by the Indian Ocean; on the eaft by the Euphrates and Gulf of Baffora; and on the weft by the Red Sea.

Arabia is divided into three parts, viz. Arabia Petræa, or the Stony; Arabia Deferta, or the Defert; and Arabia Felix, or the Happy.

Arabia the Stony is the wildernefs in which the children of Ifrael fojourned 40 years: and in it may be feen the mountains of Horeb and Sinai, mentioned in Sacred Writ.

Arabia the Defert principally confifts of a large fandy defert; it has, however, a few fpots of fruitful land, covered with verdure, which are interfperfed in different parts of the defert. It is over this defert that fome of the eaftern nations bring their commodities of traffic from the Eaft, travelling in large caravans.

Arabia the Happy is, in general, barren; but fome of the vallies between the mountains, and thofe plains which are well fupplied with water, are very fruitful. From this part great quantities of drugs are exported to Europe, and alfo Turkey coffee.

The Arabs are, in general, a wandering people: many of their tribes live wholly in tents, and fubfift partly by robbing the caravans which travel through the defert, and partly by the produce of their country, and the flefh of their cattle; raifing no grain of any kind for domeftic ufe.

Their religion is that of Mahometifm; but many of the tribes are ftill Pagans. Their language is faid to exceed even the Greek itfelf in copioufnefs. The Arabians have never yet been fubdued by any military force, though feveral attempts have been made for that purpofe.

## SECT. VIII.

### OF AFRICA.

THE continent of Africa is in the form of a peninfula, furrounded on each fide by water, except where it joins to Afia by the Ifthmus of Suez. Several countries, famous in antiquity for the arts and fciences, were fituated in the northern parts of this quarter. And in the early days of Chriftianity feveral Chriftian churches were founded here; but at the prefent period Mahometifm and idolatry degrade this moft fertile quarter of the globe. That moft inhuman commerce, trafficking in men, alfo is carried on here by the European nations.

The ancients believed the greater part of this quarter of the globe to be uninhabited, as alfo the greater part of Afia, and, indeed, all that part of the globe lying between the tropics; but modern travellers have difcovered, that the tropical countries are in general the moft fertile and beft populated; and of thefe the fouthern and interior parts of Africa are found the moft eligible, both for vegetation and population. Its fea-coafts are the only parts with which we are particularly acquainted; but travellers are now bufily employed in making difcoveries in the internal parts.

Africa is bounded on the weft by the Atlantic Ocean; on the north by the Mediterranean; on the eaft by the Red Sea; and on the fouth by the Southern Ocean. It lies between 37 degrees north, and 36 degrees fouth latitude, the equator running nearly through the middle thereof; and between 17 degrees weft, and 51 degrees eaft longitude. In length, from north to fouth, it is about 4600 miles; and in breadth, from eaft to weft, 3500 miles.

EGYPT

EGYPT is bounded on the north by the Iſthmus of Suez; on the eaſt by the Red Sea; on the ſouth by Nubia; and on the weſt by the interior parts of Africa. It lies between 30 and 36 degrees eaſt longitude; and between 20 and 32 degrees north latitude; and is divided into Upper and Lower Egypt.

The climate, during the ſummer ſeaſon, is exceſſively hot; when the ſouth winds often raiſe ſuch a cloud of ſand as to obſcure the light of the ſun, and cauſe epidemical diſeaſes.

The ſoil is exceedingly fruitful, owing to the annual overflowing of the Nile. This river, ſo famous in ancient hiſtory, has its riſe in Abyſſinia, at between 11 and 12 degrees of north latitude, and purſues a northern courſe for above 1500 miles; when it divides into two branches, about ſix miles below Grand Cairo; one branch extending eaſtward, and the other weſtward. It begins to riſe in the beginning of ſummer, and increaſes three or four inches in height each day, for the firſt week: the next fortnight it increaſes in a ſtill greater proportion: and it is near four months before it is reduced into its channel again. The principal cities and towns are built on eminences on the banks of the Nile, and, during the inundation, correſpond with each other by means of boats. When the Nile riſes to the height of 49 feet, it produces a plentiful ſeaſon, but if it exceed that height it is productive of great miſchief, ſweeping away both houſes and cattle.

In Egypt they generally have three crops in a year: the firſt, of lettuces and cucumbers; the ſecond, of corn; the third, of melons, and all the fruits common to hot climates.

Their paſtures are the richeſt in the world, the graſs being uſually as high as the cattle.

Their trade conſiſts of great quantities of flax and cotton, both prepared and unmanufactured; leather of different kinds; alſo a great variety of drugs, and roots for dying.

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The common language ſpoken here is the vulgar Arabic, as it is under the dominion of the Turks.

BARBARY extends from Egypt to the Atlantic Ocean, and from the Mediterranean Sea to the Libyan Deſerts, being 750 miles in breadth, and near 2000 in length: containing the countries of Morocco and Fez, which form one diſtinct empire; and the ſtates of Algiers, Tunis, Tripoli, and Barca, compoſing ſeveral diſtinct ſtates, united together in confederacy, under the Turkiſh government.

Its ſoil is exceedingly fruitful, producing excellent corn, cattle, and paſture, and all the variety of tropical fruits; and vaſt quantities of fiſh and fowl; alſo a great variety of tame and wild animals.

The commerce of this country is chiefly carried on by caravans: their exports conſiſt of leather, mats, handkerchiefs, carpets, elephant's teeth, oſtrich feathers, copper, tin, wool, fruits, gum, drugs, &c. for which they receive timber, artillery, gunpowder, &c.

Their religion is that of Mahometiſm. Their language varies according to the different parts of the country. That ſpoken in the inland parts, is either an African language, or a corrupt Arabic. The latter is alſo ſpoken in moſt of the ſea-port towns: but in ſome parts they uſe a mixed language, ſuch as is ſpoken in moſt of the Mediterranean ports.

Moſt of the Barbary ſtates ſubſiſt by piracy: and their ſailors fight deſperately when they meet a veſſel belonging to any power with whom they are at war.

The government is that of an abſolute monarchy. The emperor is in general both judge and executioner; and he acknowledges the Grand Seignior of Turkey to be his ſuperior. When there is a vacancy in the government, every ſoldier in the army has a vote in chooſing a new emperor, which is often attended with great bloodſhed.

The parts of Africa, from the tropic of Cancer to the Cape of Good Hope, are very little known, except the ſea-coaſt thereof. The natives in general are black, except thoſe

thofe of Abyffinia, who are of a tawney complexion, and are a mixture of Jews, Chriftians, and Pagans. The religion of the other countries in this part is generally that of Paganifm, and the form of government every where monarchical, except in a few fettlements formed by the Europeans, on the fea-coaft. Few of their princes, however, poffefs an extenfive degree of territory. As the natives are ignorant of all the arts of utility and refinement, the different kingdoms are therefore unconnected with each other, and are generally at war.

The foil of Africa is in general very fruitful: though in fome parts it is perfectly barren, particularly where there is very little water; the heat of the fun reducing the foil to a perfect fand; fuch are the countries of Anian and Zaara; but the countries of Mandingo, Ethiopia, Congo, Angola, Batua, Truticui, Monomotapa, Cafati, and Melunenrugi, are extremely fruitful, and very rich in gold and filver.

On the weftern coaft the Englifh trade is carried on at James's Fort, and other fettlements, near and up the river Gambia, where woollen and linen cloths, hardware, and fpirituous liquors are exchanged for the perfons of the natives. Many of the negroes will fell their own families for thofe fuperfluities. The natives are often trepanned by foreigners, or their own countrymen, and then fold to the Europeans: and many more are fold by the princes of the different ftates, being captives taken in war. Gold and ivory form the principal branches of commerce, next to that of the flaves.

The Portuguefe poffefs the greater part of the eaft and weft coaft of Africa, from the tropic of Capricorn to the equator. The Dutch have fome fettlements towards the fouthern parts of the continent; and Cape Town, at the Cape of Good Hope, belongs to them, and is well fortified, and where the fhips bound for India ufually put in, and trade with the natives, or Hottentots, for their cattle and

 other

other provifions, for which they give them fpirituous liquors. There are feveral iflands near the coaft of Africa, lying in the Eaftern or Indian Ocean, or in the Weftern or Atlantic Ocean, of which the chief are:

1. Zocotra, fituated in 53 degrees eaft longitude, and 12 degrees north latitude; 30 leagues eaft of Cape Guardafui, on the continent of Africa; being 80 miles in length, and 54 in breadth, and has two good harbours. It is a populous, plentiful country, governed by a prince who is tributary to the Porte.

2. Babelmandel, fituated in the ftrait of the fame name, at the entrance of the Red Sea, in 44 degrees 30 minutes eaft longitude, and 12 degrees north latitude, being a fmall fandy ifland, not five miles round.

3. The iflands of Joanna, Mayotta, Mohilla, Angezeia, and Comora, fituated between 41 and 46 degrees eaft longitude, and between 10 and 14 degrees fouth latitude: the chief of thefe is Joanna, to which the others are tributary, being 30 miles long, and 15 broad: affording excellent fruits and provifions. The natives are a friendly fet of people, and profefs the Mahometan religion.

4. Madagascar, the largeft of the African iflands, fituated between 43 and 51 degrees eaft longitude, and between 10 and 26 degrees fouth latitude; three hundred miles fouth-eaft of the continent of Africa, being near 1000 miles in length, from north to fouth; and between 2 and 300 miles in breadth. Between this ifland and the Cape of Good Hope, or the continent of Africa, the fea rolls with great force, and is exceedingly rough. In this channel, all European fhips pafs in their voyage to and from India, except the water be too rough. Madagafcar is a fertile country, abounding in all the variety of fruits and vegetables to be met with in the fame climate. The air is temperate alfo, and healthy. It is inhabited by both blacks and whites, profeffing different religions; but principally Mahometifm and Paganifm; and governed by feveral petty princes.

5. Mau-

5. MAURITIUS, or Maurice, fituated in 56 degrees eaft longitude, and 20 degrees fouth latitude; about 400 miles eaft of Madagafcar. It is of an oval form, and about 150 miles in circumference, with a large fine harbour. The climate is healthy and pleafant; and the ifland is well watered with feveral rivers: though the foil is not fo fruitful as that of the former, it neverthelefs feeds a great number of cattle, fheep, deer, and goats.

6. BOURBON, fituated in 54 degrees eaft longitude, and 21 degrees fouth latitude; about 300 miles eaft of Madagafcar, and about 90 miles in circumference. Surrounded for the moft part with blind rocks, a few feet under water. The climate is in general healthy, though hot. It affords very good pafture and cattle.

There are feveral other fmall iflands about Madagafcar, and on the eaftern coaft of Africa.

7. ST. HELENA, fituated in 6 degrees weft longitude, and 16 degrees fouth latitude; being 1200 miles weft from the continent of Africa, and 1800 eaft from South America. The whole ifland is fituated on a rock, and is about 21 miles in circumference. There is but one landing-place in the ifland, which is at the eaft fide thereof. It is very fertile, diverfified by hills and vallies, and abounds in all the conveniences and comforts of life. There are about 200 families, moftly defcended from Englifh parents.

8. ASCENSION, fituated in 7 degrees 40 minutes fouth latitude, and 600 miles north-weft of St. Helena. It is a mountainous barren ifland, and uninhabited; about 20 miles round.

9. ST. MATTHEW, fituated in 6 degrees 1 minute weft longitude, and 1 degree 30 minutes fouth latitude; and uninhabited.

10. CAPE VERD ISLANDS are fituated between 23 and 26 degrees weft longitude, and between 14 and 18 degrees north latitude. They are about 20 in number; but the principal are St. Jago, Bravo, Fogo, Mayo, Bonavifta, Sal,

B St.

St. Nicholas, St. Vincent, Santa Cruz, and St. Antonio. They moſtly belong to the Portugueſe and Spaniards. The air in general is very hot, and in ſome unwholeſome. They are inhabited by Europeans and their deſcendants.

11. Goree, ſituated in 14 degrees 43 minutes north latitude, and 17 degrees 20 minutes weſt longitude. It is a ſmall ſpot not exceeding two miles in circumference; but an important ſituation for trade.

12. The Canaries, or Fortunate Islands, are ſeven in number, and ſituated between 12 and 19 degrees weſt longitude, and between 27 and 29 degrees north latitude. Theſe iſlands have a pure temperate air, and abound in moſt delicious fruits, from whence they have thoſe rich wines called Canary, of which they export 10,000 hogſheads annually.

13. The Madeiras are three iſlands, ſituated in 32 degrees 27 minutes north latitude, and between 18 degrees 30 minutes and 19 degrees weſt longitude. Theſe iſlands are moſtly famous for producing the Madeira wine, of which no leſs than 20,000 hogſheads are annually exported.

14. The Azores, or Western Islands, are ſituated between 25 and 32 degrees weſt longitude, and between 37 degrees and 40 degrees north latitude. Being 900 miles weſt of Portugal; and lying in the midway between Europe and America. Of theſe St. Michael is the largeſt, being near 100 miles in circumference, and containing 50,000 inhabitants. Tercera is the moſt important of theſe iſlands, on account of its harbour, which is very ſpacious, and affords good anchorage. There are ſeven other of theſe iſlands: their names are Santa Maria, St. George, Gracioſa, Fayal, Pica, Flores, and Corvo.

SECT.

## SECT. IX.

### OF AMERICA *.

AMERICA, or the Great Weftern Continent, frequently called the New World, extends from the 80th degree of north latitude, to the 56th degree of fouth latitude; and where the breadth is known, from the 35th degree, to the 136th degree weft longitude; extending near 9000 miles in length, and 3690 in breadth. As it extends into both hemifpheres, it has two fummers and two winters. It is wafhed by the two great oceans, the Atlantic and Pacific; having the former on the eaft, and the latter on the weft: by thefe feas it has a direct communication with the other three quarters of the world. It is compofed of two great continents, North and South America, connected together by the kingdom of Mexico, which is an ifthmus of 1500 miles long, and in one part only 60 miles broad.

AMERICA is the beft watered of any part of the globe; even thofe vaft tracts of country fituated beyond the Apalachian Mountains, at an immenfe diftance from the ocean, are watered by inland feas, as the Lakes of Canada, which give rife to feveral large rivers, as the Miffiffippi, the Miffouri, the Ohio, and on the north, the river St. Lawrence, all of them being navigable to their heads, which is a great advantage for commerce.

SOUTH AMERICA is better watered, if poffible, than North America, having the two largeft rivers in the world: viz.—

* To fuch perfons as wifh to make themfelves thoroughly acquainted with this part of the world, I would recommend a perufal of the Rev. W. Winterbotham's Hiftorical, Geographical, Commercial, and Philofophical View of the United States of America, in four large volumes 8vo. illuftrated by a complete Atlas and other plates, price 1*l.* 16*s.*

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the river of Amazons, and the river of La Plata; the former having a courfe of about 3000 miles.

A country of fuch vaft extent on both fides of the equator muft neceffarily have all the varieties of foils and climates to be met with in every other part of the globe. It alfo produces moft of the metals, minerals, plants, fruits, trees, and wood, to be met with in the other parts of the world, and many of them in greater quantities and higher perfection.

This country likewife produces diamonds, pearls, emeralds, amethyfts, and other precious ftones; alfo cochineal, indigo, anatto, logwood, brazil, fuftic, pimento, lignum vitæ, rice, ginger, cocoa or chocolate, fugar, cotton, tobacco, the balfams of Peru, Tolu, and Chili, Jefuit's bark, mechoacan, faffafras, farfaparilla, caffia, tamarinds, and a great variety of other woods, roots, and plants, many of which were not known before the difcovery of America; befides hides, furs, and ambergris.

Though the Indians ftill live in quiet poffeffion of many large tracts of country, in the inland parts, yet America, fo far as is known, is generally claimed by four powers: viz.—the Spaniards, Englifh, Portuguefe, and American fettlers, being the defcendants of Europeans, and who have the largeft fhare of country, except the Spaniards, who poffefs the largeft and moft extenfive portion of all, extending from New Mexico and Louifiana, in North America, to the Straits of Magellan, in South America; except the large province of Brazil, which belongs to Portugal, Surinam, claimed by the Dutch, and Cayenne, the property of the French, all in South America. The United States of America poffefs all that tract of country which is bounded by the Miffiffippi, the river St. Lawrence, and the Lakes of Canada, on the north and weft; and wafhed by the Atlantic Ocean on the eaft; and on the fouth by the Gulf of Mexico.

The American Islands, commonly called the Weft Indies,

Indies, was the firſt of America diſcovered by the Europeans, and are ſituated in the gulf called the Caribbean Sea, between the continents of North and South America, extending from the coaſt of Florida, to the river Oroonoko: they are divided between five European nations: viz.—the Engliſh, French, Spaniards, Dutch, and Danes.

As all theſe iſlands lie between the tropics, their climates and ſoil are pretty much alike: the heat would be intolerable, if it were not for the trade-winds which blow during the fore part of the day, and the ſea and land breezes. Their ſeaſons are divided into the wet and dry: in the wet ſeaſons, the rain pours down with ſuch impetuoſity as to overflow the rivers, and lay the low country under water.

The principal trade of the Weſt Indies conſiſts of ſugar and rum: they alſo export cotton, indigo, chocolate, coffee, and dying and phyſical drugs, ſpices, and hard woods; for which they receive from Europe, manufactures; from the African iſlands, wine; and from the neighbouring continent, lumber and proviſions.

The Bahama Islands, which are ſaid to be 500 in number, lie to the ſouth of Carolina, between 21 and 27 degrees north latitude, and between 73 and 81 weſt longitude. There are, however, not above twelve of them of any magnitude, the reſt being little better than rocks or banks, and almoſt uninhabited, except Providence Iſland.

The Bermudas, or Summer Islands, lie in the Atlantic Ocean, about 500 leagues eaſt from Carolina, in 32 degrees north latitude, and in 65 degrees weſt longitude: theſe are ſaid to be about 400 in number; but containing not more than 20,000 acres.

The iſlands of Newfoundland, Cape Breton, and St. John, lie at the mouth of the river St. Lawrence; and are celebrated for the quantity of fiſh found on their coaſts, which is ſuppoſed to increaſe the national ſtock upwards of 300,000*l.* annually: in this branch of commerce 3000 ſmall craft are employed, and 10,000 hands.

British America, or the territories on the continent belonging to the Engliſh, are New Britain, Canada, or the province of Quebec, and Nova Scotia, or Acadia: bounded on the eaſt and ſouth by the Atlantic Ocean and the American States; on the north and weſt, their boundaries have never been defined, but are blended with the lands of the Indian nations. New Britain contains Labrador, and New North and South Wales. Canada contains the towns of Quebec, Trois Rivieres, and Montreal, all ſituated on the river St. Lawrence.

Nova Scotia contains the towns of Halifax, Annapolis, and St. John's.

The United States of America are bounded on the weſt by the Indian nations; on the north, by Britiſh America; on the weſt, by the Atlantic; and on the ſouth, by Spaniſh America; containing the following ſtates or colonies: New Hampſhire, Maſſachuſetts, Rhode Iſland, Connecticut, New York, New Jerſey, Pennſylvania, Delaware, Maryland, Virginia, North Carolina, South Carolina, Georgia, Vermont, Weſtern Territory, and Kentucky.

The United States, in the year 1776, were only 13 in number: Vermont, Kentucky, and the Weſtern Territory, have been added ſince. The Weſtern Territory is of ſuch extent, that the Congreſs have determined to divide it into ten new ſtates.

From the lateſt accounts, it appears, that the population of the United States amounts to upwards of 3,083,600 perſons, who are compoſed of almoſt all nations, languages, characters, and religions: the greater part, however, have deſcended from the Engliſh.

The language generally ſpoken through all theſe ſtates is the Engliſh, in which all their civil and eccleſiaſtical matters are performed, and their records kept.

There are, however, great numbers of Dutch, French, Germans, Spaniards, Jews, and Swedes, who retain in a great degree each their native language, and have their reſpective

ſpective places of worſhip; and in general live comfortably and unmoleſted, as to principles of conſcience.

NEW ENGLAND is bounded on the north by Canada; on the eaſt by Nova Scotia and the Atlantic Ocean; on the ſouth by the Atlantic; and on the weſt by New York; and is divided into five ſtates: viz. New Hampſhire, Maſſachuſetts, Rhode Iſland, Connecticut, and Vermont, which are ſubdivided into counties, and thoſe counties again ſubdivided into townſhips.

New England is a fine country for paſtures; the valleys are generally interſected with brooks of water, the banks of which are covered with a tract of rich meadow land.

The ſtate of NEW YORK is bounded on the ſouth-eaſt by the Atlantic Ocean; on the eaſt by Connecticut, Maſſachuſetts, and Vermont; on the north by Canada; on the ſouth and ſouth-weſt by Pennſylvania and New Jerſey: being 350 miles in length, and 300 in breadth; and containing about 44,000 ſquare miles, equal to 28,160,000 acres. The river St. Lawrence divides this ſtate from Canada. The ſettlements formed in this ſtate are chiefly upon two oblongs, extending from the city of New York eaſt and north. The eaſt is Long Iſland, which is 140 miles in length; the other, extending north, is about 40 miles in breadth. This ſtate exports to the Weſt Indies, biſcuits, peas, Indian corn, apples, onions, boards, ſtaves, horſes, ſheep, butter, cheeſe, pickled oyſters, beef, and pork: but the principal part of their trade is wheat, of which, in the year 1775, they exported 677,700 buſhels, and 2555 tons of bread, beſides 2828 tons of flour, for which they receive in exchange the commodities of the Weſt India iſlands.

NEW JERSEY is 160 miles in length, and 52 in breadth. Bounded on the eaſt by Hudſon's river and the ſea; on the ſouth by the ſea; on the weſt by Delaware bay and river, which divides it from Pennſylvania; and on the north by a line drawn from the mouth of Mahakkamak river, in lati-

tude 41 degrees 24 minutes, to a point in Hudſon's river, in latitude 41 degrees; containing about 8320 ſquare miles.

This ſtate has a great variety of ſoil, from the worſt to the beſt. But it contains a greater portion of barren land than any other ſtate, there being nearly one fourth of this ſtate unfit for cultivation. But thoſe parts which are fruitful are equal in fertility to any part of the United States.

The ſtate of PENNSYLVANIA is bounded by Delaware river on the eaſt; by the ſtate of New York on the north; by the parallel of latitude, 39 degrees 43 minutes 18 ſeconds, on the ſouth; and by a meridian line drawn from the ſaid parallel at five degrees of longitude, from a point on Delaware river weſt. This ſtate lies in the form of a parallelogram. The north ſide of Pennſylvania is the beſt ſoil, and moſt populated, owing to the great number of new roads which have lately been made.

The ſtate of DELAWARE is bounded on the north by the territorial line (which is a circle), deſcribed with a radius of 12 Engliſh miles, and whoſe centre is in the middle of the town of Newcaſtle, which divides it from Pennſylvania; on the eaſt by Delaware river and bay; on the ſouth by a line drawn due eaſt and weſt from Cape Henlopen in 38 degrees 30 minutes north latitude, to the middle of the peninſula, which line divides the ſtate from Worceſter county, in Maryland; and on the weſt by Maryland: containing about 1400 ſquare miles; being 92 miles in length, and 16 in breadth.

The ſtate of MARYLAND is bounded on the north by Pennſylvania; on the eaſt by the Delaware ſtate; on the ſouth and ſouth-eaſt by a line drawn from the ocean over the peninſula (dividing it from Accomac county in Virginia), to the mouth of Patomac river, and from thence up the river to its firſt ſource; from thence, by a due north line, till it interſects the ſouthern boundary at Pennſylvania: being 134 miles in length, and 110 in breadth. The ſoil of Maryland, where it is good, will produce from 12 to 16 buſhels of

wheat,

wheat, or from 20 to 30 bufhels of Indian corn, per acre. Their trade is chiefly with the other ftates, the Weft Indies, and fome parts of Europe: to which places they annually export 30,000 hogfheads of tobacco, befides great quantities of pig-iron, lumber, flax-feed, and provifions.

The ftate of VIRGINIA is bounded on the eaft by the Atlantic; on the north by Pennfylvania and the river Ohio; on the weft by the Miffiffippi; and on the fouth by North Carolina: being 758 miles in length, and 224 in breadth.

The ftate of KENTUCKY is bounded on the north-weft by the river Ohio; on the weft by Cumberland river; on the fouth by North Carolina; and on the eaft by Sandy river, and a line drawn full fouth from its fource to the northern boundary of North Carolina: being 250 miles in length, and 200 in breadth. The fertility of the foil is fuch, that the land, in common, will produce 30 bufhels of wheat, or rye, an acre. The beft lands are too rich for wheat, and will produce from 50 to 80 bufhels of good corn per acre; and few foils yield more and better tobacco.

The ftate of NORTH CAROLINA is bounded on the north by Virginia; on the eaft by the Atlantic; on the fouth by South Carolina and Georgia; and on the weft by the Miffiffippi: being 758 miles in length, and 110 in breadth.

The ftate of SOUTH CAROLINA is bounded on the eaft by the Atlantic; on the north by North Carolina; on the fouth and fouth-weft by the Savannah river. The weftern boundary is not afcertained. It is reckoned 200 miles in length, and 125 in breadth.

The ftate of GEORGIA is bounded on the eaft by the Atlantic; on the fouth by Florida; on the weft by the river Miffiffippi; on the north and north-eaft by South Carolina: being 600 miles in length, and 250 in breadth.

The WESTERN TERRITORY includes all that part of the United States on the north-weft of the river Ohio; bounded on the weft by the Miffiffippi river; on the north by the lakes;

on the eaſt by Pennſylvania; and on the ſouth and ſouth-eaſt by the river Ohio; containing 411,000 ſquare miles, equal to 263,040,000 acres, from which deducting 43,040,000 acres occupied by the water, there remain 220,000,000 acres, which are to be ſold by Congreſs, for the diſcharge of the national debt.

## *Territories of Spain in North America.*

The dominions of Spain, in North America, extend from 81 degrees to 120 degrees weſt longitude, and from 8 to 43 degrees north latitude. Bounded on the north by the United States and the Indian nations; on the weſt by the Pacific Ocean; on the eaſt by the Gulf of Mexico and the Atlantic; and on the ſouth terminating in the Iſthmus of Darien. They contain the following countries, viz. Eaſt Florida, Weſt Florida, New Mexico, California, and Old Mexico.

The ſoil of this extenſive tract of country is very various, but in general fertile: and it is in the moſt mountainous parts that the mines of gold and ſilver are found. The air is in general warm and pleaſant; but the northern parts have very cold winds; and the ſouthern parts, lying within the torrid zone, are exceedingly hot.

## *South America.*

South America, from the northern coaſt of Terra Firma, and the Iſthmus of Darien, to the Straits of Magellan, belongs to the Spaniards, except the province of Brazil, which belongs to the Portugueſe; and the ſettlements of the Dutch in Surinam, and thoſe of the French in Cayenne.

Brazil,

Brazil, belonging to the Portugueſe, extends from the equator to 35 degrees ſouth latitude, and from 35 to 60 degrees weſt longitude. Bounded on the north by the mouth of the river Amazons and the Atlantic; on the eaſt by the Atlantic; on the ſouth by the mouth of the river Plata; and on the weſt by the unknown country of the Amazons.

Cayenne is the only ſettlement in the ſouthern continent of America retained by the French, and is ſituated between the equator and 5 degrees north latitude; and between 52 degrees 15 minutes, and 55 degrees 30 minutes weſt longitude. It extends 240 miles along the coaſt of Guiana, and near 300 miles inland. Bounded by Surinam on the north; by the Atlantic on the eaſt; by Amazonia on the ſouth; and by the territories of the Indians on the weſt.

Surinam, or Dutch America, lies between 5 and 7 degrees north latitude; and is bounded on the north by Cayenne; on the weſt by Terra Firma; on the ſouth by the Indian nations; and on the eaſt by the river Oroonoko.

The dominions of Spain, in South America, contain the ollowing extenſive countries, viz. Terra Firma, Peru, Chili, Paraguay, Amazonia, and Patagonia; extending, as before obſerved, through the whole continent of South America.

The climate and ſoil of Spaniſh America vary greatly, from the hot burning ſand and ſmoking ſwamp in the northern parts in the torrid zone, to the cold region of the ſouthern part, near Cape Horn.

The iſlands of South America are Terra del Fuego, the Falkland Iſlands, and the iſland of Juan Fernandes; the latter of which gave riſe to the famous ſtory of Robinſon Cruſoe, from one Alexander Selkirk, mariner, native of Scotland, who was put aſhore on this iſland by his captain, in the year 1697, and diſcovered by Woodes Rogers in the year 1709, who took him on board, and brought him to Europe; after having been on this uninhabited iſland for twelve years.

The

The number of inhabitants in the known parts of the world, computed at a medium from the beſt calculations, are about nine hundred and fifty-three millions; viz.

| | |
|---|---|
| Europe contains | 153 millions |
| Aſia - - - | 500 |
| Africa - - - | 150 |
| America - - | 150 |
| | 953 |

# CHAP. XIV.

# OF ASTRONOMY.

## SECT. I.

### OF THE PRIMARY PLANETS.

Before I proceed to the deſcription of the Primary Planets, it will be neceſſary to take a view of the ſolar ſyſtem, with the order and economy of the motions and courſes of thoſe planets.

The ſyſtems which have been moſt generally received in aſtronomy, are the *Ptolomaic*, the *Copernican*, *Pythagorean*, and the *Tychonic*.

The

The *Ptolomaic* ſyſtem, ſo called from its inventor Ptolomy, ſuppoſes the earth to be placed at reſt in the centre of the univerſe, and the heavens revolving about it from eaſt to weſt, in the ſpace of twenty-four hours, and by this motion carrying the ſun, ſtars, and planets, completely round the earth in that ſpace of time.

It was this ſyſtem which Ariſtotle, Hipparchus, and moſt of the philoſophers of antiquity defended ſo ſtrenuouſly, and was followed by the whole world for many ages, and longer retained in many learned univerſities. But latter improvements and more evident demonſtrations have now utterly exploded it.

In the *Tychonic* ſyſtem, invented by Tycho, the Dane, the earth is alſo ſuppoſed to be fixed in the centre of the univerſe, and all the heavenly bodies performing a revolution round it in the ſpace of twenty-four hours, as in the Ptolomaic ſyſtem; but with this difference, that it allows a monthly motion to the moon round the earth, and alſo the proper motions of the ſatellites about Jupiter and Saturn. It alſo ſuppoſes the ſun to be the centre of all the primary planets: the primary planets being carried round the ſun in their reſpective periods, while the ſun, with all the aforeſaid planets, revolve round the earth every twenty-four hours. But this ſyſtem was ſo inconſiſtent with obſervations, that it had but few followers. It was therefore altered by Longomontanus and others, who allowed the diurnal motion of the earth on its own axis, but denied its annual motion round the ſun. This improved hypotheſis is called the *ſemi-tychonic* ſyſtem.

But theſe ſyſtems have now given place to that called the *Copernican* ſyſtem, which undoubtedly is the moſt ancient in the world. It was firſt introduced into Greece and Italy by Pythagoras, and from him called the Pythagorean ſyſtem. It was adopted by Philolaus, Plato, Archimedes, and all the moſt ancient philoſophers, but was at length loſt under the Peripatetic philoſophy, and reſtored again about the year 1500, by Nic. Copernicus.

This fyftem has been proved by the moft evident demonftrations to be the only true one. I fhall therefore confine myfelf to the defcription of this alone, and the phenomena that arife from it.

Fig. 1. Plate 15, is a reprefentation of this fyftem, where the feven concentric circles, marked Mercury, Venus, the Earth, Mars, Jupiter, Saturn, and Georgian Sidus, reprefent the orbits of thefe feven primary planets, performing each its annual rotation round the fun, which is placed in the centre. The next two circles reprefent the twelve figns of the zodiac, with all its divifions into thirty degrees in each fign. And, laftly, the two next outer circles fhow the twelve calendar months of the year, with their divifions into days. Of each of which in their order.

A planet, in the literal fenfe of the word, fignifies a wanderer, or wandering ftar; and is therefore ufed in contradiftinction to the ftars which we call fixed. It is a celeftial body, globular and opake, and revolving around the fun, or fome other planet, as a centre, at leaft as a focus to its orbit, which always has a moderate degree of eccentricity, fo that it is never much further from the fun, or centre of motion, at one time than at another, in proportion to the diameter of its orbit.

The planets are either primary or fecondary.

Primary planets, fometimes called planets, by way of eminence, are thofe feven above defcribed, which move round the fun, as their common centre, or focus of their orbits.

Secondary planets, or fatellites, are fuch as move round fome primary planet, as their focus, in the fame manner as the primary planets move round the fun. Such are the moon, which moves round the earth, and the four moons of Jupiter, the feven moons of Saturn, and the fix of the Georgian Sidus.

The primary planets are divided into fuperior and inferior: the fuperior planets are thofe that perform their revolutions round the fun at a greater diftance from the fun, than the earth

Earth is, as Mars, Jupiter, Saturn, and the Georgian. The inferior planets are thofe included within the orbit of the earth, as Venus and Mercury; as may be feen in the figure.

The planets were formerly reprefented by the fame characters which the chemifts made ufe of, to reprefent their metals by, on account of a fuppofed analogy between the planets and thofe metals; and thefe characters are ftill ufed, to avoid confufion.

Thus, *Mercury*, anciently called the meffenger of the Gods, was fignified by the character ☿; which ftood for the metal mercury, and alfo bore a rude refemblance to the heathen deity of that name, being a man with wings on his head and feet.

*Venus*, the next planet in order, fo named from the Goddefs of Love, was characterized by ♀, for the figure of a woman; and denoted the metal copper.

*Tellus*, or the Earth, was characterized by ⊖; and is the third planet diftant from the Sun.

*Mars*, or the God of War, was denoted by ♂, and reprefented iron; and fuppofed to bear a refemblance to a man holding out a fpear.

*Jupiter*, the chief of the heathen gods, was marked ♃, to reprefent thunderbolts; and fignified the metal tin.

*Saturn*, the father of the gods, was reprefented by ♄, to refemble an old man, fupporting himfelf with a ftaff; the fame character being ufed for the metal lead.

The *Georgian*, or Herfchel, is denoted by ♅, the initial of Dr. Herfchel's name, with a crofs for the Chriftian planet.

The *orbit* of a *planet* is the path it defcribes in going round the Sun; they are reprefented in the figure by concentric circles. The Earth's orbit is called the ecliptic.

Kepler was the firft aftronomer who difcovered that the orbits of the planets were not circular, but of an elliptic form, in the form of the figure (fig. 2), having the Sun in one of the foci thereof: and he further difcovered thefe two

primary laws, from hence called *Kepler's Laws*: viz. that a radius drawn from the centre of the Sun to the centre of the planet, always describes equal areas in equal times; or, which is the same thing, in unequal terms, it describes areas proportional to those times: and the squares of the periodical times of the planets are as the cubes of the mean distances of the planets from their centres.

Thus, let *d e f* (fig. 2) be the orbit of a planet, or an ellipse, in which *c* is the centre, and *a b* the two foci of the ellipse, in one of which foci the sun is placed, as suppose *b*: then if the area *b h g* be equal to the area *b g f*, the planet will be as long in describing that part of its orbit from *h* to *g*, as from *g* to *f*. A straight line *e f* drawn through the two foci of the orbit is called the line of the apsides, and the point *e*, being the farthest from the Sun, or centre of motion, is called the higher apsis, or *aphelion*; and the point *f*, being that point of the line next the Sun, or centre of motion, is called the lower apsis, or *perihelion*: therefore, when the planet is in its aphelion, its motion is the slowest of all, and when in the perihelion the quickest.

The axis of a planet is an imaginary line, supposed to be drawn through the centre, about which the planet performs a diurnal rotation.

The method of describing an ellipse is this: having fixed two pins, or points, *a b*, with a piece of thread doubled, and a pen, or pencil, *d*, in the double of the thread, describe the ellipse *d e f*, keeping the thread extended to the full length by the pencil, *d*. And note: the two fixed points *a* and *b*, which hold the thread, are always the two foci of the ellipse; and the nigher these two foci are together, the nearer the ellipse will approach to the form of a circle; and the farther they are distant from each other, the farther the ellipse departs from a circular form. The distance *a c*, or *c b*, that is, the distance from the centre to either focus, is called the eccentricity.

The

The orbits of the planets are not all in the ſame plane; neither are any two of them in the ſame plane: but the plane of the ecliptic, or Earth's orbit, interſects the plane of the orbit of every other planet in a right line, which paſſes through the Sun, called the line of the nodes; and the points of interſection of the orbits are called the nodes.

The different inclinations of the axis of a planet to the plane of its orbit are the cauſe of the different ſeaſons of the year in that planet; and the diurnal rotation of the planet round its axis produces the ſucceſſive changes of the day and night. Thus, in the Earth's orbit (fig. 3), let A repreſent the Earth in the ſummer ſeaſon, on the 21ſt of June. N S is the axis of the Earth, N being the north, and S the ſouth pole, about which the Earth performs her revolution every twenty-four hours. Let *c a* repreſent the path deſcribed by the city of London, in the rotation of the globe round its axis. Now the reaſon why the days are longer than the nights at this ſeaſon of the year in London, will be evident, from a bare inſpection of the figure. For the axis of the Earth being inclined to a perpendicular drawn to the ecliptic, in an angle of twenty-three degrees and a half, and the city of London being ſituated in the fifty-ſecond degree of north latitude, which is nigher the north pole than to the equinoctial, it will not paſs through near ſo great a ſpace of the dark ſide of the Earth, or that ſide oppoſite to the Sun, as it will through the illuminated part of the Earth, or part next to the Sun. The nights, in this ſeaſon of the year, only continue while London deſcribes that part of its track, from *c* to *b*, and from *b* to *c*, on the other ſide: but as ſoon as it arrives at *b* it will be daybreak, and the day continues while London deſcribes that part of its track from *b* to *a*, and from *a* to *b* on the other ſide of the globe; and when it arrives at *a*, the Sun will be on the meridian, or it will be then noon-day. The length of the day, therefore, will be to that of the night, as the diſtance *c b* to that of *b a*.

But when the Earth arrives at B, which is on September

the 23d, the axis of the Earth always keeping in the fame pofition, the days will be equal to the nights; for the illuminated parts of the Earth will extend exactly from the north to the fouth poles; and therefore, in the paffage of the Earth from A to B, the length of the days will gradually decreafe in the northern latitudes, but increafe in the fouthern latitudes.

*Fig.* 1. *Mercury* is the fmalleft of all the primary planets, and neareft the Sun; performing his revolution in a lefs fpace of time than any of the reft, round the Sun, and with a very rapid motion. This occafioned the Greeks to give it its prefent name and character, calling it, as before obferved, the meffenger of the gods.

The mean diftance of this planet from the Sun, compared with that of the Earth from the Sun, is as 387 to 1000; therefore, his diftance from the Sun is about 37,000,000 miles, or little more than one third of the Earth's diftance from the Sun. Hence, the diameter of the Sun, feen from Mercury, will appear near three times as large as when feen from the Earth; confequently, that planet receives about feven times as much heat and light from the Sun, as this Earth does.—This is a degree of heat fufficient to make water boil.

The diameter of Mercury is not one third of that of the Earth, or about 2600 miles; therefore, its furface is nearly one ninth, and its folidity one twenty-feventh of that of the Earth.

The orbit of Mercury is inclined to the plane of the ecliptic, or Earth's orbit, in an angle of 6 degrees 54 minutes. He performs one entire revolution round the Sun in the fpace of 87 days 23 hours 15½ minutes; therefore, the fummers and winters in that planet cannot be more than 44 days each. His greateft elongation from the Sun, that is, the greateft diftance that he is feen by us to depart from the Sun, is 28 degrees 20 minutes: the eccentricity of his orbit is one fifth of his proportional mean diftance from the Sun, which

which is far greater than that of any of the other planets; and the pace with which he moves in his orbit is at the amazing rate of about 95,000 miles in an hour.

The place of his aphelion is 14 degrees 13 minutes of Sagittary; the place of his afcending node is 15 degrees 47 minutes of Taurus, and confequently that of the defcending node, 15 degrees and 47 minutes of Scorpio.

The length of his days, or rotation on his own axis, inclination of his axis to his orbit, gravity on his furface, denfity and quantity of matter, are all unknown.

Mercury is obferved to appear with various phafes, like the Moon; varying according to his various pofitions with regard to the Earth and Sun; therefore he never appears with a full face towards us, except when he is too near the Sun to be diftinctly feen; for his enlightened face is always towards the Sun. From thefe obfervations it is plain, that Mercury, like all the other planets, is a dark opake body, having no light of his own, but what he receives from the Sun; for, if he had, he would always appear completely round.

The beft time to make an obfervation on this planet, is when it is feen on the Sun's difk, called its tranfit; at which time it paffes before the Sun, and appears like a little round black fpot on the Sun's furface, eclipfing a fmall part thereof, and is only vifible through a telefcope. And as thefe tranfits can happen only when the Earth and Mercury are both in the fame node of Mercury, (that is, on the 6th of November, and the 4th of May;) and at which time Mercury muft alfo be in an inferior conjunction, it will follow, that at thefe times, when Mercury is in the inferior conjunction with the Sun, he will appear to pafs over the Sun's difk.

But in all the other parts of his orbit, he will never make a tranfit over the Sun, though he may be in an inferior conjunction, becaufe he goes either above or below the face of the Sun. Gaffendi, in November 1631, firft took an obfervation of this kind; and Mr. Whifton has given a lift of

feveral

ſeveral tranſits of Mercury: viz.—November 12th, at 3 hours 44 minutes, afternoon, in 1782; May 4th, at 6 hours 57 minutes, forenoon, 1786; November 5th, 3 hours 55 minutes, afternoon, 1789; and the 7th, 2 hours 34 minutes, afternoon, 1799.

*Venus*, the brighteſt, and, to appearance, the largeſt of all the planets, is the next inferior planet, and is diſtinguiſhed from all the reſt by her brightneſs, and white appearance: her light is ſo conſiderable, that in a duſky place ſhe will often cauſe an object to project a ſenſible ſhadow; and her brightneſs is ſo great, as often to render her viſible in the day-time.

As Venus moves round the Sun in an orbit within the Earth's orbit, like Mercury, ſhe can never be ſeen in oppoſition to the Sun, though ſhe departs farther from him than Mercury does; her greateſt diſtance from the Sun being about 47 degrees 48 minutes, as ſeen from the Earth.

When ſhe is on the weſt of the Sun, which happens from her inferior to her ſuperior conjunction with him, ſhe riſes in the morning before the Sun, and is then called the *morning ſtar*; when ſhe is to the eaſt of the Sun, which is from her ſuperior to her inferior conjunction, ſhe ſets after the Sun in the evening, and is then called the *evening ſtar*.

The diameter of Venus is nearly equal to that of the Earth, being about 7687 miles; her apparent mean diameter, ſeen from the Earth, is 58 ſeconds; but her apparent diameter, ſeen from the Sun, or her horizontal parallax, is only 30 ſeconds: her diſtance from the Sun 68,000,000 miles; her eccentricity $\frac{7}{1000}$ of her diſtance, or near 476,000 miles, The inclination of her orbit to the plane of the ecliptic is 3 degrees 23 minutes; the points of their interſections, or nodes, are in 14 degrees of Gemini and Sagittary. The place of her aphelion is 9 degrees 38 minutes of Aquarius. Her axis is inclined to her orbit in an angle of 75 degrees: her periodical courſe round the Sun is performed in 224 days 16 hours 49 minutes. The diurnal rotation round her axis is not

not certainly known: Caffini makes it 23 hours, but others make it much more.

When Venus is obferved through a good telefcope, fhe is perceived to have various phafes and changes, like thofe of the Moon; her illuminated parts being conftantly turned towards the Sun.

Dr. Herfchel made many obfervations on this planet, between the years 1777 and 1793. The refults of his obfervations are, that this planet has a revolution about its own axis, but the periodical time of which he was not able to afcertain; that the pofition of its axis cannot certainly be known; that the planet's atmofphere is very confiderable: the furface of the planet he alfo found to be diverfified with hills and vallies, and other inequalities. But the atmofphere of the planet appeared fo denfe, or fome other obftruction in the region of the planet prevented him, as he fays, from having a particular view of the mountains, though affifted by the beft inftruments.

The tranfits of Venus happen but feldom. One of thefe tranfits was feen in England, in the year 1639, by Mr. Horrox and Mr. Crabtree: two more were feen in the laft century; the one, June the 6th, 1761; and the other in June 1769. There will not happen another till the year 1874.

In all other refpects Venus has the fame appearance to us regularly every eight years; that is, her conjunctions, elongations, times of rifing and fetting, being very nearly the fame, and on the fame days.

Some aftronomers have difcovered, or imagined they have difcovered, a fatellite belonging to Venus: of this number was Caffini, who, with a telefcope of 34 feet, in the years 1672 and 1686, thought he faw a fatellite move round this planet, at the diftance of about three fifths of Venus's diameters. And Mr. Short, in 1740, with a reflector of $16\frac{1}{2}$ inches focus, perceived a fmall ftar near Venus: and with another telefcope of the fame focus, and magnifying to the

60th

60th power, he found its diſtance from Venus about 10 minutes; and with a glaſs magnifying to the 240th power, he obſerved the different phaſes of this ſatellite; and its diameter appeared to be near one third of that of Venus. And ſeveral other acute obſervers have imagined they diſcovered the ſame thing.

The *Earth*, the next primary planet in order from the Sun, is our habitation. It performs its revolution round the Sun, at the diſtance of 95,000,000 miles. Like all the other primary planets, it has both a diurnal and annual motion:—its diurnal motion is that by which it turns round its own axis in the ſpace of 24 hours nearly, from weſt to eaſt, and thereby cauſing the continual ſucceſſion of day and night. Its annual motion is that by which it is carried round the Sun in its own orbit, and between the orbits of Venus and Mars, having the orbits of Venus and Mercury within its own orbit, or between it and the Sun, which is in the centre; and thoſe of Mars, Jupiter, Saturn, the Georgian, &c. without, or above it, which are therefore called the ſuperior planets, and Mercury and Venus the inferior ones. This annual motion is accompliſhed in the ſpace of a year, or 365 days 6 hours, or rather 365 days, 5 hours, 48 minutes, 48 ſeconds:—this is called the tropical year. But the time the Earth takes to perform its annual revolution, from any fixed ſtar to the ſame again, as ſeen from the Sun, is 365 days, 6 hours, 9 minutes, 17 ſeconds, which is the ſidereal year. The figure of the Earth's orbit, as that of all the other planets, is elliptical; the eccentricity of the orbit, or diſtance of the Sun in the focus from the centre of the orbit, is about one ſixtieth part of the mean diſtance of the Earth from the centre. The Earth, as well as all the other planets, performs its annual revolution according to the natural order of the ſigns.

By the diurnal rotation of the Earth on its axis the ſame appearance is produced as if it were fixed, and the ſun and

ſtars moved round them every 24 hours. For their turning round from weſt to eaſt, cauſes the Sun, and all the heavenly bodies, to appear to move the contrary way, or from eaſt to weſt, as is imagined to be the caſe by the vulgar and illiterate. Thus, when, by the rotation of the earth, the obſerver is brought to that part where he ſees the Sun or a ſtar juſt riſing above the horizon, in the eaſt, they are ſaid to be riſing; and as the Earth continues to move, other ſtars will appear to riſe, and advance weſtward; and when, by the motion of the Earth, the obſerver is brought directly under the Sun or ſtar, they are then ſaid to be on the meridian; after which, by a continuance of the ſame motion, the obſerver is brought to the eaſtern ſide of the Earth, when the Sun or ſtar will appear to ſet on the weſtern ſide thereof.

The diameter of the Earth is 7957¾ miles; though ſome make it 7964 miles.

The Earth, throughout its annual orbit, always keeps its axis parallel to itſelf in every part of its orbit; thereby occaſioning all the varieties of different ſeaſons of the year, and the different length of the days and nights, as ſeen in *fig.* 3; the Sun enlightening more of the north polar parts, at one ſeaſon of the year, and more of the ſouthern parts at the oppoſite ſeaſon of the year; thus producing the different degrees of heat in the different ſeaſons; for in the ſummer ſeaſon the heat of the Sun is increaſed by two cauſes, viz. 1. By the Sun's rays being more vertical to thoſe ſituated in the north parts of the globe; and, conſequently, the heat is not diminiſhed in paſſing through ſo great a portion of the atmoſphere, as when the rays come more oblique. And, 2dly, having the light, and conſequently the heat of the Sun for a longer ſpace of time in the ſummer than in the winter. All which will be evident from an inſpection of the figure.

What is here aſſerted, concerning the Earth, holds good

with regard to all the other planets, as each of them revolves about an axis, which is not perpendicular to the plane of its orbit, but inclined thereto in a greater or lefs angle; and which axis is always parallel to itfelf in every part of the planet's orbit.

The figure of the Earth, like that of all the other planets, is that of an oblate fpheroid, which Sir Ifaac Newton demonftrated to arife from the rotation of the Earth about its axis; and by the obfervation and experiments of later aftronomers, the polar diameter is to the equatorial diameter, as 178 to 179, as affirmed by that great genius.

The abfolute gravity or denfity of the whole mafs of the Earth is to that of the water as 9 is to 2, and to common ftone as 9 is to 5. Thus, we difcover the very confiderable mean denfity of the Earth, which is almoft double to that of common ftone; from whence it may be prefumed, that the internal parts of the Earth contain great quantities of metals.

Having the denfity of the Earth, its quantity of matter is eafily found; being always equal to the product of its denfity multiplied by its magnitude.

The Earth is, moreover, every where furrounded by an atmofphere, which is that large quantity of fluid matter extending over the whole furface of the Earth, confifting of air, aqueous and other vapours, electric fluid, &c. which furround the Earth to a confiderable height, and partake of all its motions, both annual and diurnal.

The atmofphere ferves for innumerable purpofes, and is even effential to both animal and vegetable life; it is this, infinuating itfelf into all the vacuities of bodies, which caufes thofe mutations of generation, corruption, diffolution, &c.

This atmofphere, like all other matter, has the properties of weight and preffure, the quantity of which is now pretty well known, and is found by the barometer to be equal to an equal column of quickfilver, of 30 inches high; therefore, becaufe a cubical inch of quickfilver is found to weigh near

near half a pound avoirdupois, a column of quickfilver of 30 inches in height, whofe bafe is one fquare inch, will weigh near 15 pounds; from whence it follows, that the weight of the atmofphere on every fquare inch of furface on the face of the Earth is alfo 15 pounds. Thus it appears, that the preffure upon the human body muft be very confiderable; for, as every fquare inch of furface fuftains a preffure of 15 pounds, every fquare foot will fuftain 144 times as much, or 2160 pounds.

The atmofphere has been proved, by late experiments, to become more rare, and of lefs denfity, the farther we remove from the Earth, and that in the following proportion: —At the height of three miles and a half from the Earth, the denfity of the atmofphere is nearly two times rarer than at the furface of the Earth: at the height of feven miles, four times rarer; at the height of 14 miles, 16 times rarer; at the height of 21 miles, 64 times rarer; at the height of 28 miles, 256 times rarer; and at the height of 35 miles, 1024 times rarer, &c.

*Mars* is the firft of the four fuperior planets, and placed immediately next above the Earth, including the orbits of the Earth, Venus, and Mercury, within that of his.

The mean diftance of Mars from the Sun is 1523 of thofe parts of which the diftance of the Earth from the Sun is 1000, or upwards of 144,000,000 miles: his eccentricity 142 of thofe parts of which the Earth is diftant from the Sun 1000: his mean diameter is 4189 miles. The length of his year, or the period of his completing one revolution round the Sun, is $686\frac{23}{24}$ days. His revolution round his own axis is performed in 24 hours 39 minutes and 22 feconds. His mean diameter, feen from the Sun, is 11 feconds. The inclination of his axis to his orbit is nothing, his axis being perpendicular. The inclination of his orbit to the ecliptic is 1 degree 51 minutes; the place of his aphelion 2 degrees 6 minutes 15 feconds of Virgo: his afcending node is 17 degrees 59 minutes of Taurus.

Dr. Herfchel has made many obfervations on the rotation of this planet about its axis, from which he inferred that the mean diurnal rotation was between 24 hours 39 minutes 5 feconds, and 24 hours 39 minutes 22 feconds. He alfo obferved feveral fmall remarkably bright fpots, near both the poles, which had a fort of motion. He alfo concludes that the inclination of his axis to the ecliptic is 59 degrees 22 minutes; and the node of the axis to be in 17 degrees 47 minutes of Pifces; the obliquity of the planet's ecliptic 28 degrees 42 minutes, and the point Aries on Mars' ecliptic to anfwer to our 19 degrees 28 minutes of Sagittary.

The figure of Mars is that of an oblate fpheroid, like the Earth, having his equatorial diameter to the polar one, as 1355 to 1272, or nearly as 16 to 15.

This planet alfo has a confiderable, but moderate atmofphere: fo that its inhabitants probably are, in their nature, fimilar to the inhabitants of this earth.

Mars always appears with a ruddy, difturbed light, occafioned by the nature of his atmofphere.

When he is in oppofition to the Sun, that is, when the Earth comes between the Sun and him, he is nearly five times nearer to us than when in conjunction with the Sun, or when the Sun is between him and us; and confequently, in the former cafe, he appears near five times more large and bright than in the latter cafe.

As he receives his light from the Sun, like the other planets, he muft neceffarily have an increafe and decreafe apparently, like the Moon; he may alfo be obferved fometimes almoft bifected, when in the quadratures; but he is never feen cornicular, as the inferior planets are; which fhows that his orbit includes that of the Earth, and that he fhines with a borrowed light.

Between the orbits of Mars and Jupiter there have lately been difcovered two new planets: viz. one named the *Pallas*, difcovered March 23, 1802: its eccentricity is greater than that of Mercury: the inclination of its orbit to the ecliptic

33 degrees 39 minutes: its periodical revolution four years five months, and real diameter 95 miles. And the *Ceres de Ferdinand*, discovered by Mr. Piazzi, at Palermo, in Sicily, on January 1st, 1801: it is not apparently larger than a fixed star of the eighth magnitude; its elements, according to M. Gauss, are all follow:—inclination of its orbit to the ecliptic 10 degrees 36 minutes 57 seconds, ascending node $2^s\ 21^\circ$, aphelion $10^s\ 26^\circ\ 27'\ 38''$, eccentricity of the orbit .082 parts of its mean distance, periodical revolution 1652 days 5 hours nearly; and real diameter, according to Dr. Herschel, 162 miles.

*Jupiter* is the next superior planet, and the largest of all the planets in the solar system; in brightness, he is next to the planet Venus: his orbit is situated between those of Mars and Saturn. His diameter is above 11 times that of the Earth; consequently his magnitude exceeds the magnitude of the Earth above 1300 times. His annual revolution round the Sun is performed in 4332 days 8 hours 51 minutes 30 seconds, or nearly 12 years. His diurnal revolution about his own axis, he performs in the short space of 9 hours 56 minutes, by which motion every part on his equator is carried round at the rate of 26,000 miles in an hour, being about 25 times faster than the equatorial parts of the Earth move.

The axis of Jupiter is nearly perpendicular to the plane of his orbit; therefore he has no sensible change of seasons, except very near the poles. This is wisely ordered by Providence. For if his axis made any considerable angle with the perpendicular of his orbit, just so many degrees as it was inclined thereto would be near six years in darkness round each of their poles, in their turn.

The orbit of Jupiter is inclined to the ecliptic in an angle of 1 degree 19 minutes 15 seconds: the place of his aphelion is 10 degrees 57 minutes 30 seconds of Libra: the place of his ascending node is 8 degrees 50 minutes of Cancer, and of his descending node 8 degrees 50 minutes of Capricorn.

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The eccentricity of his orbit is one twentieth part of his mean diſtance from the Sun.

There are ſeveral faint ſhining ſubſtances which ſurround Jupiter, of different dimenſions, called his zones or belts, which are conſtantly changing their ſize and ſituations, and are, therefore, generally believed to be clouds. They have ſometimes appeared of different breadths, at other times of the ſame breadth. Large ſpots have alſo been obſerved in theſe belts; and whenever a belt vaniſhes, as is often the caſe, the ſpots contiguous to it have alſo vaniſhed. Theſe belts have ſometimes been interrupted and broken, and the broken ends of ſuch belts have often been obſerved to revolve round the planet in the ſame time with the ſpots. The ſpots have alſo aſſumed different appearances; ſome of them changing their ſhape from a circular to an oblong form, others uniting together in one, and ſometimes one large one dividing into two or three.

The difference between the equatorial and polar diameters of Jupiter is upwards of 6000 miles, the former being to the latter as 13 to 12.

Jupiter is attended by four moons, or ſatellites, ſome of them larger than our earth, which perform their reſpective revolutions round Jupiter in different periods of time; ſo that there is ſcarcely any part of this great planet but what is enlightened by one or more of theſe moons during the whole night. The periods, diſtances in ſemidiameters of Jupiter, and the angles of the orbits of theſe moons, ſeen from the Earth, are as follow:

| Moons. | Periods round Jupiter. | | | Diſtances in Semidiameters of Jupiter. | Angles of the Orbits. | |
|---|---|---|---|---|---|---|
| | D. | H. | M. | | | |
| 1 | 1 | 18 | 27 | $5\frac{2}{3}$ | 3′ | 55″ |
| 2 | 3 | 13 | 13 | $9\frac{1}{10}$ | 6 | 14 |
| 3 | 7 | 3 | 42 | $14\frac{5}{13}$ | 9 | 58 |
| 4 | 16 | 18 | 32 | $25\frac{3}{10}$ | 17 | 30 |

The

The three neareſt moons of Jupiter fall within his ſhadow, and are eclipſed once in every revolution; but the orbit of the fourth ſatellite is ſo much inclined, that it paſſes the ſhadow of Jupiter without falling into it two years in every ſix.

*Saturn* is the next primary planet, and the outermoſt from the Sun, except the Georgian. He ſhines with a feeble light, on account of his great diſtance from the Sun, which is alſo apparently increaſed by his great diſtance from us. This planet has attracted the moſt attention of all the primary planets, on account of his wonderful ring. This ring, or rather a double ring, one within the other, ſurrounds the body of Saturn, at a diſtance equal to the diameter of the planet. Beyond the ring, ſeven moons perform their reſpective revolutions round Saturn. The rings and the moons are all in the ſame plane, and are all dark denſe bodies, and therefore caſt their ſhadows upon each other.

The phenomena of the rings have engaged the attention of all the aſtronomers, ſince their diſcovery; ſome contending it was one entire ring, others dividing it into two or more; but the obſervations of Dr. Herſchel have been more ſatiſfactory on this head than thoſe of his predeceſſors: he divides them into two rings, one within the other: their dimenſions and ſpaces, he ſtates in the following proportion to each other:—

| | *Miles.* |
|---|---|
| Inſide diameter of ſmaller ring | 146,345 |
| Outſide diameter of ditto | 184,393 |
| Inner diameter of larger ring | 190,248 |
| Outſide diameter of ditto | 204,883 |
| Breadth of the inner ring | 20,000 |
| Breadth of the outer ring | 7,200 |
| Breadth of the vacant ſpace | 2,839 |

This

This ring revolves in its own plane in 10 hours 32 minutes 15 feconds.

From the above ftatement it appears, that the outfide diameter of the larger ring is almoft 26 times the diameter of the Earth. This ring is inclined to the plane of the ecliptic in an angle of 30 degrees. When we fee the ring moft open, its fhadow upon the planet is broadeft; and from that time the fhadow grows narrower as the ring appears to do to us, until, by the annual motion of Saturn, the Sun comes to the plane of the ring, or even with its edge; which, being then directed towards us, becomes invifible.

Saturn is found to have certain zones or belts, fomewhat like thofe of Jupiter. Dr. Herfchel has difcovered and demonftrated that Saturn has a denfe atmofphere; that he has a revolution about his axis; that his axis is perpendicular to the plane of his rings; that his figure, like that of the other planets, is that of an oblate fpheroid, the polar diameter being to the equatorial as 10 to 11; that his rings have a revolution in their own plane, their axis being the fame as that of Saturn.

The annual period of Saturn about the Sun is near 30 years, or 10,761 days 14 hours 36 minutes 45 feconds; his diameter is about 79,042 miles, being near ten times that of the Earth; his diftance from the Sun is about 9½ times that of the Earth.

From his great diftance from the Sun, fome have imagined that the portion of light and heat derived from the Sun is not fufficient for animal life. But that they have a greater portion of light, and confequently heat, than is a, firft imagined, is evident from the brightnefs of this planet and its fatellites, in the night-time. Alfo, as the Sun's light to us is 45,000 times as great as that of the full moon, the Sun will afford 500 times as much light to Saturn, as

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the full moon does to us; and 1600 times as much to Jupiter. Thus, theſe two planets, without any moons to enlighten them, would receive more light from the Sun than might be at firſt imagined; their number of ſatellites, the rings of Saturn, and the nature of their atmoſpheres, may alſo have a conſiderable effect in increaſing their light and heat. For we find that, in our earth, the different degrees of heat do not entirely depend on the rays of the Sun. The inhabitants alſo of thoſe planets are, no doubt, adapted to their ſituations.

Saturn has ſeven ſatellites, or moons, performing their revolutions round him in their reſpective periods, as follows:

| Satellites. | Periods. | | | | Diſtances in Semidiamet. of Saturn. | Diſtances in Miles. | Diameters of the Orbits. | |
|---|---|---|---|---|---|---|---|---|
| | D. | H. | M. | S. | | | | |
| 1 | 1 | 21 | 18 | 27 | $4\frac{3}{8}$ | 170,000 | 1′ | 27″ |
| 2 | 2 | 17 | 41 | 22 | $5\frac{1}{2}$ | 217,000 | 1 | 52 |
| 3 | 4 | 12 | 25 | 12 | 8 | 303,000 | 2 | 36 |
| 4 | 15 | 22 | 41 | 13 | 18 | 704,000 | 6 | 18 |
| 5 | 79 | 7 | 48 | 0 | 54 | 2,050,000 | 17 | 4 |
| 6 | 1 | 8 | 53 | 9 | $3\frac{5}{9}$ | 120,000 | 1 | 14 |
| 7 | 0 | 22 | 40 | 46 | $2\frac{5}{6}$ | 91,000 | 0 | 57 |

The four firſt of theſe ſatellites deſcribe ellipſes, like thoſe of the rings, and are alſo in the ſame plane. Their inclination to the ecliptic is from 30 to 31 degrees. The fifth ſatellite deſcribes an orbit which is inclined from 17 to 18 degrees to the orbit of Saturn; the plane of the orbit of this ſatellite lies between the plane of the ecliptic and the planes of the orbits of the other ſatellites. Dr. Herſchel diſcovered that this ſatellite turns once round its own axis in the time that it makes one revolution about the planet Saturn; in which reſpect it reſembles our moon.

The *Georgium Sidus*, or *Herſchel*, is the moſt diſtant of all the primary planets from [illegible] apparent diameter is

 about

about four ſeconds. It is but ſeldom he can be ſeen by the naked eye; but, having his ſituation, he may be plainly ſeen with a good teleſcope, in a clear night.

This planet is twice the diſtance from the Sun that Saturn is, and is nearly 83 years performing his annual courſe. It is 90 times as large as the Earth. The degree of cold in this planet is ſuppoſed to be extreme, as it is computed that the light of the Sun is not above the 300th part of what we enjoy on our earth.

This planet is attended by ſix ſatellites, which perform their revolutions round him as follows:

| Satellites | Periods. | | | Diſtances in Semi-diameters of the Georgian. |
|---|---|---|---|---|
| | D. | H. | M. | |
| 1 | 5 | 21 | 25 | $12\frac{1}{2}$ |
| 2 | 8 | 17 | 1 | $16\frac{1}{2}$ |
| 3 | 10 | 23 | 4 | 19 |
| 4 | 13 | 11 | 5 | 22 |
| 5 | 38 | 1 | 49 | 44 |
| 6 | 107 | 16 | 40 | 88 |

The orbits of theſe ſatellites are nearly perpendicular to the plane of the ecliptic; and by the beſt obſervations that have been made, it is probable that their magnitudes are equal to, if not greater than, that of Jupiter. The motion of theſe ſix ſatellites is retrograde, or contrary to the order of the ſigns.

Theſe are the primary planets which conſtitute the ſolar ſyſtem, each of them moving, in his own proper orbit, round the Sun, as the common centre. The Sun, therefore, can hardly be conſidered as a planet, though reckoned as ſuch by the ancient aſtronomers; but it may rather be ranked as a fixed ſtar. He has, in every reſpect, the ſame properties, being a fixed luminous body, imparting light

and

and heat to all the planets, both primary and ſecondary, found in his ſyſtem. The reaſon he appears brighter and larger than any of the fixed ſtars, is his nearneſs to the Earth, in compariſon with the great diſtance of the former. For a ſpectator, placed as near to any fixed ſtar as we are to the Earth, would ſee that ſtar, in every reſpect, as large and as bright as the Sun appears to us; and an obſerver, as far diſtant from the Sun as we are from the neareſt fixed ſtar, would ſee the Sun as ſmall as the ſtar appears to us, and would reckon it as one of the ſtars.

Though the Sun is ſaid to be a fixed body, yet he has a revolution round his own axis, which he performs in the courſe of 27 days 12 hours and 20 minutes, which is found, by an obſervation of the ſeveral ſpots to be ſeen on the Sun's diſk, which paſs from the weſtern edge of his diſk to the eaſtern edge thereof, in the ſpace of leſs than 14 days. And theſe ſpots are found to perform one entire revolution round the Sun in the ſpace of 27 days 12 hours and 20 minutes; therefore we reaſonably ſuppoſe, that this is the Sun's proper motion from weſt to eaſt, like that of all the planets.

Philoſophers have been greatly divided in opinion concerning the matter of the Sun; ſome contending it was a ball of fire, from the property of the Sun's rays acting like fire, when collected by concave mirrors or convex lenſes; others, as Boerhaave, maintain the contrary; the particulars of which arguments I have not room to inſert; but the following properties of the Sun are demonſtrated by Sir Iſaac Newton:—1. That the denſity of the Sun's heat and light is ſeven times as great on the planet Mercury as it is with us.—2. That the quantity of matter in the Sun is to that in Jupiter nearly as 1100 is to 1; and that the diſtance of that planet from the Sun is in the ſame ratio to the Sun's ſemidiameter.—3. That the quantity of matter in the Sun is to that in Saturn as 2360 to 1; and the diſtance of Saturn from the

the Sun is in a ratio but little lefs than that of the Sun's femidiameter. And hence the common centre of gravity of the Sun and Jupiter is nearly in the fuperficies of the Sun; and that of the Sun and Saturn a little within it.—4. Hence the common centre of gravity of all the planets cannot be more than the length of the folar diameter diftant from the centre of the Sun. This common centre of gravity is always at reft; and though the Sun, by the various pofitions of the planets, may be moved every way; yet it cannot recede far from this common centre.—5. The axis of the Sun is inclined to the ecliptic in an angle of 87 degrees 30 minutes nearly. The Sun's apparent diameter being fenfibly larger in December than in June, the Sun muft be proportionably nearer to the Earth in winter than in fummer. In winter, the Earth will be in the perihelion; in the fummer, in the aphelion. This is demonftrated by the Earth's motion being quicker in December than in June, by about a fifteenth part: for the Earth and every planet defcribe equal areas in equal times; thus, when it moves fwifter, it muft be nearer the Sun. From this we find that we have about eight days more in real time from the Sun's vernal equinox to the autumnal, than from the autumnal to the vernal.—6. The Sun's diameter is equal to 100 diameters of the Earth; therefore, the body of the Sun is 1,000,000 times greater than that of the Earth.—7. The apparent mean diameter of the Sun is 32 minutes 12 feconds. The Sun's horizontal parallax is now fixed at eight feconds, five twentieths of a fecond.—8. If 360 degrees (the whole ecliptic) be divided by the quantity of time in the folar year, it will give 59 minutes 8 feconds, &c. which is the mean quantity of the Sun's diurnal motion. And if thefe 59 minutes 8 feconds be divided by 24, the number of hours in a day, the quotient is 2 minutes 28 feconds, which is the Sun's motion in one hour; and which, divided by 60, will give his motion in one minute, &c. By this

 method,

method, the tables of the Sun's mean motion are constructed.

Though the planets above described perform their periods round the Sun, or rather round the centre of gravity, yet many of the planets seen from the Earth will appear to move in a contrary motion to the order of the signs; particularly the inferior planets; and sometimes they may appear stationary, or not to move at all, for several nights together. But these appearances are nothing but optical deceptions, arising partly from the motions of the planets, and partly from the motion of the Earth on which we are placed; for we always judge a planet to be in that part of the ecliptic which is on the opposite side of the planet to us; this is called its Geocentric Longitude: but the part of the ecliptic in which the planet is seen by an observer supposed to be placed in the Sun, is called the Heliocentric Longitude. And the longitude o any planet or star is an arch of the ecliptic, counted from the beginning of Aries to the place where the ecliptic is cut by a circle perpendicular to the ecliptic, and passing through the star or planet.

## SECT. II.

### OF THE SECONDARY PLANETS.

The secondary planets, or satellites, are certain planets which perform a revolution round any other planet, as the Moon does round the Earth. They are called satellites, because

becaufe they are always found attending their primary planets, and making the tour about the Sun together with them.

There are but four primary planets that are certainly known to have fatellites: viz. the Earth, Jupiter, Saturn, and the Georgium Sidus; though fome have imagined they have difcovered fatellites attending fome of the other planets, as hath been hinted in the laft fection; but thefe obfervations have not been fufficiently confirmed.

The Earth is attended by one fatellite, called the Moon, and marked ☾. She performs her revolution round the Earth in an elliptic orbit, the mean eccentricity of which is one eighteenth part nearly of her mean diftance from the Earth, or about 13,000 miles; her mean diftance from the Earth being $60\frac{1}{2}$ femidiameters of the Earth; or about 240,000 miles.

The mean time of one revolution of the Moon about the Earth, or from one New Moon to another, when fhe overtakes the Sun again, is 29 days 12 hours 44 minutes 3 feconds 11 thirds. But the mean time in which fhe moves once round her whole orbit is 27 days 7 hours 43 minutes 8 feconds, which is at the rate of about 2290 miles in an hour. For the Moon has completed one revolution about the Earth before fhe comes again in conjunction with the Sun; becaufe, while the Moon is performing her revolution, the Earth has advanced about a 13th part of the ecliptic forward.

The Moon turns once round her own axis exactly in the time that fhe goes round the Earth. This is the reafon the fame fide of the Moon is always turned towards the Earth; and day and night in the Moon, taken together, are juft as long as a lunar month.

The diameter of the Moon is to that of the Earth, as 20 to 73; therefore it is equal to 2180 miles. The furface of the Moon is to that of the Earth as 3 is to 40, or as 1 to $13\frac{1}{4}$ nearly; therefore, the Earth reflects 13 times

as much light upon the Moon, as ſhe does upon the Earth, when ſhe is at her full. The ſolid content of the Moon is to that of the Earth as 3 is to 146: the denſity of the Moon's body is to that of the Earth as 5 is to 4; therefore, her quantity of matter is to that of the Earth as 1 is to 39 nearly. The force of gravity on her ſurface is to that on the Earth as 100 is to 293. The axis of the Moon is almoſt perpendicular to the plane of the ecliptic; therefore, ſhe has little or no difference of ſeaſons. The mean apparent diameter of the Moon is 31 minutes 16½ ſeconds.

The various phaſes and appearances of the Moon have puzzled all the aſtronomers of antiquity. Her wanings and increaſings, her various poſitions with regard to the Earth, and her frequent eclipſes, were matters of conſtant admiration. The moon being a dark ſpherical body, and ſhining only with the borrowed light of the Sun, can only have one half of her body illuminated at the ſame time, the oppoſite half remaining in its native darkneſs; therefore, as the Moon performs a revolution round the Earth, ſhe will ſometimes turn the whole of her illuminated face towards the Earth; at which time ſhe appears perfectly round, and is a *full moon:* at other times only a certain portion of her illuminated face will be turned towards the Earth; ſhe will then appear either horned, half round, or gibbous, according to the quantity of her illuminated part which is ſeen by us.

To illuſtrate this, let A B C D E F G H repreſent the orbit of the Moon, (*fig.* 9, *plate* 17.) Now, when the Moon is at A, in conjunction with the Sun, her dark ſide will be turned towards the Earth, and therefore ſhe will be inviſible, as at *a*, which is then called the New Moon. When ſhe arrives at B, or has run through one eighth part of her orbit, one quarter of her illuminated face will be turned towards the Earth; ſhe will then appear horned, as at *b*. When ſhe arrives at C, one half of her illuminated

face

face is turned towards the Earth, as at *c*, when ſhe is ſaid to be in her quadrature. When ſhe arrives at D, which is called her ſecond octant, three parts of her illuminated face will be turned towards the Earth, and ſhe will appear gibbous, as at *d*. When ſhe arrives at E, the whole of her illuminated face is turned towards the Earth, and ſhe appears quite round, as at *e*, when ſhe is ſaid to be a full Moon. As ſhe proceeds through the other half of her orbit ſhe decreaſes again from *e* to *a*, and nearly in the ſame ratio as ſhe increaſed in the former half of her orbit. And the Earth has all the ſame appearances to an obſerver in the Moon, as the Moon has to us, but in a contrary order: viz. the Earth being at the full to them, when the Moon changes to us, and *vice verſa*; as is evident from a view of the figure.

The motions of the Moon are all very irregular; the only equable motion ſhe has, is the rotation on her own axis in the ſpace of a month, being the time in which ſhe moves round the Earth; which is the reaſon that ſhe always expoſes the ſame face towards the Earth.

The orbit of the Moon is very changeable, and does not long preſerve the ſame figure; for though the orbit of the Moon be an ellipſe, having the Earth in one of her foci thereof; yet the eccentricity is ſometimes greater than at other times.

The plane of the Moon's orbit is inclined to that of the ecliptic, in an angle of five degrees.

The face of the Moon has the appearance, when viewed through a teleſcope, of being diverſified with hills and vallies; this is alſo proved to be the caſe, from the edge or border of the Moon appearing jagged, eſpecially about the line which ſeparates the illuminated part of the Moon from the dark ſide thereof. The ſpots alſo of the Moon, which are taken for mountains, are found to caſt a triangular ſhadow in the direction oppoſite to the Sun; and thoſe

parts which are taken for vallies or cavities are always dark on that fide next the Sun, and illuminated on the oppofite fide; which is agreeable to experience. Sometimes the tops of the mountains are feen illuminated by the Sun, while their bafes are in the dark fide of the Moon; and by thefe means we have a good method of taking the height of the lunar mountains.

Thus, let E D (*fig.* 14) be the Moon's diameter, E C D the line dividing the dark from the illuminated part of the Moon; and A the top of a hill in the dark part, juft beginning to be illuminated: with a telefcope, take the proportion of A E to the diameter E D, then there are given the two fides A E, E C, of the right-angled triangle A E C; the fquares of the two fides of the right-angled triangle being added together give the fquare of the hypothenufe A C, from the fquare root of which, fubtracting B C, the radius, there remains A B, the height of the mountain.

From late obfervations, Dr. Herfchel has difcovered that very few of the lunar mountains exceed half a mile in perpendicular height. The fame gentleman has alfo obferved three volcanoes in the Moon, which he thus defcribes: "I perceived (April 19th, 10 hours 36 minutes, fidereal time) three volcanoes in different parts of the dark part of the New Moon: two of them are either already nearly extinct, or otherwife in a ftate of going to break out; which, perhaps, may be decided next lunation: the third fhows an actual eruption of fire, or luminous matter; its light is much brighter than the nucleus of the comet which Mr. Mechain difcovered at Paris the 10th of this month." The following night he difcovered it burn more violently; and by meafuring, he found the fhining or burning matter to be more than three miles in diameter. The actual fire or eruption of a volcano exactly refembled a fmall piece of burning charcoal, when it is covered by a very thin coat of white afhes; and it had a degree of brightnefs,

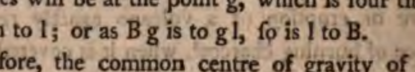
 about

about as ſtrong as that with which a coal would be ſeen to glow in faint daylight.

Theſe are the chief phenomena obſervable in the Moon. All the other ſatellites are of a ſimilar nature to this; but from their great diſtance from the Earth, we are unable to be ſo particular in our deſcription of them.

Though it be aſſerted, that the Moon and the other ſecondary planets revolve round the primary planets as their centres, and the primary planets revolve round the Sun for their centre, yet it muſt be remembered, that this aſſertion is not the real mathematical truth. For the primary planets do not regard the Sun as their exact centre; but each primary planet, and the Sun, revolve round their common centre of gravity: which common centre of gravity is that point where the two bodies, or the Sun and planet, will equiponderate each other. Thus, the centre of gravity in a common balance-beam, or ſteelyard, is the point of ſuſpenſion.

To diſcover the common centre of gravity of two bodies, is to find that point, whoſe diſtance from the greater body is leſs than its diſtance from the leaſt body, in the ſame proportion as the gravity of the leſs body is leſs than that of the greater; and in two bodies of equal gravities, their common centre of gravity is equally diſtant from their two centres.




Thus, if B be a body four times as great in magnitude as the body l, and both be ſuppoſed to be connected by an inflexible wire B l, the common centre of gravity of the two bodies will be at the point g, which is four times nearer to B than to l; or as B g is to g l, ſo is l to B.

Therefore, the common centre of gravity of the Earth and Sun is nearer to the centre of the Sun than to that

of

of the Earth, by as great a ratio as the quantity of matter in the Sun exceeds that in the Earth, which centre of gravity is in the body of the Sun. The common centre of gravity of Jupiter and the Sun is alſo within the body of the Sun, though very near its ſuperficies.

The Sun is not acted upon by our Earth only, ſo as to turn round the common centre of gravity of that and the Earth, without having regard to the other planets; but there is a common centre of gravity between the Sun and each of the primary planets; and each of theſe planets has its effect in cauſing the Sun to turn round their centres of gravity.

As the centre of gravity of Jupiter and the Sun is the fartheſt diſtant from the Sun's centre, owing to the great ſize of Jupiter, and its diſtance from the Sun, and as this centre is within the body of the Sun, it follows that the Sun is never removed above one of its own diameters out of its place.

Each of the ſecondary planets, and its primary planet alſo, turn round their common centre of gravity.

The figures 1, 2, 3, 4, 5, 6, 7, and 8, ſhow the true proportions of the planets Mercury, Venus, Mars, the Earth, Jupiter, Saturn, Georgium Sidus, and the Moon.

Jupiter has four ſatellites; the times of whoſe periods and diſtances have been noticed in the laſt ſection. Their periods were found from their conjunction with Jupiter, after the ſame manner as the periods of the primary planets were diſcovered. Their diſtances from the body of Jupiter are meaſured by a micrometer, and computed in ſemidiameters of Jupiter, and then reduced into miles.

The ſatellites of Jupiter are of very great uſe in aſtronomy; for, by an obſervation of the eclipſes of theſe ſatellites, we derive three great advantages:—1ſt, in determining the diſtance of Jupiter from the Earth; 2dly, we find the progreſſive motion of light: for by theſe eclipſes, it is evident that light does not come to us from Jupiter

in an inſtant; for if the motion of light were inſtantaneous, it is evident we ſhould ſee the commencement of the eclipſes of the ſatellites at the ſame moment they really happen, whatever diſtance they might be from us; but, on the contrary, if light have a progreſſive motion, it is evident, the farther we are from a planet, the later we ſhould be in ſeeing the beginning of its eclipſe; and ſo it is found to happen: the ſatellites of Jupiter appear to be eclipſed later than the true computed time, and always proportionally later, as the Earth is removed farther from the planet. When the Earth and Jupiter are neareſt to each other, that is, when they are both in conjunction, on the ſame ſide of the Sun, then the eclipſes of Jupiter's ſatellites are ſeen to happen ſooner than when the Sun is directly between Jupiter and the Earth; in which laſt caſe, the diſtance of Jupiter from the Earth is greater than it is in the former caſe, by the whole diameter of the Earth's annual orbit, or by double the Earth's diſtance from the Sun: in this laſt caſe, we cannot obſerve an eclipſe of Jupiter's ſatellites, till near a quarter of an hour after the time we could have diſcovered it in the former caſe, that is, when Jupiter was at his leaſt diſtance from the Earth. From hence it follows, that the light is near a quarter of an hour in paſſing through a ſpace equal to the diameter of the Earth's orbit, or near eight minutes in paſſing from the Sun to the Earth; which is at the rate of about twelve million of miles in a minute.

But the third and greateſt advantage derived from the obſervation of theſe eclipſes, is the diſcovery of the longitude of different places on the Earth: for having the difference of time between two obſervations of the ſame eclipſe, taken in two different places, we have the difference of longitude between the two places. For example, ſuppoſe there be two obſervers of an eclipſe, the one at London, the other at Barbadoes, the eclipſe will appear at the ſame moment of *real time* to each perſon; but being under different meridians, the hour of the day will be different at each place; thus, if it be 12

o'clock

o'clock at noon at London, it will be 8 o'clock in the morning at Barbadoes, by which the obfervers find the difference of longitude between the two places to be fixty degrees, or four hours in time.

The planet Saturn has feven fatellites, the fixth and feventh of which were difcovered by Dr. Herfchel, in the years 1787 and 1788. Their periods, diftances, &c. have been defcribed in the laft fection.

The Georgian planet, or Herfchel, is alfo found to have fix fatellites revolving round him, like thofe of Jupiter and Saturn. Thefe fatellites were difcovered by Dr. Herfchel, for which fee the laft fection.

Thefe are the only primary planets which we are certain are attended by fatellites. Some aftronomers have imagined they difcovered a fatellite belonging to Venus; but the many repeated obfervations which have been made by others to obferve it, and without effect, leave us room to fufpect they were deceived.

## SECT. III.

### OF THE FIXED STARS.

THE fixed ftars, generally denominated *ftars* by way of eminence, are thofe heavenly bodies which ufually keep the fame diftance with regard to each other. All the heavenly bodies, except the primary and fecondary planets, and comets, are of this clafs.

The

The diftances of the fixed ftars are fo great, that we have no diftance in the planetary fyftem with which we can compare them; for the diameter of the Earth's orbit, which is nearly 190,000,000 miles, bears no fenfible proportion to the diftance of the neareft fixed ftar from the Earth.

The diftances of the fixed ftars have been the fubject of inveftigation to feveral aftronomers. Various methods have been devifed for the purpofe of difcovering the diftances of thefe heavenly bodies, but without fuccefs, on account of their almoft infinite diftance; the moft accurate obfervations only gave us a diftant approximation; but by the beft obfervations, however, we can fafely conclude that the neareft fixed ftar is upwards of forty thoufand diameters of the Earth's orbit diftant from us, or eighty thoufand times farther diftant from us than the Sun is.

The magnitudes of the fixed ftars appear very different in different ftars, owing, in fome fort, to their real magnitudes, which are different, but principally owing to their different diftances from us.

The ftars are generally divided, according to their apparent magnitudes, into fix, and by fome into eight claffes. The firft clafs, called ftars of the firft magnitude, are thofe that appear largeft: ftars of the fecond magnitude appear fomewhat lefs; and thus every following clafs comprehends thofe ftars next in fize to the former clafs; the ftars of the fixth magnitude containing the fmalleft ftars vifible to the naked eye. All others that cannot be perceived, but by the help of telefcopes, are called telefcopic ftars. It is not to be inferred from hence that Nature has divided the ftars into thofe claffes; for there are almoft as many claffes as there are ftars, fo great is their variety of magnitude and brightnefs!

The number of ftars is alfo very great, and appears to be almoft infinite; but aftronomers have deduced all that are vifible to the naked eye into catalogues. Mr. Flamftead reduced 3000 ftars into a catalogue, which contains all that are vifible to the naked eye at any time of the year, and a

3 great

great number that are only viſible through a teleſcope. The number of ſtars that are viſible at one time, in the cleareſt heaven, ſeldom exceeds 1000; their appearing ſo much more numerous ariſes from their twinkling, and from viewing them confuſedly, without reducing them into any order. But a good teleſcope will, nevertheleſs, bring great numbers to our view: and the more the magnifying power of the teleſcope is increaſed, the greater will be the number of ſtars diſcovered; till the number becomes ſo great as to baffle our computation.

From the great diſtance of the ſtars, we are at a loſs to diſcover many of their properties; but from their phenomena we can with certainty deduce the following theorems concerning them:—1. That they are much greater than our Earth; for, if that were not the caſe, they could not be viſible at ſuch a diſtance.—2. They are farther diſtant than the moſt diſtant of the planets; for we often find them hidden behind the body of the planets.—And, 3. They ſhine with their own natural light; for though they be much farther from the Sun than Saturn is, and appear much ſmaller to us, yet they ſhine much brighter than that planet. And it is known, that the more the teleſcope magnifies, the leſs is the angle under which the ſtar is ſeen; becauſe the teleſcope deſtroys all the adventitious rays. Thus a teleſcope magnifying 200 times will ſhow a ſtar leſs in magnitude than it appears to the naked eye, inſomuch, that it will appear to be only an indiviſible point.

From hence we conclude, that the fixed ſtars are ſo many Suns; and that, in all probability, they are not much ſmaller than our Sun, but perhaps larger.

Therefore, it is generally believed that every ſtar is the centre of a ſyſtem, and has planets revolving round it in the ſame manner as the Earth and the other primary planets revolve round the Sun; for our Sun, together with the orbits of all the planets, would be almoſt inviſible from the neareſt fixed ſtar.

To

To imagine that the stars are formed only to afford us a faint light, would be absurd, as we have incomparably more light from the Moon than from all the fixed stars taken together.

The fixed stars have two apparent motions; one called the first, common, or diurnal motion; the other called the second or proper motion. The former of these motions arises from the Earth's motion round its axis; by which the stars appear to be carried round the Earth, from east to west, in the space of 24 hours. The latter is that motion by which they appear to go backwards from west to east round the poles of the ecliptic with a very slow motion, describing only one degree of a circle in the space of 71½ years. This apparent motion is owing to the precession of the equinoxes; or, in other words, the axis of the Earth is directed to different parts of the heavens every year, describing a circle, one degree of which it describes in 71½ years.

The *Zodiac* is an imaginary ring or zone in the heavens, in the space of which all the primary planets revolve in their orbits: its breadth is made different by different astronomers, but is from eight to ten degrees on each side the ecliptic; and is divided into twelve parts, called the Twelve Signs of the Zodiac, and each sign is subdivided into 30 degrees; the degrees are each again divided into minutes, seconds, &c. But as the stars have a motion from west to east, these constellations, or signs of the Zodiac, do not now correspond to their proper signs; for the vernal equinox formerly happened when the Sun was in the first degree of Aries, and the Earth in the opposite degree of the Zodiac, or first degree of Libra; whereas now, the Sun has advanced a whole sign from that point at the vernal equinox.

The twelve signs of the Zodiac are distinguished by the following names and characters, viz. ♈ *Aries*, ♉ *Taurus*, ♊ *Gemini*, ♋ *Cancer*, ♌ *Leo*, ♍ *Virgo*, ♎ *Libra*, ♏ *Scor-*

*pio*,

*pio*, ♐ *Sagittarius*, ♑ *Capricornus*, ♒ *Aquarius*, ♓ *Piſces*: or, according to the Engliſh names, the Ram, the Bull, the Twins, the Crab, the Lion, the Virgin, the Balance, the Scorpion, the Archer, the Goat, the Water-bearer, and the Fiſhes.

Beſides theſe conſtellations in the Zodiac, the ſtars in every other part of the heavens are reduced into conſtellations of ſome certain figures, to which it is ſuppoſed each ſet of ſtars bears ſome reſemblance. In the northern hemiſphere are 21 conſtellations, of which the following are the names:—the Little Bear, Great Bear, the Dragon, Cepheus, Boötes, the Northern Crown, Hercules, the Harp, the Swan, Caſſiopeia, Perſeus, Auriga, Serpentary, the Serpent, the Arrow, the Eagle, the Dolphin, the Horſe, Pegaſus, Andromeda, and the Triangle. In the ſouthern hemiſphere are 15 conſtellations, viz. the Whale, Orion, the Eridanus, the Hare, the Great Dog, the Little Dog, the Ship, the Hydra, the Cup, the Raven, the Centaur, the Wolf, the Altar, the Southern Crown, and the Southern Fiſh.

This diviſion was introduced by Ptolemy, and to theſe Bayer added 12 more, about the ſouthern pole, viz. the Peacock, the Tucan, the Crane, the Phœnix, the Dorado, the Flying Fiſh, the Hydra, the Chamelion, the Bee, the Bird of Paradiſe, the Triangle, and the Indian.

To theſe, Mr. Royer has added 11 other conſtellations, viz. the Giraffe, the River Jordan, the River Tigris, the Sceptre, and the Flower-de-Luce, being on the north. The following ſix are on the ſouth part, viz. the Dove, the Unicorn, the Croſs, the Great Cloud, the Little Cloud, and the Rhomboid.

Hevelius alſo added the following new conſtellations, compoſed of ſome unformed ſtars, viz. the Unicorn, the Camelopardalis, the Sextant of Urania, the Dogs, the Little Lion, the Lynx, the Fox and Gooſe, Sobieſki's Crown, the Lizard, the Little Triangle, and the Cerberus, to which Gregory has added the Ring and the Armilla.

It muſt, however, be remarked, that ſome of theſe conſtellations introduced by Hevelius anſwer to ſome of thoſe of Royer; as the Camelopardalis to the Giraffe, the Dogs to the River Jordan, and the Fox to the River Tigris. The foregoing is the number of the conſtellations as they ſtand at preſent; but an attempt has lately been made by Dr. Hill, to introduce fourteen new ones; they are, however, not yet adopted by mathematicians.

Beſides the ſtars in the foregoing conſtellations, there are a great number of ſtars not included in any conſtellation, and therefore called unformed ſtars.

The Galaxy, or Milky Way, is that long, white, luminous tract which ſeems to encompaſs the heavens, and is eaſily ſeen in a clear night, when the Moon is not up. It is of a conſiderable breadth, and in ſome parts double. Its luminous appearance is owing to the great number of ſmall ſtars with which it is every where beſpangled, and which by a good teleſcope may be plainly diſcovered.

## *Of Comets.*

A comet is a wandering body, appearing ſuddenly, and as ſuddenly diſappearing; and moves in its own proper orbit, like a planet.

It is uſually furniſhed with a long train of light, called its tail, which is always oppoſite to the Sun. Comets are divided into three kinds; viz. *bearded*, *tailed*, and *hairy* comets: which diviſion ariſes from the different ſituation of the comet. Thus, when the comet is eaſtward of the Sun, and moves from him, it is ſaid to be a bearded comet, becauſe the light precedes it in the manner of a beard: when the comet is weſtward of the Sun, and ſets after him, it is ſaid to be a tailed comet, becauſe the train of light follows it in the manner of a tail: and when the Sun and comet are in oppoſition to each other, the Earth being between them,

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the train of the comet is hid behind its body; except the extremities of the train, which being broader than the body of the comet, appear, as it were, round the edges of it like a border of hair, from which it is called a hairy comet.

The comets make a part of the folar fyftem, and move in elliptical orbits, having the Sun in one of their foci, and defcribe areas proportional to the times of their motions, like the planets. The reafon why they fometimes appear vifible, and fometimes not, is the great eccentricity of their orbits, which is very confiderable, for when they are in that part of the orbit moft remote from the Sun, they are much beyond the orbit of Jupiter; and in their perihelion they frequently defcend within the orbit of Mars, and fometimes within thofe of the inferior planets.

## SECT. IV.

### OF ECLIPSES.

An eclipfe is the privation of the light of one of the luminaries by the interpofition of fome opake body, either between the luminary and the eye, or between it and the Sun.

The *duration* of an eclipfe is the time of its continuance.

The *immerfion*, or incidence of an eclipfe, is the moment when the eclipfe begins; or when part of the luminary firft begins to be obfcured.

The *emerfion*, or expurgation of an eclipfe, is the time

when the eclipſed luminary begins to reappear, or emerge out of the ſhadow.

The *quantity* of an eclipſe, is the part of the luminary eclipſed. To determine this quantity, the diameter of the eclipſed body is divided into 12 equal parts, called digits; and the eclipſe is ſaid to be of ſo many digits as are contained in that part of the diameter which is eclipſed.

Eclipſes are either thoſe of the Sun, the Moon, or of ſome of the ſatellites, and are either *total*, *partial*, *annular*, *central*, &c.

A *total* eclipſe, is when the whole body of the luminary is darkened.

A *partial* eclipſe, is when only a part of the luminary is eclipſed.

A *central* eclipſe, is when the centres of the two luminaries and the Earth come in a ſtraight line, and is always total.

An *annular* eclipſe, is when the whole body is eclipſed, except a ring or annulus, which appears round the border or edge.

An eclipſe of the Moon, is a privation of the light of the Moon, and occaſioned by the body of the Earth being directly between the Sun and the Moon, and ſo intercepting the Sun's rays, that they cannot arrive at the Moon; conſequently the Moon paſſes through a part of the conical ſhadow of the Earth, as ſeen in *fig.* 12, *plate* 17, where D E C repreſents the Earth, and D G F C the conical ſhadow thereof, in which is the Moon in an eclipſe. The dotted ſpaces D G *s*, and F C *r*, ſhow thoſe parts of the ſhadow called the *penumbra*, in which the Moon is deprived only of part of the Sun's light.

An eclipſe of the Sun, is an obſcuration of the Sun's body, occaſioned by the Moon's coming between the Earth and the Sun, and thus intercepting the light of the Sun from us, on which account ſome have conſidered it an eclipſe of the Earth.

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The ſolar eclipſe is repreſented *fig.* 11, where *m* repreſents the Moon, C D the Earth, and *r m s o* the Moon's conical ſhadow, travelling over that part of the Earth C *o* D, and cauſing a complete eclipſe of the Sun to all the inhabitants who reſide in the tract C D. The ſpaces C *r o* and D *s o* include the penumbra, and all the inhabitants within thoſe ſpaces will perceive a faint ſhadow of the eclipſe.

Hence, an eclipſe of the Moon can happen only at the time of the full Moon, or when ſhe is oppoſite to the Sun; and an eclipſe of the Sun will take place only at the time of a new Moon, or when the Moon is between the Sun and Earth.

From hence ſome may imagine that there may be two eclipſes, viz. one of the Sun and another of the Moon, in every lunation, which would really be the caſe, if the Moon moved in the ſame plane with the ecliptic; but the orbit of the Moon not being in the plane of the ecliptic, but inclined thereto in an angle of 5 degrees 35 minutes, and paſſing through the plane of the ecliptic, it muſt neceſſarily follow, that an eclipſe can only take place when the Moon is near that part of its orbit which paſſes through the plane of the ecliptic. Theſe two oppoſite points where the Moon's orbit intercepts the ecliptic, are called its Nodes.

That point where the Moon aſcends from the ſouth to the north ſide of the ecliptic, is called the *aſcending node*, or dragon's head, and marked ☊; and the oppoſite point, where the Moon deſcends from the north to the ſouth ſide of the ecliptic, is called the *deſcending node*, or dragon's tail, and marked ☋; and a line drawn from one node to the other, is called the line of the nodes. Thus, if (*fig.* 13) *a b c d* be the orbit of the Moon, and *e g* the ecliptic, the points *a*, *c*, where the orbit cuts the ecliptic, are the two nodes, and the dotted line *a c* the line of the nodes. From a view of the figure, it is plain, when the full or new Moon happens when the Moon is at the points *b* or *d*, there

can

can be no eclipſe, the ſhadow of the Moon or Earth falling either above or below the other luminary; but when the full or new Moon is at the points *a* or *c*, or within 17 degrees of theſe points, there will be an eclipſe of one of the luminaries.

In order to calculate an eclipſe, it is neceſſary to know how to take the parallax of the Sun, or any heavenly body; as alſo to take the parallactic angle.

The parallactic angle, called alſo the *parallax*, is the angle E S T (*fig.* 1, *plate* 18), made at the centre of a ſtar, or other bodies, by two lines, one drawn from the centre of the Earth T, and the other from its ſurface E; or, which is the ſame thing, it is the difference of the two angles C E A and B T A.

Parallax is an arch of the heavens intercepted between the true and apparent place of any ſtar, or heavenly body.

The true place of a ſtar, S, is that point of the heavens, B, where it would be ſeen by an obſerver placed in the centre of the Earth T: and the apparent place of the ſame ſtar is the point C in the heavens, where it would appear to an obſerver on the ſurface of the Earth, at E. This difference of the two places of the ſame ſtar is the parallax, ſometimes called, for diſtinction ſake, the parallax of altitude; and is an angle formed by two viſual rays, the one drawn from the centre, and the other from the circumference, of the Earth, and traverſing the body of the ſtar; the meaſure of it being an arch of a great circle, intercepted between the points of the true and apparent places, B and C.

The parallax B C is alſo the difference between the true diſtance of the ſtar from the zenith A, and the apparent diſtance A C. Hence the parallax diminiſhes the altitude of a ſtar, or increaſes its diſtance from the zenith.

The parallax is greateſt in the horizon, which is therefore called the horizontal parallax, as E F T. From the horizon the parallax decreaſes all the way to the zenith A, where the true and apparent places of the ſtar coincide.

The *parallax of the annual orbit of the Earth*, is the angle under which the ſemidiameter of the Earth's orbit is ſeen.

To find the parallax of a celeſtial body, obſerve when the body is in the ſame vertical line with a fixed ſtar which is near it; and while it is in that poſition, meaſure its apparent diſtance from the ſtar; then obſerve when the ſtar and body are at equal altitudes from the horizon, and there meaſure their diſtances again, and the difference of theſe diſtances will be the parallax.

## *The Aſtronomy of Eclipſes.*

To calculate a lunar eclipſe it is neceſſary, firſt, to find the length of the Earth's conical ſhadow, which may be found by finding the diſtance between the Earth and Sun, and the proportion of their diameters. Thus, ſuppoſe the ſemi-axis of the Earth's orbit to be 95,000,000 miles, and the eccentricity of the orbit 1,377,000, which, added together, make 96,377,000 miles, or 24,194 ſemidiameters of the Earth; and the Sun's ſemidiameter being to that of the Earth as 112 to 1; then, as A D is to B E, ſo is D B to E C (*fig.* 2), that is, as 111 is to 1, ſo is 24,194 to 218 ſemidiameters of the Earth, equal to E C, the length of the Earth's ſhadow.

To find the apparent ſemidiameter of the Earth's ſhadow, in the place where the Moon paſſes through it, add together the parallaxes of the Sun and Moon, and from the ſum ſubtract the apparent ſemidiameter of the Sun, and the remainder will be the apparent ſemidiameter of the ſhadow at the place where the Moon paſſes through it.

*Note.* The Sun's parallax may very well be omitted in this calculation; and the apparent ſemidiameter of the ſhadow increaſed by adding one whole minute.

It is alſo neceſſary to have the true diſtances of the Moon from the node at the mean oppoſition; alſo the true time of the oppoſition, with the true place of the Sun and Moon reduced

reduced to the ecliptic, and the Moon's true latitude at the time of the true oppofition; likewife the angles of the Moon's way with the ecliptic, and the true horary motions of the Sun and Moon; from which every particular concerning the eclipfe may be computed by common arithmetic and trigonometry.

The method of conftructing an eclipfe of the Moon is as follows:—Let E W (*fig.* 3) reprefent a part of the ecliptic, C the centre of the tranfverfe fection of the Earth's fhadow. Draw the line C N perpendicular to the ecliptic, and towards the north, if the Moon have north latitude; but if fhe have fouth latitude, draw a line C S. Make the angle N C D equal to 5 degrees 35 minutes, which is the angle the Moon's orbit makes with the ecliptic. Bifect this angle by the right line C F, and in this line the true equal time of oppofition between the Sun and Moon falls by the tables.

Take the Moon's latitude at the true time of full Moon from a fcale of equal parts, which is fuppofed to reprefent minutes of a degree, and fet this diftance from C to G on the line C F. Through the point G draw a line H I at right angles to C D, which line reprefents a portion of the Moon's orbit. Then L is the point where the Moon's centre is at the middle of the eclipfe; G the place of her centre at the tabular time of her being full; and K the point of her centre at the inftant of her ecliptic oppofition; I is the Moon's centre at the moment of her immerfion, and H her centre at the end of the eclipfe, or emerfion.

From the fame fcale take the Moon's femidiameter, and defcribe three circles on the points I G H, which reprefent the Moon in the beginning, middle, and end of the eclipfe.

Then, to find the length of time or the duration of the eclipfe, meafure the line I H on the fame fcale, and fay, as the Moon's horary motion from the Sun is to H I, fo is one hour, or 60 minutes, to the whole duration of the eclipfe.

From

From the above figure the eclipſes may be computed. For, firſt, in the right-angled triangle C G L, right-angled at L, there is given the hypothenuſe C G, which is the Moon's latitude at the time of the full Moon: alſo we have the angle G C L equal to the half of 5 degrees 35 minutes, wherefore the legs C L and G L may be found. Secondly, in the right-angled triangle C L H, or C L I, are given the legs C L, and C H or C I, which latter is the ſum of the ſemidiameter of the Moon and the Earth's ſhadow; therefore L H or L I may be found, which is equal to half the difference of the motions of the Sun and Moon during the eclipſe. Thirdly, as the difference of the horary motions of the luminaries is to one hour, or ſixty minutes, ſo is H L to the ſemiduration of the eclipſe; and ſo is G L to the difference between the oppoſition and middle of the eclipſe. This laſt, therefore, taken from the time of full Moon, gives the time of the middle of the eclipſe; from which, ſubtracting the time in L I, before found, gives the beginning of the eclipſe; and, added to the ſame, gives the end of the eclipſe.

Laſtly, from C O, the ſemidiameter of the Earth's ſhadow, ſubtract C L, and the remainder L O, added to L P, gives O P, the quantity eclipſed.

A ſolar eclipſe, or an eclipſe of the Sun, would be diſcovered in the ſame manner as a lunar eclipſe, if the Moon had no ſenſible parallax; but as the Moon has a parallax, the method is ſomewhat different.

1. Add the apparent ſemidiameters of both Sun and Moon together, both in the aphelion and perihelion, which gives 33 minutes 6 ſeconds for the greateſt ſum, and 30 minutes 31 ſeconds for the leaſt ſum.

2. As the parallax diminiſhes the northern latitude and increaſes the ſouthern, let the greateſt parallax in the latitude be added to the former ſums, and alſo ſubtracted from them: thus may be had, in each caſe, the true latitude, beyond which there can be no eclipſe. Having this latitude, the

Moon's diſtance from the nodes is found in the ſame manner as for the lunar eclipſe.

To find the digits eclipſed, add the apparent ſemidiameters of the luminaries together, from which ſubtract the Moon's apparent latitude; the remainder is the part of the diameter eclipſed. Then ſay, as the ſemidiameter of the Sun is to the ſcruples eclipſed, ſo are ſix digits reduced into ſcruples (that is, 360 ſcruples or minutes) to the digits eclipſed.

To determine the duration of a ſolar eclipſe, find the horary motion of the Moon from the Sun for an hour before the conjunction, and for one hour after it. Then ſay, as the former horary motion is to the ſeconds in an hour, ſo are the ſcruples of half the duration to the time of immerſion; and as the latter horary motion is to the ſame number of ſeconds, ſo are the ſcruples of half the duration, to the time of emerſion. Then adding the times of immerſion and emerſion together, the ſum is the whole duration.

To find the beginning, middle, and end of a ſolar eclipſe, find the arch G L from the Moon's latitude, for the time of conjunction. Then ſay, as the horary motion of the Moon from the Sun before the conjunction is to one hour, ſo is the diſtance of the greateſt darkneſs to the interval of time between the greateſt darkneſs and the conjunction. Subtract this interval in the firſt and third quarter of the anomaly from the time of the conjunction, but in the other quarters add it to the ſame, and the reſult is the time of the greateſt darkneſs. Laſtly, from the time of the greateſt darkneſs, ſubtract the time of incidence, to which is to be added the time of emerſion; the difference in the firſt caſe will be the beginning, and the ſum in the latter caſe will be the end of the eclipſe.

To calculate eclipſes of the Sun, it is neceſſary, firſt, to find the mean new Moon, and from thence the true one, with the place of the luminaries for the apparent time of the

the true new Moon.—2. For the apparent time of the true new Moon, compute the apparent time of the new Moon obſerved.—3. For the apparent time of the new Moon ſeen, compute the latitude ſeen.—4. Thence determine the number of digits eclipſed.—5. Find the times of the greateſt darkneſs, emerſion, and immerſion.—6. And from thence determine the beginning and ending of the eclipſe.

From this it is evident that the trouble in the calculation of eclipſes ariſes from the parallaxes of longitude and latitude, without which the calculation of ſolar eclipſes would be the ſame as that of lunar ones.

## SECT. V.

### OF TIME.

Time is a mode of duration marked by certain periods, meaſures, and motions; and the chief method we have of meaſuring time is by the revolution of the two luminaries, the Sun and Moon, but particularly that of the Sun.

Mr. Locke obſerves, that the idea we have of time is acquired by conſidering any part of infinite duration, as ſet out by periodical meaſures. The idea of any particular time, or length of time, as a day, an hour, &c. is acquired by obſerving certain appearances of ſome bodies, moving with

with a regular motion, and at regular and ſeeming equidiſtant periods. Now, by being able to repeat theſe lengths or meaſures of time as often as we pleaſe, we can imagine duration where nothing really endures or exiſts, and thus we imagine to-morrow, or next year, &c.

Time is alſo the duration of a thing which has both a beginning and an end, and in this ſenſe it is diſtinguiſhed from eternity.

Thus, time is the duration of ſome motion; for without ſome regular and uniform motion we ſhould have no methods to compute time, or diſtinguiſh it from eternity.

Time may be divided into *abſolute* and *relative*.

Abſolute time is time conſidered in itſelf, and without any relation to motion.

Relative or apparent time is the ſenſible meaſure of any duration, by means of motion, as, by the motions of the luminaries, the hands of a clock, watch, &c.

Relative time is ſubdivided into *aſtronomical* and *civil*.

Aſtronomical time is that which is meaſured by the motions of the heavenly bodies.

Civil time is formed for civil purpoſes, and diſtinguiſhed into years, months, days, hours, &c.

The year, in the full extent of the word, is a ſyſtem of ſeveral months, or a ſpace of time meaſured by the revolution of ſome celeſtial body in its orbit. Thus, the time in which the fixed ſtars make one revolution is called the great year; and the times in which Jupiter, Saturn, &c. complete their revolutions, and return to the ſame point again, are reſpectively called the years of Jupiter, Saturn, &c. For a year originally denoted a revolution, and is not limited to that of the Sun; therefore we find ſome ancient nations at different times called the revolutions of the Moon, or the ſpace of a month, a year; which occaſions ſuch ſtrange accounts in the chronology of ſome very ancient nations, as in thoſe of the Egyptians, Babylonians, &c.

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The ſolar year, called alſo *year* by way of eminence, is that ſpace of time in which the Sun moves through the twelve ſigns of the Zodiac. This year, by the beſt obſervations, is found to contain 365 days 5 hours 48 minutes 48 ſeconds; but the quantity, according to the authors of the Gregorian calendar, is 365 days 5 hours 49 minutes. In the civil account, this year is ſaid to contain only 365 days; and one day is added to every fourth year, to make up for the odd hours, which is therefore called leap-year.

The Julian year, ſo named from Julius Cæſar, who eſtabliſhed it, conſiſts of 365 days 6 hours, which exceeds the true ſolar year by upwards of 11 minutes, which exceſs amounts to a whole day in near 131 years. And one day is added to the end of February every fourth year, which is compoſed of the odd ſix hours every year. This year is, therefore, called Biſſextile, or leap year.

The Gregorian year, introduced by Pope Gregory XIII. in 1582, is the Julian year corrected by this rule, viz. That inſtead of every hundredth year being a leap year, as it would be in the Julian calendar, in this way, only one hundredth year out of four is a leap year, the other three being common years. By this omiſſion of three days in every 400 years, the civil year would nearly keep pace with the ſolar year for time to come.

Yet this year is not quite perfect, for, as in four centuries the Julian year gains 3 days 2 hours 40 minutes, and as there are only three days omitted in the Gregorian account, there is ſtill an exceſs of 2 hours 40 minutes in 400 years, which amounts to a whole day in 3600 years.

In the year 1752 this ſtyle was adopted in England, and the eleven days were thrown out after the 2d of September, by accounting the 3d the 14th of that month. This was called the New Style, in diſtinction from the former, which was called the Old.

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The ſolar year is either aſtronomical or civil. The aſtronomical ſolar year is that which is determined preciſely by aſtronomical obſervations, and is of two kinds, viz. *tropical*, and *ſideral* or *aſtral*.

The tropical or natural year is the time the Sun takes to paſs through the twelve ſigns of the Zodiac, and is the only proper natural ſolar year, becauſe the ſeaſons always fall in the ſame months.—The ſideral or aſtral year is the ſpace of time the Sun takes in paſſing from any fixed ſtar till his return to the ſame again, and is 20 minutes 29 ſeconds longer than the true ſolar year.

The lunar year is the ſpace of twelve lunar months, and is either aſtronomical or civil.

The lunar aſtronomical year conſiſts of twelve lunar ſynodical months; and is, therefore, 354 days 8 hours 48 minutes 38 ſeconds, being 10 days 21 hours 10 ſeconds ſhorter than the ſolar year.

The lunar civil year is either common or emboliſmic. The common lunar year conſiſts of twelve lunar civil months, and contains 354 days. The emboliſmic lunar year conſiſts of thirteen lunar civil months, and contains 384 days.

The civil or legal year, in England, formerly began on the 25th of March, or the day of the Annunciation of the Virgin Mary: but the hiſtorical year began on the 1ſt of January. The part of the year between theſe two terms was uſually expreſſed thus, 1735-6, or 173⅚. But according to the new ſtyle, the civil year now begins on the 1ſt of January.

The ancient Roman year, as firſt ſettled by the Romans, contained only ten months, and in all 304 days.

The Egyptian year, called alſo the year of Nabonaſſar, from the epoch of that name, contains only 365 days, divided into twelve months of 30 days each, with five intercalary days added at the end. Thus the Egyptian year loſes a whole day of the Julian year every four years, and after the

the ſpace of 1460 years it begins with the Julian year, which length of time is called the Sothic Period.

The ancient Greek year conſiſted of twelve months, which at firſt were divided into 30 days each, but afterwards each month contained 29 and 30 days alternately; and this year was computed from the firſt appearance of the new Moon, with the addition of an emboliſmic month of 30 days every 3d, 5th, 8th, 11th, 14th, 16th, and 19th year, in order to keep the new and full Moons to the ſame ſeaſons of the year.

The ancient Jewiſh year conſiſted of twelve months, containing 29 and 30 days alternately. To which were added eleven or twelve days to make it agree with the ſolar year.

The Syrian year was the ſame in quantity as the Julian year, but commenced in the beginning of October, according to the Julian year.

The Perſian year contained twelve months, of 30 days each, with five intercalary days added.

The Arabic, Mahometan, or Turkiſh year, called alſo the year of the Hegira, conſiſts of 354 days 8 hours 48 minutes, divided into twelve months, containing 29 and 30 days alternately: though ſometimes it contains 13 months; and intercalary days alſo added every 2d, 5th, 7th, 10th, 13th, 15th, 18th, 21ſt, 24th, 26th, and 29th year. The months commence with the firſt appearance of the new Moon.

The year is divided into twelve parts, called months, from the Moon, by whoſe motions it was regulated: the month is properly the time in which the Moon paſſes through the Zodiac, and is of ſeveral kinds; as,

1. The *illuminative month*, which is the interval between the appearance of one new Moon and that of the next; and always varies in quantity. This month is uſed by the Turks and Arabs.

2. The *lunar periodical month*, or the exact time in which the Moon runs through the Zodiac, and conſiſts of 27 days 7 hours 43 minutes and 8 ſeconds.

3. The

3. The *lunar ſynodical month*, called a lunation, is the time between two new Moons, as ſeen from the Earth, and conſiſts of 29 days 12 hours 44 minutes 3 ſeconds and 11 thirds.

4. The *ſolar month* is the time the Sun runs through one ſign of the ecliptic, and conſiſts at a mean rate of 30 days 10 hours 29 minutes 5 ſeconds.

5. The *civil* or *common month* is an interval of a certain number of whole days—ſuch are the calendar months.

6. The *civil lunar month* conſiſts alternately of 29 and 30 days. Thus two of theſe months are equal to two aſtronomical months, and the new Moon will be kept to the firſt day of ſuch civil months for a long time together. This month was in moſt common uſe till the time of Julius Cæſar.

7. The *civil ſolar month*, which conſiſted alternately of 30 and 31 days, excepting one month which had 29 days, introduced by Julius Cæſar. But under Auguſtus, the ſixth month, till then called Sextilis, received the name of Auguſtus, from thence called Auguſt; and one day more was added to it, which was taken from February. This is the regular civil month in uſe in England.

A week is a ſpace of time contained in ſeven days, and originated from the diviſion of the lunar month into four parts.

The diviſion of the month into weeks was uſed by the Syrians, Egyptians, and moſt of the Oriental nations. The Roman week conſiſted of nine days, and the ancient Greeks uſed decades, or a ſyſtem of ten days.

But the Jews uſed the week of ſeven days. The days of the week they denominated, the firſt, ſecond, third, fourth, fifth; and the ſixth day they called the preparation of the Sabbath, the Sabbath being the ſeventh day: and this diviſion is ſtill obſerved by the Chriſtians, Arabs, Perſians, Ethiopians, &c.

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The ancient heathens denominated the days of the week from the feven planets, calling each day after that planet which they fuppofed governed the firft hour of the day. Thus, the firft day was called Dies Solis, or Sunday, from the Sun; the fecond, Dies Lunæ, or Monday, from the Moon, &c.

But our Saxon anceftors, before their converfion to Chriftianity, named the days of the week from the Sun and Moon, and alfo from fome of their deified heroes, to whom they were peculiarly confecrated, which names we ftill retain. Thus, Sunday was dedicated to the Sun; Monday to the Moon; Tuefday to Tuifco, or Mars; Wednefday to Woden, or Mercury; Thurfday to Thor, the Thunderer, or Jupiter; Friday to Friga, or Friya, the wife of Thor, or Venus; and Saturday to Seater, or Saturn. And the days of the week are often expreffed by modern aftronomers by the characters of the planets, as, ☉ for Sunday, and ☽ for Monday, &c.

A *day* is that fpace of time which arifes from the appearance or difappearance of the Sun, and is either natural or artificial.

The *natural* day is the portion of time in which the Sun apparently performs one revolution round the Earth; that is, the time in which the Earth makes a rotation on its own axis.

The *artificial* day is the time from Sun rifing to Sun fetting.

The *natural* day is either aftronomical or civil.

The *aftronomical* day begins at noon, or when the Sun's centre is on the meridian, and contains 24 hours to the following noon.

The *civil* day is the time allotted for the fpace of a day in civil purpofes, and includes one entire rotation of the Earth on its axis. This day begins at different times in different nations: at Sun rifing among the ancient Babylonians, Perfians, Syrians, and moft other Eaftern

nations, and the prefent inhabitants of the Balearic Iflands, the Greeks, &c. It began at Sun fetting among the ancient Athenians and Jews; it is alfo ufed by the Auftrians, Bohemians, Marcomanni, Silefians, modern Italians, and Chinefe. With all modern aftronomers, and the ancient Umbri and Arabians, the day is begun at noon; and at midnight among the ancient Egyptians, Romans, and with the modern Englifh, French, Dutch, Germans, Spaniards, and Portuguefe.

An *hour* is the twenty-fourth part, but fometimes only the twelfth part of a day.

There are various kinds of hours, as, 1. Equal hours, which are the twenty-fourth part of a natural day. They are called equinoctial hours, becaufe they are meafured on the equinoctial; and aftronomical, becaufe ufed by aftronomers. 2. Babylonifh hours, of which there are 24 equal ones in the day, and reckoned from Sun rifing. 3. European hours, ufed in civil computations, and are reckoned from midnight; 12 hours from thence till noon, and 12 more from noon to midnight. 4. Jewifh, or planetary, or ancient hours, which are the twelfth parts of the artificial day, and the fame parts of the artificial night. They are called ancient or Jewifh hours, becaufe ufed by the ancients, and ftill ufed by the Jews: they are called planetary hours, becaufe ancient aftrologers pretended that a new planet prefided over every hour. 5. Italian hours, of which there are 24 equal ones to a day, reckoning from funfet.

The hour is divided into 60 minutes; and each minute into 60 feconds; each fecond into 60 thirds, &c.

As time, for the purpofes of chronology, is calculated by years, it is neceffary to have fome certain fixed point of time from which calculations can be made with certainty, which fixed point of time is called an *epocha*, or epoch.

Different nations ufe different epochs or æras; the Chriftians chiefly ufe that of the nativity of Jefus Chrift; the Mahometans, that of the Hegira, or flight of Mahomet;

the Jews, that of the creation of the world, or that of the Deluge; the ancient Greeks, that of the Olympiads; the Romans, that of the building of Rome; the ancient Perſians and Aſſyrians, that of Nabonaſſar, &c.

The doctrine and uſe of epochs is of great importance in chronology. And to find what year of one epoch correſponds with that of another, a period of years has been invented, which commenced before all the epochs, and is a common ſtandard of them all, and called the Julian period. To this period all the epochs are reduced; that is, the year of this period when each epoch commences is determined. Thus, adding the given year of one epoch to the year of the period correſponding with its beginning, and from the ſum ſubtracting the year of the ſame period correſponding to the other epoch, the remainder is the year of that other epoch.

# A TABLE

OF THE

## *Years of the most remarkable Epochs.*

| N. B. *The years before Christ, are those before the reputed year of his birth, and not reckoned back from the first year of his age, as is usually done.* | *Julian Period* | *Years of the World.* | *Years before Christ.* |
|---|---|---|---|
| Creation of the World — — — | 706 | 0 | 4007 |
| The Deluge, or Noah's Flood — — | 2362 | 1656 | 2351 |
| Assyrian Monarchy, founded by Nimrod — | 2537 | 1831 | 2176 |
| Kingdom of Athens, founded by Cecrops — | 3157 | 2451 | 1556 |
| Entrance of the Israelites into Canaan — — | 3262 | 2556 | 1451 |
| The Destruction of Troy — — — | 3529 | 2823 | 1184 |
| Solomon's Temple founded — — — | 3701 | 2995 | 1012 |
| The Argonautic Expedition — — — | 3776 | 3070 | 937 |
| Lycurgus formed his Laws — — — | 3829 | 3123 | 884 |
| Arbaces, first King of the Medes — — | 3838 | 3132 | 875 |
| Olympiads of the Greeks began — — | 3938 | 3232 | 775 |
| The Building of Rome — — — | 3961 | 3255 | 752 |
| Æra of Nabonassar — — — | 3967 | 3261 | 746 |
| First Babylonish Captivity, by Nebuchadnezzar | 4107 | 3401 | 606 |
| The Second Babylonish Captivity, and Birth of Cyrus | 4114 | 3408 | 599 |
| Solomon's Temple destroyed — — | 4125 | 3419 | 588 |
| Cyrus began to reign in Babylon — — | 4177 | 3471 | 536 |
| Peloponnesian War began — — — | 4282 | 3576 | 431 |
| Alexander the Great died — — | 4390 | 3684 | 323 |
| Captivity of 100,000 Jews, by Ptolomy — | 4393 | 3687 | 320 |
| Archimedes killed at Syracuse — — | 4506 | 3800 | 207 |
| Julius Cæsar invaded Britain — — | 4659 | 3953 | 54 |
| He corrected the Calendar — — — | 4667 | 3961 | 46 |
| The true Year of Christ's Birth — — | 4709 | 4003 | 4 |

The

| *The Christian Æra.* | *Julian Period* | *Years of the World.* | *Years since Christ.* |
|---|---|---|---|
| Dionyſian, or vulgar Æra of Chriſt's Birth — | 4713 | 4007 | 0 |
| Chriſt crucified, Friday, April 3d — — | 4746 | 4040 | 33 |
| Jeruſalem deſtroyed — — — | 4783 | 4077 | 70 |
| Adrian's Wall built in Britain — — | 4833 | 4127 | 120 |
| Diocleſian Epoch of Martyrs — — | 4997 | 4291 | 284 |
| The Council of Nice — — — | 5038 | 4332 | 325 |
| Conſtantine the Great died — — | 5050 | 4344 | 337 |
| The Saxons invited into Britain — — | 5158 | 4452 | 445 |
| Hegira, or Flight of Mahomet — — | 5335 | 4629 | 622 |
| The Death of Mahomet — — — | 5343 | 4637 | 630 |
| The Perſian Yeſdegird — — — | 5344 | 4638 | 631 |
| The Moon, and all the Primary Planets, ſeen in the Sign Libra, from the Earth — — | 5899 | 5193 | 1186 |
| Art of Printing diſcovered — — — | 6153 | 5447 | 1440 |
| The Reformation begun by Martin Luther — | 6230 | 5524 | 1517 |
| The Calendar corrected by Pope Gregory — | 6295 | 5589 | 1582 |
| Sir Iſaac Newton born, December 25th — | 6355 | 5649 | 1642 |
| Oliver Cromwell died — — — | 6371 | 5665 | 1658 |
| Sir I. Newton made Preſident of the Royal Society | 6416 | 5710 | 1703 |
| Died March 20th — — — | 6440 | 5734 | 1727 |
| The New Planet diſcovered by Dr. Herſchel — | 6494 | 5788 | 1781 |
| The Ceres de Ferdinand diſcovered by M. Piazzi | 6514 | 5808 | 1801 |

SECT.

## SECT. VI.

### OF ASTRONOMICAL PROBLEMS, AND THE USE OF THE GLOBES.

The celeftial globe differs from the terreftrial one in having the images of the conftellations and figures of the ftars upon it, inftead of the feveral parts of the Earth. The meridian circle drawn through the poles and the point Cancer reprefents the folftitial colure; and that meridian, drawn through the point Aries, reprefents the equinoctial colure.

### PROBLEM I.

*To exhibit a true Reprefentation of the Face of the Heavens for any given Time and Place.*

Rectify the globe for the latitude of the place (as taught in Geography), placing the north pole of the globe towards the north pole of the world. Having found the Sun's place in the ecliptic, and brought it to the meridian, fet the index to 12 o'clock at noon; then turn the globe on its axis till the index points to the given hour. In this pofition the globe exactly reprefents the face of the heavens as it appears at that time; every conftellation and ftar in the heavens correfponding in fituation to thofe on the globe.

### PROBLEM II.

*To find the Declination and right Afcenfion of any Star.*

Bring the ftar to the brazen meridian; and the number of degrees on the meridian between the equator and the ftar is its declination; and the degree of the equator, cut by the meridian, is the right afcenfion of the ftar. Thus the right afcenfion

afcenfion of any ftar is an arch of the equator, intercepted between the firft degree of Aries, and that point where the meridian or circle paffing through the ftar cuts the equator.

## PROBLEM III.

### *To find the Latitude and Longitude of any Star.*

Bring the folftitial colure to the meridian, and then fix the quadrant of altitude over the pole of the ecliptic, in the fame hemifphere with the ftar, and bring its graduated edge to the ftar; then the degree on the quadrant, cut by the ftar, is its latitude, counted from the ecliptic; and the degree of the ecliptic, cut by the quadrant, is the ftar's longitude.

## PROBLEM IV.

### *To find the Place of any Star or heavenly Body, having its Declination and right Afcenfion.*

Find the point of right afcenfion on the equinoctial by Problem II. and bring it to the meridian; then count the degrees of declination upon the meridian from the equinoctial, and there make a mark upon the globe, which will be the place of the ftar, &c.

## PROBLEM V.

### *To find the Place of a Star, Planet, Comet, &c. having the Latitude and Longitude.*

Bring the pole of the ecliptic to the meridian, and there fix the quadrant of altitude, which turn round till its edge cut the given longitude on the ecliptic; then count the given latitude from the ecliptic upon the quadrant of altitude, and there make a mark upon the globe, which will be the place of the ftar, planet, &c. The place of any ftar, planet,

tar, &c. being found by this or the foregoing Problem, its rifing, fetting, or any other circumftance concerning it, may be found by the proper Problems, as thofe of the Sun are found.

## PROBLEM VI.

*To find the rifing, fetting, and culminating of a Star, or any celeftial Body, and confequently its Continuance above the Horizon for any Place and Day; alfo its oblique Afcenfion and Defcenfion, with its eaftern and weftern Amplitude and Azimuth.*

Adjuft the globe to the ftate of the heavens at 12 o'clock at noon on the given day; bring the ftar, &c. to the eaftern fide of the horizon, which will give its eaftern amplitude and azimuth, and the time of rifing; as for the Sun. Again, turn the globe till the fame ftar comes to the weftern fide of the horizon; fo will the weftern amplitude and azimuth, with the time of fetting, be found. Then the time of rifing fubtracted from that of fetting, leaves the continuance of the ftar above the horizon; which, fubtracted from 24 hours, leaves the time it is below the horizon. Laftly, bring the ftar to the meridian, and the hour to which the index then points is the time of its culminating, or fouthing.

## PROBLEM VII.

*To find the Altitude of a Star, &c. for any given Hour.*

Adjuft the globe to the pofition of the heavens, and turn it till the index point to the given hour; then fix the quadrant of altitude at 90 degrees from the horizon, and turn it to the place of the ftar: then the degrees of the quadrant intercepted between the horizon and the ftar will be the altitude fought.

PROB

## PROBLEM VIII.

*Having the Altitude of a Star by Night, or the Altitude of the Sun by Day, to find the Hour of the Day or Night.*

Rectify the globe as in the foregoing Problem, and turn the globe and quadrant till that degree of the ecliptic where the Sun is, or the ſtar itſelf, cut the quadrant in the given degree of altitude; then the index will point to the hour required.

## PROBLEM IX.

*Having the Azimuth of a Star, or the Sun, to find the Time of the Night or Day.*

Rectify the globe as before, and bring the quadrant to the given azimuth in the horizon; then turn the globe till the ſtar or Sun come to the quadrant, and the index will then ſhow the hour of the night or day.

| Motions, Distances, &c. | Mercury. | Venus. | Earth. |
|---|---|---|---|
| Greatest elongation of inferior and parallax of superior planets | $28^{\circ}\ 20'$ | $47^{\circ}\ 48'$ | * |
| Periodical revolutions round the Sun | *d. h. m.* 87 23 15½ | *d. h. m.* 224 16 49½ | *d. h. m.* 365 6 9½ |
| Diurnal rotation upon their axes | * | *h. m.* 23 22 | *h. m. s.* 23 56 4 |
| Inclinations of their orbits to the ecliptic | $7^{\circ}\ 0'$ | $3^{\circ}\ 23\frac{1}{3}'$ | * |
| Place of the ascending node | $1^{s}\ 15^{\circ}\ 46\frac{3}{4}'$ | $2^{s}\ 14^{\circ}\ 44'$ | * |
| Place of the aphelion, or point farthest from the Sun | $8^{s}\ 14^{\circ}\ 15'$ | $10^{s}\ 9^{\circ}\ 38'$ | $9^{s}\ 9^{\circ}\ 15\frac{1}{2}'$ |
| Greatest apparent diameters seen from the Earth | $11''$ | $58''$ | * |
| Diameters in English miles — that of the Sun being 883217 | 2600 | 7687 | 7964 |
| Proportional mean distances from the Sun | 38710 | 72333 | 100000 |
| Mean distances from the Sun in semidiameters of the Earth | 9210 | 17210 | 23799 |
| Mean distances from the Sun in English miles | 37 millions | 68 millions | 95 millions |
| Proportional eccentricities, or distances of the focus from the centre | 7960 | 510 | 1680 |
| Proportion of light and heat — that of the Earth being 100 | 668 | 191 | 100 |
| Proportion of bulk — that of the Sun being 1380000 | $\frac{1}{15}$ | $\frac{8}{9}$ | 1 |
| Proportion of density — that of the Sun being $\frac{1}{4}$ | 2 | 1¼ | 1 |

| Mars. | Pallas. | Ceres de Ferdinand. | Jupiter. | Saturn. | Herschel, or Georgian. |
|---|---|---|---|---|---|
| 7° 24′ | * | * | 11° 51′ | 6° 29′ | 3° 4$\frac{1}{4}$′ |
| *h. m.* 23 30$\frac{3}{4}$ | *d.* 1594 | *d. h.* 1652 5 | *d. h. m.* 4332 8 51$\frac{1}{2}$ | *d. h. m.* 10761 14 36$\frac{3}{4}$ | *d. h.* 30445 18 |
| *m. s.* 39 22 | * | * | *h. m.* 9 56 | * | * |
| ° 51′ | 33° 39′ | 10° 36′ 57″ | 1° 19$\frac{1}{4}$′ | 2° 30$\frac{1}{3}$′ | 46′ 26″ |
| 17° 59′ | * | 2ˢ 21° | 3ˢ 8° 50′ | 3ˢ 21° 48$\frac{1}{4}$′ | 3ˢ 13° 1′ |
| 2° 6$\frac{1}{4}$′ | * | * | 6ˢ 10° 57$\frac{1}{2}$′ | 9ˢ 0° 45$\frac{1}{2}$′ | 11ˢ 23° 23′ |
| 25″ | * | * | 46″ | 20″ | 4″ |
| 4189 | 95 | 162 | 89170 | 79042 | 35109 |
| 52369 | * | * | 520098 | 953937 | 1903421 |
| 36262 | * | * | 123778 | 227028 | 453000 |
| millions | * | * | 490 millions | 900 millions | 1800 millions |
| 4218 | * | * | 25277 | 53163 | 4759 |
| 43 | * | * | 3.7 | 1.1 | 0.276 |
| $\frac{7}{24}$ | * | * | 1$\frac{2}{3}$ | 1000 | 90 |
| .7 | * | * | .23 | .02 | * |

# AN EXPLANATION

OF THE

## *Principal Terms uſed in Aſtronomy.*

*Æras*, certain periods of time from whence chronologers and aſtronomers begin their computations.

*Altitude*, the height of the Sun, Moon, or ſtars above the horizon; and it is always reckoned upon a vertical circle.

*Amplitude*, an arc of the horizon contained between the eaſt or weſt point of the heavens, and the centre of the Sun, or a ſtar, at the time of its riſing or ſetting.

*Anomaly* (true), the diſtance of a planet in ſigns, degrees, &c. from that point of its orbit which is the fartheſt from the Sun.

*Anomaly* (mean), is that which would take place, if the planet moved uniformly in the circumference of a circle.

*Antecedentia*, the motion of any heavenly body when it is contrary to the order of the ſigns, as, through Aries, Taurus, Gemini, &c. towards Piſces, &c.

*Aphelion*, that point in the orbit of a planet which is the fartheſt diſtant from the Sun.

*Apogeon*, that point in the orbit of a planet, in which it is at its greateſt diſtance from the Earth.

*Apſides*, the two points in the orbit of a planet, which are its greateſt and leaſt diſtance from the Sun: the line joining theſe points is called the line of the apſides.

*Armillary*

*Armillary ſphere*, an inſtrument having the principal circles which are uſually drawn upon the artificial globe.

*Aſcenſional difference*, an arc of the equinoctial contained between that point of it which riſes with the Sun, Moon, or ſtar, and that point which comes to the meridian with them; or it is the time the Sun riſes or ſets before or after ſix o'clock.

*Atmoſphere*, that collection of vapours, or body of air, that ſurrounds the Earth.

*Axis*, of the Earth or any planet, is an imaginary line paſſing through the centre, from one pole to the other.

*Azimuths*, great circles paſſing through the zenith and nadir, and they are perpendicular to the horizon. The azimuth of any celeſtial object is an arc of the horizon, contained between the eaſt or weſt point of the heavens, and a vertical circle paſſing through the centre of that object.

*Biſſextile*, or *leap year*, every fourth year, ſo called becauſe the Romans reckoned the ſixth day of the calends of March, in this year, twice over.

*Cardinal points*, the Eaſt, Weſt, North, and South points of the compaſs.

*Cardinal points of the ecliptic*, the firſt points of the ſigns Aries, Cancer, Libra, and Capricorn.

*Centrifugal force*, that force by which any body, revolving in a circular orbit, endeavours to fly off from the centre of motion in a right line, or tangent to the circle.

*Centripetal force*, that force which attracts any heavenly body towards the centre of its orbit, and which, together with the centrifugal force, preſerves the body in the proper path of its orbit.

*Colures*, two great circles or meridians, one of which paſſes through the ſolſtitial points Cancer and Capricorn, and is called the ſolſtitial colure; the other paſſes through the equinoctial points Aries and Libra, and is called the equinoctial colure. *Conjunction*,

*Conjunction*, is when two ſtars or planets, ſeen from the Sun or Earth, appear in the ſame point of the heavens.

*Conſtellation*, ſeveral ſtars lying near each other, which aſtronomers, for the ſake of remembering with more eaſe, ſuppoſed to be circumſcribed by the outlines of ſome animal, or other figure.

*Coſmical* riſing or ſetting of a ſtar, is when they riſe with the Sun in the morning, or ſet with him in the evening.

*Conſequentia*, the motion of the planets according to the order of the ſigns, as from Aries towards Taurus, &c.

*Culminating* of the Sun or a ſtar, is when they come to the meridian of any place.

*Cycle* of the Moon, a period of 19 years, in which time the changes and eclipſes of the Moon are nearly the ſame, and happen at the ſame time.

*Day*, that portion of time in which the Earth performs an entire revolution upon its axis; and is either natural, artificial, or aſtronomical.

*Declination* of any celeſtial body, is its diſtance north or ſouth from the equator, reckoned in degrees, minutes, &c. upon a circle perpendicular to the equator.

*Degree*, the 360th part of a circle.

*Direct*, a planet is ſaid to be direct when it moves according to the order of the ſigns.

*Diſk* of the Sun, or Moon, is its round face, which, on account of its diſtance from us, appears flat, like a plane ſurface.

*Digit*, is the twelfth part of the Sun or Moon's diameter.

*Eccentricity*, the diſtance between the centre of an ellipſe and either of its foci.

*Ecliptic*, that great circle in which the Sun appears to move.

*Elevation* of the pole, is an arc of the meridian contained [...]en the pole and the horizon, and is always equal to the

the diftance of the zenith from the equator, that is, the latitude of the place.

*Elongation*, the angular diftance of a planet from the Sun, as it appears to a fpectator upon the Earth.

*Ellipfis*, a figure formed by cutting a cone obliquely. The orbits of all the planets are of this form.

*Emerfion*, the time when any planet that is eclipfed begins to recover its light again.

*Epact*, the Moon's age at the end of the year, or the difference between the folar and lunar year.

*Equations*, certain quantities by which are eftimated the inequalities in the motion of a planet: the Moon being fubject to many irregularities, has a great number of equations.

*Equation of Time*, the difference between equal and apparent time, or that fhown by a clock and a fun-dial.

*Equinoxes*, the two points where the ecliptic cuts the equator.

*Galaxy*, or the Milky Way, a large irregular zone in the heavens, illuminated with a great number of ftars.

*Geocentric* place of a planet, is that part where it is feen from the Earth.

*Heliacal* rifing of a ftar, is when it appears above the horizon, before the Sun in the morning: and heliacal fetting of a ftar, is when it is not feen after the Sun in the evening.

*Heliocentric* place of a planet, is that part in which the planet is feen from the Sun.

*Hemifphere*, the half of a globe or fphere, and is either celeftial or terreftrial.

*Horizon*, is the circle which feparates the vifible from the invifible hemifphere, and is either fenfible or rational. The former paffing over the furface of the Earth, and the latter through the centre.

*Hour circles*, are great circles paffing through the poles of the world.

*Immerfion*,

*Immerſion*, the moment when an eclipſe begins on a planet.

*Inclination*, the angle which the orbit of one planet makes with that of another.

*Latitude* of a ſtar or planet, is its diſtance from the ecliptic, reckoned in degrees, minutes, &c. upon the arc of a great circle.

*Longitude* of a ſtar or planet, is its diſtance from the firſt point of Aries, in degrees, minutes, &c. upon the ecliptic.

*Luminaries*, the Sun and Moon, ſo called by way of eminence.

*Lunation*, the time between one new Moon and the next.

*Magnitudes*, the different claſſes of the ſtars, of which there are uſually reckoned ſix or eight.

*Mean motion* of a planet, is that which would take place if it moved in a perfect circle, and an equal ſpace every day.

*Meridian*, that great circle which paſſes through the poles and the zenith of any place.

*Minute*, the 60th part of an hour in time, or the ſame part of a degree of ſpace.

*Nadir*, that point in the heavens directly under our feet.

*Nodes*, the two points where the orbit of a planet interſects the ecliptic.

*Northern ſigns* of the ecliptic, are thoſe ſix on the north of the equinoctial, viz. Aries, Taurus, Gemini, Cancer, Leo, and Virgo.

*Nucleus*, the head of a comet, or the central part of a planet.

*Oblique aſcenſion* of the Sun, or a ſtar, is an arc of the equinoctial contained between the firſt degree of Aries, and that point of it which riſes with the Sun or ſtar.

*Oblique*

*Oblique Sphere*, is that poſition of the globe when either pole is above the horizon leſs than 90 degrees.

*Oppoſition*, when two ſtars or planets are 180 degrees diſtant from each other.

*Orbit*, the path a planet deſcribes in its courſe round the Sun.

*Orbis magnus*, the orbit of the Earth.

*Parallax*, the difference between the places of any celeſtial body, as ſeen from the centre, and from the ſurface of the Earth.

*Parallax of the Earth's annual orbit*, is the angle at any planet which is ſubtended by the diſtance between the Sun and Earth.

*Parallels of latitude*, are ſmall circles of the ſphere, parallel to the equator.

*Perigeon*, that point of a planet's orbit in which it is neareſt the Earth.

*Perihelion*, that point of a planet's orbit neareſt the Sun.

*Pole ſtar*, a ſtar of the ſecond magnitude in the tail of the Greater Bear, ſo called from being ſituated near the North Pole of the world.

*Poles of the world*, the two points at the extremities of the Earth's axis.

*Preceſſion of the equinoxes*, a ſlow motion of theſe two points, whereby they are found to go backwards about 50 ſeconds in a year.

*Quadrant*, the fourth part of a circle; alſo an inſtrument for meaſuring angles.

*Retrograde*, is that motion by which ſome of the planets ſeem to go backwards, or contrary to the order of the ſigns.

*Right aſcenſion*, is that degree of the equator which comes to the meridian with any celeſtial body, reckoning from the firſt point of Aries.

*Satellites*, the ſecondary planets.

*Second*, the sixtieth part of a minute, either of time, or space.

*Solstitial points*, are the two points in the ecliptic through which the solstitial colure passes.

*Stationary*, a planet is said to be stationary when it has no apparent motion.

*System*, a number of bodies revolving round a common centre; as the solar system.

*Syzigies*, those points of the Moon's orbit where she is at the new and full.

*Telescopic stars*, are stars only discoverable by means of a telescope.

*Transit*, is the passing of celestial bodies before one another.

*Twilight*, that faint light we perceive before the rising and after the setting of the Sun, occasioned by the Earth's atmosphere.

*Vector radius*, a line supposed to be drawn from any planet to the Sun, which moving with the planet, describes equal areas, in equal times.

*Zenith*, that point of the heavens directly over our heads.

*Zodiac*, that zone surrounding the heavens on each side of the ecliptic, in which all the planets perform their motions.

# CHAP. XV.

## OF MECHANICS.

### *Definitions.*

1. THE *mechanical powers* are certain ſimple machines uſed for raiſing greater weights, or overcoming greater reſiſtances, than the natural ſtrength of man can perform without them.

2. Theſe ſimple machines are reckoned ſix in number: viz. 1. The *lever*; 2. the *wheel*; 3. the *pulley*; 4. the *ſcrew*; 5. the *wedge*; 6. the *inclined plane*.

3. *Force* is a power exerted on a body to move it; if it act inſtantaneouſly, it is called *percuſſion*, or impulſe; if conſtantly, it is an *accelerative* force.

4. *Gravity* is that force wherewith the body endeavours to fall downwards: it is called *abſolute gravity* when in an empty ſpace, and *relative gravity* when immerſed in a fluid.

5. *Specific* gravity is the proportion which the weight of one body bears to that of another.

6. The *centre of gravity* is a certain point in a body, upon which the body, when ſuſpended, will reſt in any poſition.

7. The *centre of motion* is a fixed point round about which a body moves. And the axis of motion is that fixed line about which it moves.

8. *Power* and *weight*, when oppoſed to each other, ſignify the body that moves another, and the body that is moved;

 the

the body which communicates the motion is the power, and that which receives the motion is the weight.

9. *Friction* is the refiftance which any machine fuffers by the parts rubbing againft each other.

In the practice of mechanics, though all bodies are rough in fome degree, and all engines imperfect; yet it is neceffary to confider all planes as perfectly even, all bodies perfectly fmooth, and all bodies and machines to move without friction or refiftance, all lines ftraight and inflexible, all cords very pliable, &c.

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## SECT. I.

### ON THE SIX MECHANICAL POWERS.

THE whole principles of relative motion in mechanics depend upon this one fingle rule:—*That the whole force of a moving body is the refult of its quantity of matter multiplied by the velocity of its motion.* Thus, when the product arifing from the multiplication of the particular quantities of matter in any two bodies by their refpective velocities are equal, the entire forces are fo too. For example:—fuppofe a body, A, which weighs 40 pounds, to move at the rate of two miles in a minute; and another body B, which weighs only four pounds, to move 20 miles in a minute: the entire forces with which thefe two bodies will ftrike againft any other would be equal to each other, and therefore it would require equal powers to ftop them; for 40 multiplied by 2, gives 80, the force of the body A: and 80 is

is alſo the product of 4, multiplied by 20, the force of the body B. Thus, the heavier any body is, the greater is the power required, either to move or ſtop it. And again, the ſwifter it moves, the greater is its force; therefore, when two bodies are ſuſpended on any machine, ſo as to act contrary to each other, if the machine be put in motion, and the perpendicular aſcent of one bedy, multiplied into its weight, be equal to the perpendicular deſcent of the other body, multiplied into its weight; thoſe bodies, how unequal ſoever in their weights, will balance one another in all ſituations; for as the whole aſcent of one is performed in the ſame time with the whole deſcent of the other, their reſpective velocities muſt be directly as the ſpaces through which they move; and the exceſs of weight in one body is compenſated by the exceſs of velocity in the other. Upon this principle the power of any machine may be eaſily computed; for it is only finding how much ſwifter the power moves than the weight does (that is, how much farther in the ſame time), and juſt ſo much power is gained by the engine.

A *lever* is a bar, either of iron or of wood, one part of which is ſupported by a prop, as its centre of motion. And the velocity of every part or point in the lever is directly as its diſtance from the prop.

There are four kinds of levers:—1. The *common lever*, where the prop is placed between the weight and power, but much nearer the weight than the power. 2. Where the prop is at one end of the lever, the power at the other end, and the weight between them. 3. Where the prop is at one end, the weight at the other end, and the power applied between them. 4. The *bended lever*, which differs from a lever of the firſt ſort only in being bent. Levers of the firſt and ſecond kind are often uſed in mechanical engines; but the third kind are ſeldom uſed, as no power can be gained by them.

When the power is at the ſame diſtance from the prop as the

the weight is, and the power and weight are both alike, the machine will remain in equilibrium, and no power can be gained. This is the principle upon which the common balance is formed. Let C D (*fig.* 14, *plate* 19) be a beam or lever, E the middle point, or centre of motion, which may be confidered as the prop; A B two weights hanging at the ends C and D; then, when the machine is fufpended at the point E, and put in motion, the points C and D, being equidiftant from E, will defcribe equal arches, therefore their velocities will be equal; and if the bodies A and B be alfo equal, then the motion of A will be equal to that of B, as the velocities and quantities of matter are equal; and confequently, if the machine be at reft, neither of the weights can move the other, but they will remain in equilibrium.

The ufe of the balance or common pair of fcales, is to compare the weights of diffetent bodies; for any body, whofe weight is required, put into one fcale, will be balanced by a body of the fame weight, put into the other fcale.

In order to have a pair of fcales perfect, they fhould poffefs the following properties:—1. The points of fufpenfion of the fcales, and the centre of motion in the beam C, E, D, muft be in a right line. 2. The arms C E and D E, muft be of equal length. 3. The centre of gravity muft be in the centre of motion E. 4. There fhould be as little friction as poffible. 5. The fcales muft be in equilibrium when empty.

If the centre of gravity of the beam be above the centre of motion, and one end of the balance be put lower than the other, that end will continually defcend, till it be ftopped at the handle H; but if the centre of gravity of the beam be below the centre of motion, the balance will preferve an equilibrium.

Hence, to examine a pair of fcales, let the weights in the two fcales be in equilibrium, then change the weights to the contrary fcales, and if they remain in equilibrium, the balance is true, otherwife it is falfe.

Let

Let A B C (*fig.* 1, *plate* 19) reprefent a lever of the firft kind, fupported by the prop D; the parts A B and B C, on each fide of the prop D, are called the arms of the lever; the end A of the fhorter arm A B is applied to the weight to be raifed, and the power is applied to the end C, of the other arm B C. The principal ufe of this lever is to loofen large ftones which are fixed in the ground, or to raife great weights to a fmall height, in order to place rollers under them, or ropes for raifing them higher by other machines.

In this lever, the fhorter arm A B fhould be as much thicker than the longer arm B C, as will be fufficient to balance it on the prop D. Thus, if P reprefent a power whofe weight is equal to one ounce, and W a weight of twelve ounces, and if the power be twelve times farther from the prop than the weight is, they will exactly counterpoife each other; and a fmall addition applied to the power P will raife the weight W; and the velocity with which the power defcends will be to the velocity with which the weight rifes, as 12 is to 1; that is, directly as their diftances from the prop; and confequently, as the fpaces through which they move. Thus, it is evident, that if a man, by his natural ftrength, could lift an hundred weight, he will, by a lever of this fort, be able to raife twelve hundred weight. If the weight be lefs, or the power greater, than in the foregoing cafe, the prop may be placed fo much farther from the weight, and then it can be raifed to a proportionably greater height by the fame addition of force: but if the weight be greater, or the power lefs, the prop fhould be placed fo much nearer the weight. For, univerfally, if the gravity of the weight, multiplied by its diftance from the prop, be equal to the gravity of the power, multiplied by its diftance from the prop, the power and weight will exactly balance each other. Thus, if the weight W be twelve ounces, and its diftance from the

the prop 1 inch; the product of 12 multiplied by 1 is 12; and if the power P be 1 ounce, and its diftance from the prop 12 inches, the product of thefe two is alfo 12; therefore they counterpoife each other. If a power equal to 2 ounces be applied at 6 inches diftance from the prop, it will alfo balance the weight W, for 6 multiplied by 2 is 12. And if the power be 3 ounces, and placed at 4 inches diftance from the prop, it would alfo balance the weight W, for 3 times 4 is 12. And the like in any other proportion. A poker ftirring a fire is a lever of this kind; the bar upon which it refts is the prop, the hand applied to the end of it is the power, and the incumbent coals on the other end the weight. Several forts of inftruments are formed of two levers of this kind, as, fciffars, fnuffers, pincers, &c.; the prop, or centre of motion, is the pin which holds them together.

The *ftatera*, or Roman fteelyard, is a lever of this kind, and is ufed to find the weight of any body by one fingle weight, placed at different diftances from the prop. G X (*fig.* 13) is a fteelyard, fufpended by the hook O, from the centre of motion D; the fhorter arm D G is of fuch a weight as exactly to counterpoife the longer arm D X; if this longer arm be divided into as many equal parts as it will contain, and each part equal to O D, the fingle weight P will weigh any body as heavy as itfelf, or as many times heavier as there are divifions in the arm D X. Thus, if the weight P be one pound, and placed at the firft divifion 1, in the arm D X, it will balance one pound in the fcale W; if it be removed to the fecond divifion at 2, it will balance two pounds in the fame fcale; if to the third, three pounds, &c. And if each of thefe integral divifions could be divided into as many equal parts, as a pound contains ounces, then the weight P, placed at any of thefe fubdivifions, would fhow the odd ounces, over and above the number of pounds of the body in the fcale.

The

The ſecond kind of levers have the weight between the prop and the power (*fig.* 2). In this, as well as the former, the advantage gained, is as the diſtance of the power from the prop, to the diſtance of the weight from the prop; and the rules for computing the force of this lever are the ſame with thoſe of the former. Thus, if W be a weight of ſix ounces, hanging at the diſtance of one inch from the prop G, and P a power or weight of one ounce, hanging at the end B, ſix inches diſtant from the prop (by the cord C D running over the fixed pulley E), the power will juſt ſupport the weight: and a ſmall addition to the power will raiſe the weight one inch for every ſix inches that the power deſcends. Thus the power acts with the ſame force upon the weight, as it would do, if the weight were at the ſame diſtance from the prop, and on the other ſide thereof, in which caſe it would be a lever of the firſt ſort.

Two men carrying a burden upon a ſtick exhibit a ſpecimen of a lever of this kind; and the portion of weight borne by each man, is in proportion to his diſtance from the weight. In yoking two horſes of an unequal ſtrength to draw any load, the point of taction is placed as much nearer to the ſtronger horſe than to the weaker, as the ſtrength of the former exceeds that of the latter.

Of this kind of levers are oars, rudders of ſhips, doors turning upon hinges, cutting knives fixed at the point, &c.

The third kind of lever has the power applied between the weight and the prop, in which, in order that the power may counterpoiſe the weight, the gravity of the power muſt exceed that of the weight, as much as the diſtance of the weight from the prop exceeds the diſtance of the power from the prop. Thus, if E (*fig.* 3) be the prop of the lever A B, and W a weight of one pound, which is placed three times as far from the prop as the power P, which acts at F by the cord C going over the pulley D; the power P muſt be three pounds, to counterpoiſe the weight of W of one pound;

 and

and for every inch the power P deſcends, the weight W will aſcend three inches, &c.

Levers of this kind are very little uſed, becauſe they give no advantage in point of force, though they give an advantage in point of motion; but in ſome caſes they become neceſſary, as in raiſing a ladder againſt a wall, in which caſe the foot of the ladder, which is fixed againſt the ground, is the prop, the man's hand who raiſes it, the power, and the whole length of the ladder, from the hand to the upper end, is the weight.

The bones of a man's arm are likewiſe levers of this kind; for the muſcle which raiſes the arm, is fixed to the bone about a tenth part as far below the elbow as the hand is. Therefore the elbow may be conſidered as the prop upon which the lower part of the arm turns, and the muſcle muſt conſequently exert a force ten times as great as the weight which is raiſed in the hand.

The fourth kind of lever has all the properties of the firſt kind, and differs from it only in being bent, which is done for the ſake of convenience. A C B (*fig.* 10) is a lever of this ſort, bent at C, its prop or centre of motion; W the weight, and P the power acting at A, over a pulley by means of the cord D. As the mechanical power of this lever is the ſame with that of the firſt ſort, it need not be repeated. A hammer drawing a nail is a lever of this ſort.

The ſecond mechanical power is the *wheel* and *axle* (*fig.* 4), in which the power is generally applied to the circumference of the wheel, and the weight W to that of the axle; the weight being raiſed by a rope winding round the axle as the wheel turns round. In this inſtrument it is evident, that the velocity of the power muſt be, to that of the weight, as the circumference of the wheel is to the circumference of the axle: and the power gained is in proportion to the circumference of the wheel to that of the axle. Therefore, when the gravity of the power is to that of the weight, as the circumference of the axle is to the circumference of the wheel, the power and weight

weight will balance each other. Again, let A B be a wheel, E D the axle, and the circumference of the wheel eight times as great as that of the axle; then a power P of one pound weight hanging by the cord I, which goes round the wheel, will balance the weight W of eight pounds hanging by the rope K, which goes round the axle; and a ſmall addition to the power will cauſe it to deſcend and raiſe the weight; but the weight will riſe with only an eighth part of the velocity wherewith the power deſcends; and conſequently will move through only an eighth part of an equal ſpace in the ſame time. If the wheel be pulled round by the handles S S, the power will be increaſed in proportion to their length.

In this mechanical power the radius of the wheel and the oppoſite radius of the axle may be conſidered as the longer and ſhorter arms of a lever of the firſt kind, the centre of the axle being the prop.

Sometimes the wheel, or the axle, is indented, or cut into teeth, which have another wheel working in them, as in jacks, clocks, mill-work, &c. by which means they give a much greater mechanical force. To compute the power of a combination of wheels, multiply the radii of all the axles continually together, as alſo the radii of all the wheels; then, as the former product is to the latter, ſo is a given power applied to the circumference of a wheel to the weight it can ſuſtain. For example: in a combination of five wheels and axles, to find the weight a man can ſuſtain or raiſe, whoſe force is equal to 150 pounds, the radii of the wheels being 30 inches, and the radii of the axles three inches. Here 3 multiplied four times into itſelf, produces 243; and 30 multiplied four times into itſelf, produces 24,300,000; therefore, as 243 is to 24,300,000, ſo is 150 to 15,000,000 pounds, the weight he can ſuſtain, which is more than 6696 tons, or above 100,000 times as great a weight as he could ſuſtain by his own natural force.

But here it muſt be obſerved, that though there is a prodigious gain of power in theſe combinations of wheels, yet there is a great loſs of time; that is, the weight in this caſe will move 100,000 times ſlower than the power; and this is true in all mechanical caſes whatever.

The third mechanical power is the *pulley*, or ſometimes a ſyſtem of pullies. Sometimes theſe are fixed in a block or caſe, which is alſo fixed; at other times they are in a block which is moveable, and riſes with the weight. The ſingle pulley A (*fig.* 9) gives no mechanical advantage, though it may ſerve to change the direction of the power; but is only as the beam of a balance, whoſe arms are of equal length and weight; and is, in fact, but another form of the balance.

The ſyſtem of pullies is repreſented *fig.* 12, where the four pullies are faſtened to an immoveable block above; three of them, A, B, C, by the three diſtinct cords running under them. The power of this ſyſtem of pullies is diſcovered by ſuppoſing W a weight of 16 pounds, ſuſpended from the pulley C, which is alſo ſuſpended by the cord C, one end of which is faſtened to the block above, and the other end ſupported by the pulley B; therefore the pulley B ſuſtains only half the weight of the weight W, or eight pounds; the other half being ſuſtained by the cord C, fixed to the block. Then the cord B, which goes under the ſecond pulley, ſuſtains the weight of eight pounds, which is alſo divided, four pounds being ſuſtained by the cord B, fixed in the block above, and the other four pounds by the next pulley A. This next pulley A alſo has its weight divided, one half being ſupported by the cord A fixed to the block, and the other half ſupported by the ſmall pulley, which ſmall pulley again divides the weight it ſupports: ſo that the power P is equal to only one pound, which will counterpoiſe the weight W of 16 pounds.

The velocity of the weight to that of the power is as the

gravity

gravity of the power is to that of the weight. Thus if P defcend eight inches, A will afcend four inches, B two inches, C one inch, and W half an inch.

A, B, C, D, (*fig.* 11,) are four pullies, two of which, A and B, are in a fixed block X; the two others, C and D, in a moveable block. Here the weight W is raifed by pulling the cord at P, which goes fucceffively over the four pullies, and is faftened at the end to the fixed block at f. The purchafe of this machine is feen to be as 4 to 1, for P is fuftained by the fingle cord; but W by four folds of the cord, viz.—*o*, *f*, *u*, *k*, fo that if P be one pound, W will be four.

The velocity of the power to that of the weight is alfo, as in the former cafe, as the gravity of the weight to that of the power, or as 4 to 1; for when P defcends four inches, the parts of the cord at k will afcend four inches towards e, and all the other parts of the cord will equally follow each other; and as there are four folds in the cord, viz. *o*, *s*, *u*, *k*, they will each of them be fhortened one inch, and C or W will be fo much raifed.

In the fame manner the purchafe of any combination of pullies may be determined; for the momenta of the weight and power will always be equal; as in the other mechanical powers.

The fourth mechanical power is the *inclined plane*. In this machine the advantage gained is as great as its length exceeds its perpendicular height. Let A B (*fig.* 5) be a plane parallel to the horizon, and C D a plane inclined to it; if the length C D be three times as great as the perpendicular height G F, the cylinder E will be fupported on the plane C D, by a power equal to a third part of the weight of the cylinder; or, it may be rolled up the plane with a third part of the power, which would be fufficient to draw it up the fide of an upright wall. If the plane were four times as long as the perpendicular height, it would require only the fourth

fourth part of the power; and ſo on in proportion. The uſe of this power is to raiſe a great weight to any eminence, which is uſually done by puſhing it up a ſtout plank, ſet ſloping to the place deſigned; and ſuch plank, or other contrivance ſimilar thereto, is called an inclined plane.—Now it is evident, the ſteeper the aſcent is, the more difficult it is to puſh any weight up it; and the more the aſcent inclines to the horizon, the eaſier the weight may be puſhed up. This is evident from the eaſe with which a rolling weight is forced up a hill, that riſes gently, while it is ſo difficult to roll the ſame weight up a hill which is very ſteep.

The force wherewith a rolling body deſcends upon an inclined plane is to the force of its abſolute gravity, as the height of the plane is to its length. Thus, if the perpendicular height G F of the plane be equal to half its length A B, the cylinder E will roll down the plane with a force equal to half its weight; and it would require a power equal to half its weight to ſuſtain it. If the plane be ſo much elevated as to be perpendicular to the horizon, the cylinder E would deſcend with its whole weight, becauſe the plane contributes nothing to its ſupport or hindrance.

In an inclined plane, a power acts to the greateſt advantage, when its direction is parallel to the ſurface of the plane.

The *wedge* conſtitutes the fifth mechanical power, and may be conſidered as two equally inclined planes, A D F (*fig.* 6) and C B F joined together at their baſes F O; then D C is the whole thickneſs of the wedge A B C D, or the back of the wedge, where the power is applied; E F the height or depth, D F the length of one of its ſides, equal to C F, the length of the other ſide, and O F its ſharp edge, which is driven into the wood intended to be ſplit by the force of a hammer or mallet ſtriking on its back. Thus, A B is a wedge (*fig.* 7) driven into the cleft C E D of the wood F G.

The

The power gained by the wedge is in proportion to the length of the flant fide to half the thicknefs of the back. Thus, if the back of the wedge be two inches thick, and the fide 20 inches long, any weight performing on the back will balance 20 times as much acting againft the fides. To ufe a wedge to the greateft advantage, it fhould be forced, not by preffure, but by percuffion, as by the blow of a hammer or mallet; by which means a wedge may be driven in below any weight, and fo made to lift it up, as the largeft fhips, &c.

The wedge has a very great mechanical force, and effects what would be impoffible by the lever, wheel and axle, or pulley; for the force of the blow fhakes all the adjacent parts, and thereby makes them feparate more eafily; fo that not only wood, but even rocks, can be fplit by it.

To the wedge may be referred the axe or hatchet, the chifel, the fpade and fhovel, knives of all kinds; as alfo the bodkin and needle, and all forts of inftruments which, beginning from an edge or point, become gradually thicker.

The fixth, and laft mechanical power, is the *fcrew*, which is not properly a fimple machine, becaufe it cannot be ufed without a lever to turn it, called the winch or handle. It is a compound engine of very great force, and is a kind of perpendicular or endlefs inclined plane, ftill farther affifted by the power of the handle or lever: and the gain of power is in proportion of the circumference defcribed by the power to the diftance between one thread and the next in the fcrew.

Thus, let C be a wheel (*fig.* 8), having a fcrew *a b*, on its axis, working in the teeth of the wheel D, which fuppofe to be 48 in number. Then it is evident, that for one revolution of the wheel C, fcrew *a b*, and winch A, the wheel D will be moved one tooth by the fcrew: and therefore in 48 revolutions of the winch A, the wheel D will be turned once round.

round. Then if the circumference of the circle defcribed by the handle of the winch A be equal to the circumference of a groove round the wheel D, the velocity of the handle will be 48 times as great as the velocity of any given point in the groove; confequently, if a line G goes round the groove D, and has a weight of 48 pounds fufpended from it, below the pedeftal E F, a power equal to one pound at the handle will fupport this weight; or, if a groove be made in the wheel C, equal in radius to the circle defcribed by the handle, the weight H of one pound, fufpended therefrom by a line in the groove, will balance the 48 pounds, as before. If the line G, inftead of going round the groove of the wheel D, go round its axle I, the power of the machine will be as much increafed, as the circumference of the groove exceeds that of the axle, as fhewn under the wheel and axle. And if a fyftem of pullies were applied to the cord H, the power could be increafed to an amazing excefs.

The ufes to which the fcrew is applied are various; it is chiefly ufed for preffing bodies clofe together, as the preffes for bookbinders, packers, hot-preffers, &c.

The friction in the fcrew is very confiderable, as it is alfo in the wedge, which generally requires a third part more of the power to work them when loaded, than what is fufficient to conftitute a balance between the weight and power.

If machines or engines could be made without friction, the leaft degree of power above what is fufficient to balance the weight, would be fufficient to raife it. In the lever the friction is little or nothing: in the wheel and axle it is but fmall: in pullies it is confiderable: and in the inclined plane, wedge, and fcrew, it is very great.

Wood greafed, or metal oiled, have nearly the fame friction; and the fmoother they are, the lefs is their friction, provided they be not too highly polifhed. In polifhed fteel moving upon polifhed fteel or pewter, the friction is about a

fourth

fourth part of the weight, on copper a fifth part, and on brafs a fixth part of the weight: iron or fteel running in brafs has the leaft friction of any. And metals of the fame fort have more friction than different forts; and in general the friction increafes in the fame proportion with the weight, but is greater with a greater velocity.

The friction in pullies is now almoft reduced to nothing, by the contrivance of Mr. Garnett, in his patent friction rollers, which produce a great faving of labour and expenfe, as well as wear of the materials, both when applied to pullies and the axles of wheel-carriages. By this contrivance, there is a hollow fpace left between the nave and axle, or centre and pin-box, which is filled up by folid equal rollers, nearly touching each other, and furnifhed with axles, each of which is inferted into a circular ring at each end, by which their relative diftances are preferved; and they are kept parallel by means of wires faftened to the rings between the rollers, and to which the wires are rivetted.

It is a general property in all the mechanic powers, that when the weight and power balance each other, if they be put in motion, the power and weight will be to each other reciprocally as the velocities of their motion; or the power is to the weight as the velocity of the weight is to the velocity of the power; fo that their two momenta are equal: viz.—The product of the power, multiplied by its velocity, is equal to the product of the weight multiplied by its velocity. And hence the general rule: viz. *That what is gained in power is loft in time.* For the weight moves as much flower as the power is lefs.

## SECT. II.

### OF THE APPLICATION OF THE POWERS TO MILLS AND MACHINES.

In order to discover the properties of any machine consisting of the mechanical powers, it is necessary to consider the weight that is to be raised, or the resistance to be overcome; and also the power required to raise the weight, or overcome the resistance. For this purpose, there are two principal problems, the resolution of which is requisite to show the powers of any engine.—The first problem is, *To determine the proportion that the power and weight ought to have to each other, that they may be in the just equilibrium.*—The second is, *To determine what the proportion should be between the power and weight, that the machine may produce the greatest effect in a given time.*

The first problem is solved by this general rule, viz.—That the power and weight sustain each other, or are in equilibrium, when the power and weight are reciprocally proportional to the distances of the directions in which they act from the centre of motion; or when the product of the power multiplied by the distance of its direction is equal to the product of the weight multiplied by the distance of its direction. This is the proportion of the weight and power when they are in equilibrium, so that the one would not prevail over the other if the engine were at rest; and if it be set in motion, it would continue to proceed uniformly if there were no friction of its parts and other resistances. And in general

general the effect of any power or force is as the product of that force multiplied by the diſtance of its direction from the centre of motion; or the product of the power, and its velocity when in motion, for the velocity is proportional to the diſtance from that centre.

The ſecond general problem in Mechanics, is of the greateſt importance, though it has been little attended to by mechanical writers, viz.—To determine the proportion between the power and weight, ſo that when the power prevails, and the machine is in motion, the greateſt effect poſſible may be produced by it in a given time. When the power is only a little greater than what is ſufficient to ſuſtain the weight, the motion is uſually too ſlow; and though a greater weight be raiſed in this caſe, it is not ſufficient to compenſate for the loſs of time. And when the power is much greater than what is ſufficient to ſuſtain the weight, the weight is raiſed in leſs time; but it often happens, that this is not ſufficient to compenſate for the loſs which ariſes from the load being reduced; therefore, the only general rule that can be given is, to find when the product of the weight, multiplied by its velocity, is the greateſt; for this product meaſures the effect of the machine in a given time, which is always greater in proportion as both the weight and velocity are greater.

In the conſtruction of compound machines, where it is neceſſary to alter the direction of the motion, recourſe muſt be had to what is called *bevel geer*, the principle of which is as follows:—

Let A and B (*fig.* 15) be two cones revolving on their centres a c and a b; if their baſes be equal, they will each perform their revolution in the ſame time; and any two points in each cone equally diſtant from the centre, as d 1, d 2, d 3, &c. will revolve in the ſame time as f 1, f 2, f 3, &c. reſpectively. But if one cone be twice the diameter of the other, as the cone a d e (*fig.* 20), which is twice the diameter of the cone f a d, then as they turn upon their

 centres,

centres, when the cone a f d has made one revolution, the cone a d e will have made but half a revolution, and every part in each cone, equally diſtant from the centre a, will have the ſame proportion in their revolutions to each other, as f 1, f 2, f 3, &c. will have made two revolutions to the points e 1, e 2, e 3, &c. for one revolution of the other cone reſpectively, &c. Now, if the cones are fluted, or have teeth cut in them, diverging from the centre a to the baſes d e, d f (*fig.* 16), they would then become bevel geer. The teeth at the point of the cone being ſmall, and of little uſe, may be cut off; or, inſtead of the two cones, may be uſed two ſhafts, with bevel wheels fixed to them, as the ſhaft a b (*fig.* 18), with the bevel wheel c d, which turns the bevel wheel e f, with its ſhaft b g, and the teeth work freely into each other, as in figure 16. The teeth may be made of any dimenſions, according to the ſtrength required, and by this means a motion may be communicated in any direction, or to any part of a building, with very little trouble and friction.

The method of conſtructing the wheels for any proportion, is as follows:—Draw the line a b (*fig.* 21) to repreſent a ſhaft of a wheel; draw the line e d to interſect the line a b, in the direction that the motion is to be conveyed, and the line e d will repreſent the other ſhaft of the motion.

Then ſuppoſe the ſhaft e d is to revolve three times in the time that the ſhaft a b revolves once; draw the parallel line i i at any diſtance, from a ſcale (ſuppoſe one foot); then draw the other parallel line k k at three feet diſtance; after which, draw the line w x through the interſections of the two ſhafts a b and e d, and likewiſe through the interſections of the two parallel lines i i and k k, in the points x y, which will be the pitch line of the two bevel wheels, or the lines where the teeth of the two wheels act on each other, as may be ſeen in figure 19, where there are three wheels.

Where it is required to communicate a continued uniform motion, and where the angle does not exceed 40 degrees, and

and alſo where the equality of the motion is not regarded, the univerſal joint may be uſed (*fig.* 22) inſtead of the bevel geer. This joint may be conſtructed by a croſs, as ſhown in the figure; or with four pins faſtened at right angles upon the circumference of a hoop, or ſolid ball. This is of great uſe in ſome machines, where the tumbling ſhafts are continued to a great diſtance from the moving power, as it is in cotton-mills. The ſhafts, by applying this joint, may alſo be cut to any length, which is a great advantage where there is much reſiſtance.

# CHAP. XVI.

# OF ELECTRICITY.

## SECT. I.

### THE PRACTICAL PART OF ELECTRICITY.

THE earth, air, and all terreſtrial bodies are ſuppoſed to contain a certain quantity of an elaſtic ſubtle fluid, called by philoſophers, the *electric fluid;* and when any body poſ-
ſeſſes

feffes more or lefs of this fluid than what naturally belongs to it, feveral effects are vifible in it, and the body is faid to be electrified.

This certain quantity of electric fluid found in all bodies could never be increafed or diminifhed, if all bodies admitted the paffage of this electric fluid through their pores or along their furfaces; but there are many bodies which will not fuffer this fluid to pafs through them, while others freely permit it. Thofe bodies through which the electric fluid can pafs are called *conductors* of electricity, of which the moft perfect are metals of all kinds. And thofe bodies through which the electric fluid cannot pafs are called *non-conductors* of electricity, of which the moft perfect are glafs, refin, fealing-wax, fulphur, bees-wax, and baked wood, among folids; and oils and air, among fluids. But all fubftances become conductors when they are made very hot. Conducting fubftances are alfo called *non-electrics*, and non-conducting fubftances are called *electrics*. Into thefe two claffes all bodies are divided by electricians.

When any body has acquired an additional quantity of electric matter, and is furrounded with other bodies through which the electric fluid cannot pafs, or non-conductors, it muft remain overloaded; or if it have loft part of its natural fhare of electric matter, it muft remain exhaufted; becaufe the bodies which furround it prevent any of the electric fluid from entering or coming out of it, and the body is then faid to be infulated.

There are two principal theories of electricity, each of which has had its advocates. The one is, that of two diftinct electric fluids, repulfive with refpect to themfelves and attractive of one another, adopted by M. Du Fay, on difcovering the two oppofite fpecies of electricity, viz. the *vitreous* and *refinous*, which is fince new-modelled by Mr. Symmer. Upon this hypothefis thefe two fluids are equally attracted by all bodies, and exift in intimate union in their pores; and in this ftate they fhow no mark of their exiftence.

But

But the friction of an electric body againſt a rubber ſeparates theſe fluids, and cauſes the vitreous electricity of the rubber to paſs to the electric, then to the prime conductor of the machine, while the reſinous electricity of the conductor and electric is communicated to the rubber: thus the quality of the electric fluid poſſeſſed by the conductor and the rubber is changed, while the quantity remains the ſame in each. In this ſeparated ſtate the two electric fluids will exert their reſpective powers; and any number of bodies charged with either of theſe may repel each other, attract thoſe bodies that have leſs of each particular fluid than themſelves, and be ſtill attracted more by bodies that are either only deſtitute of it, or loaded with the contrary. In this theory the electric ſpark makes a double current; one fluid paſſing to the electrified conductor from any ſubſtance preſented to it, while the ſame quantity of the other fluid paſſes from it; and when each body receives its natural quantity of both fluids, the balance of the two powers is reſtored, and both bodies are unelectrified.

The other theory, and that which is commonly received, is that diſtinguiſhed by the name of *poſitive* and *negative electricity*, ſuggeſted by Dr. Watſon, and demonſtrated by Dr. Franklin; in which it is ſuppoſed, that all bodies poſſeſs a certain ſhare of one and the ſame fluid, which is extremely ſubtle and elaſtic, by which the particles of it are ſtrongly attracted, as they are repelled by one another. When bodies poſſeſs their natural ſhare of this fluid, they are ſaid to be in an unelectrified ſtate; but when the equilibrium is deſtroyed, and they have an additional quantity *from* other bodies, or when they loſe part of their natural ſhare by the communication *to* other bodies, they then become electrified, and exhibit electrical appearances; which are generally the ſame in both caſes. In the former caſe they are ſaid to be electrified poſitively, or *plus*; and in the latter caſe negatively, or *minus*. It is alſo ſuppoſed, that electrics always contain

contain an equal quantity of this fluid; ſo that there can be no increaſe on one ſide without a proportional decreaſe or loſs on the other, and *vice verſa*. And as the electric will not ſuffer the fluid to paſs through its pores, there will be an accumulation on one ſide, and a correſponding deficiency on the other; then, connecting both ſides together by proper conductors, the equilibrium will be reſtored, by the ruſhing in of the redundant fluid from the overcharged ſurface to the exhauſted one. Thus, if an electric be rubbed by a conducting ſubſtance, the electricity is only conveyed from one to the other, the one giving what the other receives: and if one be electrified poſitively, the other will be electrified negatively, unleſs the loſs be ſupplied by other bodies connected with it, as in the caſe of the electric and inſulated rubber of a machine. Thus, bodies differently electrified will naturally attract each other, till they mutually give and receive an equal quantity of the electric fluid, and then the equilibrium between them will be reſtored.

The method of diſturbing the equilibrium of the electric fluid in bodies, or of making it paſs from one to another, is chiefly friction, or a ſlight rubbing of them one againſt the other; when the electric fluid will generally leave the rougher ſurface, and paſs upon the ſmoother; or it leaves the leaſt perfect electric, and paſſes to the more perfect one of the two. Thus, if a ſmooth glaſs tube (*fig.* 1, *plate* 20) be drawn through the hand, the effect of the friction makes the electric fluid leave the hand and paſs to the glaſs tube, which is the more perfect electric of the two, where it will remain in addition to its natural quantity. For the electric fluid cannot poſſibly leave the glaſs, becauſe neither the glaſs nor the ſurrounding air are conductors of electricity; but if a conducting ſubſtance, as the finger, or a piece of metal, be preſented to any part of the glaſs, the electric fluid will leave the glaſs and paſs into them; and if the finger, or metal, be preſented to every part of the tube ſucceſſively, the whole of the redundant fluid will leave the tube, and it will retain only its natural

natural ſhare. Here the glaſs is ſaid to be *excited*, becauſe the friction ſeems to excite the electric power which was in the glaſs.

In the ſame manner the friction of the glaſs globe againſt the rubber in the electrical machine makes the electrical fluid which was in the rubber paſs to the glaſs, from whence it is conveyed to the prime conductor, the points of which are preſented to every part of the globe in ſucceſſion, as it is turned in the machine; and as the friction is continued, there will be a conſtant ſupply of electric fluid to the prime conductor (though other bodies be preſented to it), and keep diſcharging all the while in viſible ſparks. The hand, in the former of theſe caſes, and the rubber, in the latter, part with their natural ſhare of electric fluid to the glaſs againſt which they are rubbed, but receive an immediate ſupply from the conducting ſubſtances to which they are connected; and theſe are again ſupplied by the general maſs of fluid that is in the earth.

Again, if a ſtick of ſealing-wax, a piece of ſulphur, or a tube of rough glaſs, be drawn through the hand, the electric fluid belonging to them will paſs from them to the hand, and being ſurrounded by the air, which is a non-conductor, they remain exhauſted, and are ready to take ſparks of electric fire from any bodies preſented to them. The ſulphur, ſealing-wax, &c. in this caſe are ſaid to be excited, as well as the glaſs, which was overloaded with fluid, though the ſtate they are in be the reverſe of one another. It is impoſſible to diſtinguiſh by the eye the courſe of the electric matter, its velocity is ſo great.

There are a variety of inventions for the conſtruction of the electrical machine, but the moſt ſimple is that repreſented in figure 2, which, by reaſon of its ſimplicity, is not liable to be put out of order, as it has neither wheel nor ſtring, though both might be attached thereto, if required. It may alſo be fixed firm on a table, and eaſily taken off: the

the globe may alfo be taken out with the greateft eafe, in order to be packed up. This machine is the fame as that ufed by Dr. Prieftley; and when the infide of the globe is lined with his compofition following, it will produce more fire than any of thofe in common ufe.

A is the bafe, which is a piece of mahogany about nine inches fquare, and 1¼ thick, in which is fixed the pedeftal B, to fupport the globe G, which is fixed in an iron axle C, to which is fixed a brafs cap. The globe is turned by the handle H running in the brafs focket E; R is the rubber, made of wood, cut to the curve of the globe, and covered with a leather covering, which is at a little diftance from the wood in the middle of the curve, that it may the better yield to the preffure of the globe.

Over this leather is another leather, made to take off by moving a pin. On this leather the amalgam is rubbed; and as it is eafily taken off, it is more readily brought into order than thofe which are fixed to the rubber. To this leather is fixed a piece of black filk, which extends half round the globe, and greatly increafes the fire; fo that this machine will give fire well, if the rubber fcarcely touch the globe. This machine will alfo fuit any kind of conductor.

For thofe who do not choofe to have the rubber infulated, there is a fpring S; but the more curious may have them made with the rubber well infulated by a glafs pillar that will hold the rubber to the back of the globe, as in figure 3.

## *Dr. Prieftley's Compofition for lining the Infide of Globes, or Cylinders.*

This compofition confifts of an equal quantity of linfeed oil and refin, which is boiled over a gentle fire for two hours. When the globe or cylinder is to be lined, it muft be put into an oven, with a fufficient quantity of the compofition broken and put into the infide; and when it is melted, the globe is turned round every way, in order to fpread it all over equally. Inftead

Inftead of the above compofition, fome ufe a mixture made of four parts of Venice turpentine, one part of refin, and one of bees-wax; which is prepared and ufed in the fame manner as the former.

There are alfo other methods for making amalgam: as, 1. By four parts of fpelter and fix parts of mercury; 2. alfo by adding fix ounces of quickfilver to one pound of molten tin, which, when cold and reduced into powder, is to be mixed with feven ounces of fulphur, and fix ounces of fal ammoniac: the whole is fublimated in a mattrafs.

The parts of the machine which are infulated fhould be varnifhed over with a varnifh made of highly rectified fpirits of wine and fealing-wax; as alfo the glafs pillars, in order to keep off the moifture they would imbibe from the damp air.

It is neceffary for the young practitioner to attend to the following rules in the performing of his experiments, as it will often happen, that though he be in poffeffion of very good inftruments, yet, through fome inadvertencies, his experiments will not fucceed according to his expectation, for want of a fufficient practice in the art.

1. The electrical machine, coated jars, and every part of the electric apparatus, fhould be kept clean, and free from duft and moifture.

2. In clear weather, when the air is dry, and particularly in frofty weather, the machine will always work well: but in hot weather, and damp weather (except it be brought in a warm room, and the apparatus made thoroughly dry), it will not work fo well.

3. The cylinder fhould always be wiped clean with a foft dry linen cloth that is warm, and then with a clean hot flannel, before the machine be ufed; then applying a little amalgam, turn the winch of the machine, and the electric fluid will come like a wind from the cylinder to the knuckle, and fome fparks and cracking will foon follow. This indicates that the machine is in good order. But if thefe appearances

 be

be not produced, there is a fault, which is generally in the rubber; to remedy which, remove the rubber from the glafs pillar, and dry the filk part before the fire, then greafe the leather with a bit of tallow or mutton fuet.

4. When the table on which the machine ftands, and to which the chain of the rubber is connected, is very dry, it is a bad conductor, and hinders the operation of the machine. The floor, and walls of the room alfo, in very dry weather have the fame effect on the machine. In this cafe the chain of the rubber fhould be connected by a long wire, with fome moift ground, or with the iron-work of a water-pump; by which means the rubber will be fupplied with a fufficient quantity of electric fluld.

5. If there be too much amalgam upon the leather of the rubber, the machine will not work well until a little be fcraped off.

6. If the globe or cylinder contract any black fpots, as is often the cafe, they fhould always be wiped off.

7. In charging electric jars, they fhould be made a little warm before they are ufed, and they will produce a greater effect.

8. When a large battery is to be difcharged, never difcharge it through a good conductor, except the circuit be at leaft five feet long, otherwife fome of the jars would be found broken.

### *To fhow the Effects of electrical Attraction and Repulfion.*

Sufpend a plate of metal F (*fig.* 2) from the conductor, which is fupported by two glafs pillars, and fupplied with electricity from the globe; and at the diftance of three or four inches below this, put another plate P of the fame fize; upon the bottom plate lay a feather, or fmall flips of paper; and when the machine is fet in motion, the feather or the papers will be attracted to the upper plate F, from which they will be immediately repelled, and will fly to difcharge themfelves

themſelves upon the lower plate P, which is ſupported on the pedeſtal G H; after which they will be attracted and repelled again as before, and fly from one plate to the other with great rapidity, if the electrification be ſtrong. It is uſual to cut the pieces of paper into the figures of men and women, when they exhibit a kind of dance, which affords ſome entertainment to the beholders.

The electrical bells are often uſed in concert with the above experiment, and depend on the ſame principle. Theſe are four bells *a*, *b*, *c*, *d*, which hang from the ends of two braſs rods (*fig.* 5), communicating with the prime conductor, and with another bell *e*, fixed on a pedeſtal A, reaching to the ground. Between the four bells hang four braſs balls, ſuſpended by ſilken ſtrings; each of theſe balls hangs between the centre bell *e* and one of the outermoſt bells. The outermoſt bells being connected with the prime conductor by braſs chains, are electrified, and attract the braſs balls which hang between them and the centre bell; and the attraction being ſtrong, each ball ſtrikes its outer bell with ſome violence, and makes it ring: being then loaded with electricity, it is immediately repelled, and flies to unload itſelf by ſtriking upon the centre bell, which is inſulated by the glaſs pillar B upon the pedeſtal A, and from which pillar the electric matter paſſes to the floor, by means of the braſs pedeſtal A. The balls are then again attracted by the outermoſt bells, as before; and thus the ringing may be continued as long as neceſſary.

When a perſon is to be electrified, and ſtands upon a ſtool with glaſs feet, or baked wood (*fig.* 7), having the chain in his hand, faſtened to the prime conductor, he is then ſaid to be inſulated, and may be conſidered as part of the prime conductor: for every part of his body will exhibit the ſame appearance as the prime conductor. For if the finger of any perſon, ſtanding upon the floor, be preſented to him, a ſpark of fire will be ſeen to iſſue from him, and both he and the perſon who receives it will feel a painful ſenſation; and a ſnapping

a ſnapping noiſe will be heard. Every part of his body will then attract all light ſubſtances, as feathers, bits of paper, &c.: the hairs of his head alſo, or of his wig, if they happen to be looſe, will repel each other, and many of them ſtand upright.

Pointed bodies have a remarkable property in electricity; for the more acutely pointed any body is, the more eaſily does it take, or part with, electric matter. Thus, if a needle, or ſharp-pointed wire, be faſtened to the prime conductor, it will retain but a ſmall degree of electricity, and conſequently will give but a ſmall ſpark when the finger or a piece of metal is preſented to it. Again, if the needle or wire be held in a perſon's hand ſtanding upon the floor, and preſented to the conductor, it will be found to receive but a ſmall degree of electricity. In the former caſe, the needle being in contact with the prime conductor, the electric fluid goes off at the point, and is diſperſed in the air. In the latter caſe, the needle being preſented towards the conductor, receives the electric fluid from it at a conſiderable diſtance.

If the ſharp-pointed wire be giving out the electric fluid, the flame will be larger (for a flame will be ſeen at the point of the needle or wire, if the experiment be made in the dark), the parts of which it conſiſts will be fewer, and a ſnapping noiſe will be heard, if the point be not very acute; whereas, if the pointed wire be receiving the electric fluid, the flame will be much ſmaller and more globular; the parts of which it conſiſts will be more in number, and inſtead of the ſnapping noiſe there will be a kind of hiſſing. The flame iſſuing from a body is called a pencil, on account of its oblong ſhape; and when the rays come to a point, they project more equally from the centre; and it is then called a ſtar.

As pointed bodies tranſmit the electric fluid with ſo much eaſe, it affords an opportunity of proving the identity of lightning and the electric fluid; for if a long rod or pole, having a ſharp-pointed wire at the end of it, be ſupported by electric

electric ſubſtances, the point projecting towards the clouds, will draw the electric matter from them, and become ſenſibly charged with electricity, as if it had been connected with the prime conductor of an electrical machine. It will attract all light bodies, and ſparks of electric matter may be drawn from it; and in ſhort, it will exhibit every appearance of common electricity; and on the other hand, by common electricity may be produced, in miniature, all the known effects of lightning.

This diſcovery was effected by Dr. Franklin, by raiſing a kite, called the *electrical kite*; and formed of a large thin ſilk handkerchief, extended and faſtened at the four corners to two ſlender ſtrips of cedar, and accommodated with a tail, loop, and ſtring, ſo as to riſe in the air like a common paper kite. To the top of the upright ſtick of the croſs was fixed a ſharp-pointed wire, extending a foot upwards, above the wood; and to that end of the twine which is next to the hand, was tied a ſilk riband. At the junction of the twine and ſilk was ſuſpended a key, from which, when the kite is raiſed during a thunder-ſtorm, a phial may be charged and electric fire collected, as by means of an electrical machine. From which it appears, that points have a remarkable property, both of throwing off, and receiving the electric fluid; from whence has riſen that uſeful invention of applying metallic conductors to houſes or buildings, in order to preſerve them from the dreadful effects of lightning, as will be hereafter ſhown.

SECT

## SECT. II.

### ELECTRICAL EXPERIMENTS.

1. *The Electrical Star.*—Cut a piece of tin in the form of a ſtar, and let it be ſupported on its centre by a wire projecting from the prime conductor; then, as ſoon as the machine is ſet in motion, and the ſtar electrified, a flame will appear at the extremity of every point of the ſtar, which will have a beautiful appearance, if the experiment be made in the dark. And if the ſtar be made to turn ſwiftly on its centre, an entire circle of fire will be ſeen. This experiment may be rendered more diverting, if the operator now and then touch the prime conductor with his finger, or a piece of metal; for by theſe means he will make it diſappear, and appear again at pleaſure; for in every experiment, if the prime conductor be touched, the effect of the experiment will be ſtopped. And if, inſtead of the ſtar, two ſharp-pointed wires be uſed, with the four ends at right angles, and in the ſame plane, but pointed different ways, and turning upon a centre, when it is electrified, a flame will be ſeen at each of the points; and what is more ſurpriſing, is, that the wires will begin to turn round of themſelves, and in the direction oppoſite to that to which the points are turned; and if the electrification be continued, the motion will become more rapid. It is by this experiment that what is called the *electric horſe-race* is performed; which is done by cutting the figures of horſes in paper, and faſtening them ſo that the points of the wires may be in their tails; by which they will ſeem to purſue one another, though without a poſſibility of any one overtaking another.

2. *The*

2. *The dancing Balls.*—Prefent the point of a wire, which is fixed on the prime conductor, to the infide furface of a glafs tumbler, grafping it on the outfide with the hands. The glafs then will foon become charged with electric matter; its infide furface acquiring the electricity from the point of the wire, and its outfide furface lofing its natural quantity of electric fluid through the hands, and which, in this cafe, ferves as a coating to the glafs. Then put a few pith balls upon the table, and cover them with this glafs tumbler, and they will immediately jump up by the fides of the glafs (*fig.* 9), and continue in motion for fome time, being attracted, and repelled, by the electric fluid upon the furface of the glafs, which they gradually conduct to the table, the outfide of the glafs acquiring the electric fluid from the furrounding air.

3. *Electrics become Conductors when made hot.*—Tie the filk ftring G (*fig.* 10) to a crooked glafs tube A E F B, by which it may be held; fill the middle part of this tube E F with refin, fealing-wax, or any other electric fubftance; then fix the two wires A E, B F, in the fealing-wax, &c. Hold the tube over a clear fire to melt the refin or fealing-wax within it; at the fame time connecting one of the wires A or B with the outfide of a charged jar, and touching the other wire with the knob of the jar. Then endeavour to make the difcharge through the refin, wax, &c. and it will be obferved, that while the refin is cold, no fhock can be tranfmitted through it, but as it melts it becomes a conductor; and when perfectly melted, the fhocks pafs through it freely: by which it may be feen, that glafs and other electrics become conductors, when they are made very hot.

4. *The Thunder Houfe.*—A (*fig.* 11) reprefents the fide of a houfe, being a board about three quarters of an inch thick. This is fixed perpendicularly upon the bottom board B, upon which is alfo fixed, in a hole in the fame board, the glafs pillar C D, about eight inches diftant from the board A. In the board A is a fmall fquare hole I L M K, about a

quarter of an inch deep, and an inch wide, which is filled by another small board nearly of the same dimensions; and made to fit easy in the hole, so that it may drop out by any sudden shock. This small piece of wood represents a shutter, or door, in the side of the house A; L K is a wire fastened diagonally to this piece of wood. I H is another wire of the same thickness, having a brass ball H screwed on its upper point: M N is another wire, turned into a ring at O. These three wires are all fixed to the board A. From the upper extremity of the glass pillar C D proceeds a crooked wire, having a spring socket F, through which is a double-knobbed wire; the lower knob G falls just above the knob H of the conductor. The glass pillar D C must be fixed in the board loose, so that it may be easily moved round; by which the brass ball G may be brought nearer or further from the ball H, without touching the part E F G with the hand. Now, when the square piece of wood M L I K is fixed in the hole, so that the wire L K may stand in the dotted line I M, then the metallic communication from H to O is complete; and the instrument exhibits a house furnished with a proper metallic conductor: but if the square piece of wood be so fixed, that the wire L K stands as represented in the figure, then the metallic conductor H O, which goes from the top of the house to the bottom, is interrupted at I M; in which case the house is not properly secured. Then let the ball G be about half an inch in perpendicular distance from the ball H; and by turning the glass pillar D C, the former ball will be removed from the latter; then by a wire or chain connect the wire E F with the wire Q of the jar P, and let another wire or chain, fastened to the hook O, touch the outside coating of the jar P. Let the wire Q be connected with the prime conductor of the machine, and charge the jar; then by turning the glass pillar D C, bring the ball G gradually near the ball H, and when they approach sufficiently near one another, the jar will explode, and the piece of wood M L I K will be pushed out of the hole to a considerable distance. In this

experiment,

experiment, the ball G reprefents a thunder-cloud, which being arrived fufficiently near the top of the houfe A, the electricity ftrikes it; and as the houfe is not fecured with a proper conductor, the explofion will break part of it, by knocking out the piece of wood I M.

Again, let the piece of wood I M be fo fituated that the wire L K may ftand in the direction I M; in this cafe, the conductor H O is not difcontinued; and repeating the experiment as before, it will be feen, that the explofion will have no effect upon the houfe, as the piece of wood L M will remain in the hole unmoved; which fhows the ufefulnefs of a metallic conductor. The inftrument ufed in this experiment is called the thunder-houfe, as it fhows the effect thunder has upon a houfe, both fecured and unfecured.

5. *The electrical Battery*—is the moft formidable and entertaining part of electricity; and is formed of a number of glafs-coated jars connected together, fo that their whole force may be united. And if a battery of no great power is required, as containing about eight or nine fquare feet of coated glafs, common pint or half-pint phials will anfwer the purpofe very well; but when a large battery is required, it is neceffary to have cylindrical glafs jars, of about fifteen inches high, and four or five inches in diameter.

The beft method of coating thefe jars, is to coat them with tin foil on both fides, which may be fixed upon the glafs with pafte, made of wheat flour; but in coating the infide of phials or jars, whofe mouths are not large enough to admit the tin foil, brafs filings are ufed, mixed with gum-water or bees-wax, &c. And the coatings fhould not come within two inches of the mouth of the jar, otherwife the jar may difcharge of itfelf. Some kind of glafs is not capable of holding any charge: the jars or phials fhould therefore be examined, before any experiment be performed.

A very good battery may be formed of twelve jars, coated on both fides with tin foil, containing in the whole about twelve fquare feet of coated glafs. In the middle of each

jar is a cork that fuftains a wire, which at the top is faftened to the wire E, which is knobbed at each end (*fig.* 12), and which connects the infide coatings of three jars; and by four fuch wires the infide coatings of all the jars are connected together. Each of the wires F has a ring at one end, through which one of the wires E paffes; and the other end has a brafs knob, refting on the next wire E. If the whole force of the battery be not required, one, two, or three rows of jars may be ufed at pleafure. The wooden box that contains thefe jars is lined at the bottom with tin foil. It has a hole on one fide, through which an iron hook paffes, communicating with the metallic lining, and confequently with the outfide coating of the jars; to this hook is faftened a wire, the other end of which is connected with the difcharging rod.

The difcharging rod confifts of two curved wires, B B (*fig.* 4), which move by a joint C, fixed to the brafs cap of the glafs handle A. The wires are pointed at the ends, on which points are fcrewed the two knobs D D, fo that it may be ufed either with the points or knobs. When a large battery is required, it is better to ufe two, three, or more fmall batteries, and their force may be united by a wire or chain: but the beft method of uniting their force, is to have a wire from every jar, connected at the top with a ball, in the form of a wire bird-cage.

The force of electricity, thus accumulated by feveral jars or batteries, is aftonifhing. Metals, which refift the greateft effect of chemical fire, are inftantly made red hot, and melted. But in performing experiments of this kind, the operator fhould be careful that no perfon touches, or even comes too near, any part of the apparatus; otherwife it may produce ferious confequences. And it is to be obferved, in charging a battery, a fmall conductor is more proper than a large one, as the diffipation of the electricity is not fo great.

6. *The animated Spider.*—The experiment called animating the fpider by electricity, is performed by fufpending a piece

of

of cork B (*fig.* 14) by a ſilk thread; in the cork a few ſhort threads are drawn, to repreſent its legs. It is to be hung in the midway, between the knob E of the wire D E, which is connected with the jar A D, and the knob A: then the jar being charged, by connecting its knob A with the prime conductor, the ſpider will be attracted by the knob A, and then repelled by it to the knob E, where it diſcharges its electricity; and is then again attracted by the knob A, and again repelled to the knob E; and will continue this motion till it has completely diſcharged the jar.

7. *To repreſent any luminous Figures.*—The ſpiral tube is compoſed of two glaſs tubes C D (*fig.* 18), one within the other, the ends being cloſed with two braſs caps A and B. On the outſide of the innermoſt tube is ſtuck a ſpiral row of ſmall round pieces of tin foil, about a twelfth of an inch diſtant from each other; then holding it by one end, and preſenting the other end to the prime conductor, ſmall ſparks will appear between all the pieces of tin foil, and in the dark it will have the appearance of a ſpiral line of fire. If, inſtead of the ſpiral tube, the tin foil be ſtuck upon a flat plate of glaſs A B C D (*fig.* 19), it may be ſo formed as to repreſent any other figures, letters, flowers, &c.

8. *To prove that Electricity prefers a ſhort Paſſage through the Air to a long one through good Conductors.*—A B D (*fig.* 16) is a wire about ten feet long, at the ends of which is fixed a piece of glaſs G, to keep the ends A B at a proper diſtance, and to let them ſlide within half an inch of each other, if required; then connect the chains belonging to the ſliding wires with the hook of the battery, and the diſcharging rod, and ſend the charge of the battery through them. On making the exploſion, a ſpark will be ſeen between A and B; which proves that the electric fluid chooſes a ſhort paſſage through the air, rather than a long one through good conductors; for very little of the electric fluid will paſs through the bent wire A D B.

9. *The*

9. *The electric Spark swells Clay.*—Roll a piece of tobacco-pipe clay in the form of a small cylinder, and in the two ends insert two wires A and B (*fig.* 22), so that their ends within the clay may be within a fifth part of an inch of each other. Then if a shock be sent through this clay, by connecting one of the wires with the outside of a charged jar, and the other wire with the inside, it will be inflated by the spark that passes between the two wires, as represented in figure 23. If the shock be too strong, and the clay not very moist, it will be broken by the explosion, and its fragments scattered in every direction; as may be proved by using a piece of the tube of a tobacco-pipe instead of the clay.

10. *The electric Spark visible in Water.*—Immerse two knobbed wires, A and B (*fig.* 17), in a glass of water, so that the knobs of the wires may be within a little distance of each other. Then, if one of these wires be connected with the outside coating of a jar, and the other wire be touched with the knob of it, the explosion in passing through the water, from the knob of one wire to that of the other, will break the glass with a surprising violence. Great caution is necessary in performing this experiment, as it is sometimes attended with danger. If, instead of a drinking-glass, a glass tube be used, stopped with a cork at each end, through which the wires are inserted, and the charge be very weak, the electric spark will appear in the water passing between the wires.

11. *The electrical Thermometer.*—A B (*fig.* 24) is the electrical thermometer, and consists of a glass tube about ten inches long, and nearly two inches in diameter, and closed air-tight at both ends by two brass caps. H A is a small tube, open at both ends, passing through a hole in the upper cap, and immersed at the bottom in some water at B, in the bottom of the large tube. F G and E I are two wires inserted through the middle of each of the brass caps, and having a brass knob at the head of each, within the brass tube.

tube. This inſtrument is faſtened to the pillar C D, by a braſs ring C. When the air within the tube A B is rarefied, it will preſs upon the water at the bottom of the tube, and ſo cauſe it to riſe in the ſmall tube; and the riſe and fall of the water ſhow the rarefaction of the air in the glaſs tube A B, which has no communication with the external air.

If the knobs G I of the two wires be brought into contact with each other, and the ring E or F be connected with one ſide of the charged jar, and the other ring with the other ſide, and a ſhock be made to paſs through the wires, the water in the ſmall tube will not be at all moved; which ſhows that the paſſage of the electric fluid, through conductors ſufficiently large, occaſions no rarefaction of the air. But if the knobs G I be placed a little diſtant from each other, and the ſhock ſent through the wires as before, the ſpark between the two knobs will conſiderably rarefy the air, and the water will be ſuddenly forced up the ſmall tube quite to the top.

12. *To ſhow the Courſe of the electric Fluid in the Diſcharge of a Jar, and to make it viſible by the Star and Pencil.*—For this purpoſe, the jar muſt be charged; then taking the diſcharging rod (*fig.* 4) without its knobs, preſent one point within an inch of the knob A (*fig.* 15), and the other point at an equal diſtance from the outſide coating of the jar. By theſe means the jar will be diſcharged ſilently; and if its inſide be electrified poſitively, the point C of the diſcharging rod will be illuminated with a ſtar, becauſe it receives the electric fluid; and the point B with a pencil, becauſe it gives out the electric fluid to the outſide of the jar; and if the jar be electrified negatively on the inſide, the pencil will appear upon the point C, and the ſtar upon the point B.

13. *The univerſal Diſcharger.*—This is an inſtrument of very extenſive uſe, and is compoſed of the following parts: A is a flat board, about fifteen inches long, four broad, and one

one thick (*fig.* 8); B B are two glaſs pillars, cemented in two holes in the board. At the top of each is a braſs cap, having a turning-joint, and a ſpring tube, through which ſlides the wire C D. Thus, each of theſe wires has two other motions, viz. an horizontal and vertical one; each wire is alſo furniſhed with an open ring at one end C, and a braſs ball at the other end D, which ball may be taken off at pleaſure. E is a ſtrong circular piece of wood five inches in diameter, on the ſurface of which is a ſlip of ivory; and furniſhed with a ſtrong cylindrical foot, which fits the ſocket, and which, by means of the ſcrew G, may be made faſt, and alſo raiſed higher, or brought down lower.

The *Leyden phial* is an inſtrument to prove the hypotheſis of a ſingle electric fluid, and is formed by coating a ſmall phial about three inches up the outſide with tin foil (*fig.* 20). To the top of the neck a braſs cap is cemented, having a hole with a valve; from this cap proceeds a wire, being blunted at the point, and terminating a few inches within the phial. When the phial is exhauſted of air, a glaſs ball is ſcrewed on the braſs cap, to prevent any air from getting into the phial. This phial ſhows the direction of the electric fluid, both in charging and diſcharging; for if it be held by its bottom with the braſs knob preſented to the prime conductor, which is poſitively charged, the electric fluid will cauſe the pencil of rays to proceed from the wire within the phial, as in figure 21; but if it be diſcharged, a ſtar will appear inſtead of the pencil, as in figure 20. But if the wire be held by the braſs cap, and its bottom be touched by the prime conductor, the point of the wire on its ſide will appear illuminated with a ſtar when charging, and with a pencil when diſcharging. If it be preſented to the prime conductor, electrified negatively, all theſe appearances, both in charging and diſcharging, will be reverſed.

Inflammable air, that will take fire by the electric ſpark, is thus made: A D (*fig.* 25) repreſent two bottles; in the bottle

bottle D are put two or three ounces of filings of iron, and ſome oil of vitriol, mixed with four times its quantity of water. The bottle A is filled with water, and the bent glaſs pipe C is fixed with one end, air-tight, into the neck of the bottle D, and the other end a little way up the neck of the bottle A; in a ſhort time the mixture will boil, and emit a fluid, which will paſs through the tube C, into the bottle A, and at length fill it, expelling the water into the baſin B. The bottle A is then to be quickly corked up for uſe.

The *electrical piſtol* is repreſented *fig.* 26, where c a is of thin braſs; to the mouth a b is fitted a cork, and a perforated piece of braſs d ſcrews on the bottom of the piſtol at c, having a glaſs tube, with a wire cemented into it, bent over the glaſs tube, ſo as to reach within one eighth of an inch of the braſs. When the piſtol is to be charged, uncork the inflammable air bottle before mentioned, likewiſe the piſtol, and place the mouth of the piſtol upon the top of the bottle; and the common air which is within the piſtol will deſcend while the other aſcends. Having held the piſtol in this ſituation a few ſeconds, in order to fill it with inflammable air, cork both it and the bottle expeditiouſly, and it is then charged. When it is to be diſcharged, fill a ſmall jar, or a hollow handle, and apply it to the knob of the wire e; it will then explode, and drive out the cork to a conſiderable diſtance, with a report as loud as that of a piſtol filled with gunpowder.

Figure 28 repreſents an inſtrument to cure the tooth-ach, in which A is a flat piece of box wood, about an inch broad, and a quarter of an inch thick. Near its oppoſite edges are made two longitudinal holes, through which are put two braſs wires, a b c and d e f, and fixed in with ſealing-wax, and then bent at c and f, as in the figure, which two points ſerve to receive the tooth and gum between them. When the inſtrument is uſed, hook two chains, g and h, on the lower end of the wires, holding the tooth and gum between

the other end of the wires c and i; put the end of the chain g round the bottom of the electric jar, and let a person hold the chain h hanging down from his hand, both chains being clear of the table, and not touching each other. Then having charged the jar, let the person who holds the chain h ſtrike the end of it againſt the prime conductor: this will diſcharge the jar, and give the patient a ſhock only in his tooth and gum, which ſeldom fails to cure the tooth-ach, if the cauſe be a cold.

The *electrometer* is an inſtrument to ſhow the kind and quantity of electricity, of which there are ſeveral ſorts; the moſt ſimple is that which conſiſts of a linen thread, having a ſmall cork or pith-ball at each end (*fig.* 13). This electrometer is ſuſpended by the middle of the thread on any conductor proper for the purpoſe. And if the conductor be charged poſitively, by applying a ſtick of ſealing-wax, excited, the balls collapſe together; and by applying an excited ſmooth glaſs tube, they will recede further aſunder; and if the conductor be charged negatively, the reverſe will take place.

But the moſt perfect of theſe inſtruments is that called the quadrant electrometer, which ſhows the exact degree to which any body is electrified, and is as follows:—A is a fine rod, that turns on B, the centre of a ſemicircle (*fig.* 17), ſo as always to keep near its graduated limb, which is divided into 180 degrees. At the end of the rod is a cork ball C. The pillar D may be fixed either to the prime conductor, or to the braſs knob of a jar or battery, or be ſet on a ſtand by itſelf. The inſtrument ſhould be made of box wood, and the ſemicircle of ivory.

When this inſtrument begins to be electrified, the rod A is repelled by the pillar D, and conſequently begins to move over the edge of the ſemicircle, and ſhows very exactly the degree to which the conductor is electrified, or how high any jar or battery is charged. This inſtrument ſhould always

always be made very dry before the fire, when it is ufed, taking care that it be not heated.

If the jar or battery be charged with pofitive electricity, and it be required to know the exact time that it becomes difcharged, while you are attempting to charge it negatively, obferve the moment the index comes to the perpendicular ftation, and at that moment there will not be the leaft fpark left in the jar. If the operation be continued, the index will again advance along the femicircle; and thus fhow the exact quantity of negative electricity which the jar has acquired.

## SECT. III.

### OF MEDICAL ELECTRICITY.

Electricity was no fooner brought to any degree of perfection, than it was applied to medical purpofes. For by late obfervations, it has been found to poffefs the invariable properties of increafing the fenfible perfpiration, quickening the circulation of the blood, and promoting all the glandular fecretions. And among all the variety of cafes in which it has been ufed, there are none in which it has been found prejudicial, except thofe of pregnancy, and venereal difeafe; and there are a number of cafes, in which it has been applied with confiderable fuccefs. In moft diforders where it has been ufed with perfeverance, it has given at leaft a temporary and partial relief; and in fome cafes it has effected a total cure. Of which, numerous inftances may be feen in

the Philofophical Tranfactions, and the writings of Meffrs. Lovet, Fergufon, Weftley, Cavallo, &c. &c.

To know what cafes are proper to be electrified, experience fhows in general, that all kinds of obftructions, whether of motion, of circulation, or fecretion, are very often removed, and in general alleviated, by electricity. Likewife, nervous diforders have very often been cured; and rheumatic diforders, even of a long ftanding, are always relieved, and very often quite cured, by only drawing the electric fluid with a point from the affected part, or by drawing fparks from the conductor. It has alfo been found very beneficial in difeafes of a long ftanding; and has not unfrequently been found a powerful remedy in mufcular contractions.

There are three inftruments generally ufed for adminiftering medical electricity, befides the electrical machine, viz.—An electric jar, with Mr. Lane's electrometer; an infulated chair or ftool, upon which a common chair may be occafionally fet; and the directors.

The jar ufed on this occafion fhould be coated with tin foil, and fhould be about four inches in diameter, and fix in height, which would contain above 72 fquare inches of coated furface. Through the covering of the jar paffes a brafs wire B (*fig.* 1, *plate* 21), touching the infide coating of the jar, and having a brafs ball F, to which the electrometer F D C is faftened, and terminating at the top in a brafs ball B, which is to touch the prime conductor, and which is fuppofed to ftand before the electrical machine. The electrometer C E D F confifts of a piece of glafs F D, cemented to the two brafs caps D and F; from the former of which proceeds a ftrong perpendicular brafs wire, having at the top an horizontal fpring focket, through which flides the wire C E, having the brafs ball C at one end, and an open ring E at the other end; and fo fixed, that the ball C is exactly the fame height as the ball B, and may be fet at any required diftance from the ball B. This diftance feldom exceeds half

an

an inch, therefore the electrometer may be made very ſmall. Sometimes there is a ſcale on the wire C E, which ſerves to ſet the balls B C to any given diſtance from each other, with more certainty. When this inſtrument is uſed, the jar is ſo placed, that the ball B may touch the conductor. Then, ſuppoſe the ball C to be ſet at one tenth of an inch diſtance from the ball B, and the electrometer E be connected to the outſide coating of the jar at I by a chain E I. In this caſe, if the electrical machine be put in motion, the jar will be charged; and when the charge is ſo high, that the electric fluid accumulated within the jar, can paſs from the ball B to C, which is here ſuppoſed to be one tenth of an inch; the diſcharge will take place, the ſpark will appear between the balls, and the ſhock will paſs through the chain E I from E to I; for the part F D being of glaſs, and generally covered with ſealing-wax, is impervious to the electric fluid; therefore the electric fluid has no way to paſs from the inſide to the outſide of the jar, but from the ball B to the ball C, and along the wire C E, and from thence along the chain E I.

When the electrical ſhock is to be adminiſtered to any part of the human body (as, for example, to the arm), inſtead of the chain, which muſt now be taken away, two ſmall pliable wires E L, I L, are to be faſtened, one to the ring E, and the other to a hook I, of the ſtand H I, which communicates with the outſide coating of the jar (if the jar have not the ſtand H I, the extremity of the wire I may be put in contact with the outſide coating of the jar in any other convenient manner); the other end of the ſaid wires is faſtened to the braſs wires L L of the directors K L. Each director conſiſts of a knobbed braſs wire L, connected to a glaſs handle K, by means of a braſs cap. Then the operator, holding the directors by the extremities of the glaſs handles, brings their balls into contact with the extremities of that part of the patient's body through which the ſhock is to be ſent. Then it is evident, from a view of the figure, that

that the discharge of the jar must be made through that part of the patient's arm which lies between the two knobs of the directors, as, in the former case, the discharge was made through the chain E I. Thus, the operator has nothing more to do, but to hold the knobs of the directors to the extremities of that part of the body through which the shock is to pass, while an assistant keeps the machine in motion. Care must be taken that the two wires E L and I L do not touch each other; otherwise the shock will not pass through the patient's body. By these means, any number of shocks, precisely of the same strength, may be given without altering any part of the apparatus. And when it is required to increase or diminish the force of the shock, it is only necessary to increase or diminish the distance between the balls B and C, which is done by moving the wire C E through the socket.

It is of little consequence whether the patient stands upon the ground, upon the insulated stool, or in any other situation. Neither is it necessary to remove the clothes from the part that is to be electrified, for the shocks will readily go through them, except there be too many coverings.

In the application of electricity, the chief difficulty consists in distinguishing the proper strength of the electric force that is requisite for a given disorder. For this purpose, it is impossible to give any general rules, the circumstances being of so complex a nature, that nothing but long experience, and strict attention to every particular phenomenon, can direct the operator. It need hardly be said, that regard must be had to the sex and age of the patient. The surest rule that can be given, is to begin with more gentle treatment, at least such that, considering the circumstances, may be thought rather weak than strong. If, after a few days trial, this gentle treatment be found ineffectual, then the operator may gradually increase the force of the electricity, until he finds the proper degree. But when any limb of the body is deprived of motion, it must be observed, that

the

the cauſe is ſometimes a contraction of the muſcles; in which caſe, electricity has often proved an effectual remedy: but the loſs of motion is ſometimes occaſioned by a relaxation, as well as contraction: as, when the hand is bent inwardly, and the patient has no power to ſtraighten it. In theſe caſes it is often difficult to diſcover the real cauſe: but the ſureſt method is, to electrify both thoſe muſcles which are contracted, and alſo their antagoniſts; for no injury can attend electrifying a ſound muſcle. In rheumatic diſorders, the electric fluid ſhould be drawn from the parts afflicted; or the ſparks may be drawn from the conductor.

The operation ſhould be continued for four or five minutes, and may be repeated once or twice a day. When the ſhocks are ſtrong, their greateſt number at one operation ſeldom exceeds a dozen or fourteen, except they be given to different parts of the body.

The *electrophorus (fig. 2)* conſiſts of two plates, from ſix to 18 inches diameter, in general, and ſometimes much larger. The upper plate is generally made of braſs; but a tin plate will ſerve the purpoſe, having a wire turned upon its edge, in the common manner; on the centre of this plate is fixed the ſocket O, in which the glaſs handle I is fixed, which is nine or ten inches long. When the electrophorus is to be of a large diameter, a thin board, covered with tin foil, and ſuſpended by ſilken ſtrings, will anſwer exceedingly well.

The lower plate (only the edge of which could be ſhown in the figure) may be made either of glaſs, ſealing-wax, or the following compoſition, viz.—reſin four parts, pitch three parts, ſhell-lac three parts, Venice turpentine two parts, melted together over a gentle fire; then poured and ſpread upon a thin linen cloth, about a quarter of an inch thick; the linen cloth is then ſtretched upon a hoop, and made as tight as poſſible.

To charge a jar with this machine, rub the coated ſide of the under plate with a piece of fine new flannel, or a hare's or

cat's

cat's ſkin; and when it is excited as much as poſſible, ſet it upon a table, and place the upper plate B upon it, and put your finger upon the upper plate; then, taking your finger off, take hold of the glaſs handle I, and apply it to the knob of a coated jar. When this operation is repeated 30 or 40 times, the jar will become charged.

It was with a machine of this kind that Mr. Cavallo charged a coated phial ſeveral times by only once exciting, and ſo ſtrong, as to pierce a hole through a card at every diſcharge. If a plate of glaſs be coated with ſealing-wax, and excited, and then laid with the wax ſide downwards; then, on making the above experiment, by putting the plate upon it, and taking the ſpark with the finger, and applying it to the glaſs handle, &c. it will have the contrary electricity to what it had before.

When it is required to diſcover whether a ſmall degree of electricity be poſitive or negative, or to know how the charge advances in uſing large batteries, and of what ſtrength it is, the moſt uſeful electrometer is Mr. Canton's balls, which are made of pith of elder, turned perfectly globular, and ſuſpended by fine threads from the conductor (*fig.* 3).

To know whether the inſide of a jar or battery be charged poſitively or negatively, the balls are to be preſented to the jar or battery which ſtands upon the table, and they will be immediately attracted by the wire, and diverge from each other. This is always the caſe in both poſitive and negative electricity. And the greater the diſtance to which the balls ſeparate, and the more they repel one another, the higher is the charge. To determine whether the electricity is poſitive or negative, rub a ſmall piece of glaſs againſt the hand or coat, which will excite it poſitively, and then preſent it to the balls in their diverging ſtate, and if it makes the balls converge, it ſhows they are electrified poſitively; but if it increaſes their divergency, it ſhows their electricity to be negative. And it muſt be obſerved, that the electricity of the balls

balls is always contrary to that with which they are charged, for they do not receive any electricity from the wires of the jar or battery; for all bodies placed within the influence of electrified bodies are affected with the contrary electricity.

But to difcover the kind of electricity, when the charge is very fmall, inftead of the pith-balls, a piece of downy feather fhould be ufed, fufpended by a fingle filken thread, as it comes from the worm, or at leaft by a very few of thofe threads, to render it as light as poffible. If any electrified body be prefented to this, the feather will be repelled by it, if it be of the fame kind with its own, and attracted by it, if the electricity be contrary to it. For this light body, when once electrified, either pofitively or negatively, will retain its virtue a long time, with very little lofs.

Notwithftanding electricity might be rendered fo generally ufeful in the application of it to medical purpofes, yet it is frequently found to be ineffectual, where it might be expected to prove the moft falutary, which is very often the effect of ignorance in the operator; for many perfons, particularly in the metropolis, undertake to adminifter electricity, who are entirely deftitute of any medical knowledge, and, confequently, of the caufe and fituation of diforders; hence, the failure of it is owing, generally, to an error in the application.

## *The technical Terms ufed by Writers on Electricity.*

*Battery*, electrical, a number of jars combined together, to be all charged and difcharged at the fame time (*fig.* 12, *plate* 20).

*Charging*, throwing an additional quantity of electric fluid upon one fide of a plate of glafs, or a jar, while the other fide is exhaufted in the fame proportion. All electric fubftances may be charged as well as glafs.

*Circuit*, thofe conducting fubftances ufed to connect the

two coatings of a jar or battery together, and through which the electric fluid muft pafs.

*Conductor*, a piece of metal furnifhed with points, to receive the electric matter from the globe. It muft always be infulated, or unconnected with the earth, by means of electric fubftances, as glafs, baked wood, &c. Whenever it is indefinitely mentioned, the prime conductor is underftood.

*Difcharging*, is reftoring the equilibrium of the electric fluid, after it has been difturbed by charging. It is effected by forming a communication between the overloaded and exhaufted fides of a jar, battery, &c. by fome conducting fubftance.

*Difcharging rod*, a brafs rod, or any other inftrument (*fig.* 4 and 8, *plate* 20), ufed to effect a difcharge.

*Electric matter* or *fluid*, that fubtle fluid inherent in all bodies, and fuppofed to be the caufe of all thofe appearances which we term electric.

*Electrics*, thofe bodies in which the electric powers of attraction, repulfion, &c. may be excited by friction. They are called *non-conductors*, becaufe the electric fluid cannot pafs through them. And *non-electrics* are called *conductors*, becaufe the electric matter may pafs through them; but no electric powers can be excited in them.

*Electrometers*, inftruments to meafure the quantity of electric matter.

*Excitation*, calling forth the electric powers from electric fubftances by friction.

*Infulating*, placing bodies where they are not in contact with any conducting fubftance; as, by fufpending them in the air by filken ftrings, placing them on glafs ftands, &c.

*Negative electricity*, a lefs quantity of the electric fluid than is natural to any body.

*Pofitive electricity*, a greater quantity of the electric fluid than its natural fhare.

*Rubber*, or *Cufhion*, a piece of leather, or any other fubftance, againft which the glafs globe or other electric body is rubbed, in order to excite them.

*Pencil*,

*Pencil*, the appearance of the electric fluid iſſuing from the point of a body electrified poſitively.

*Star*, the appearance of the electric fluid iſſuing from the point of a body electrified negatively.

*Shock*, *electric*, the convulſion given to the animal muſcles by the diſcharge of a jar or battery.

*Wire of a jar*, &c. the wire or metal rod which touches the inſide coating of a jar, &c.

# CHAP. XVII.

# OF PNEUMATICS.

## SECT. I.

### OF THE PROPERTIES OF AIR.

PNEUMATICS is that part of natural philoſophy which treats of the weight, preſſure, and elaſticity of the air, with the effects ariſing from them.

The air is that thin, tranſparent, fluid body which ſurrounds the whole earth to a conſiderable height; and which, together with the clouds and vapours that float in it, is called the atmoſphere. That the air is a fluid, is evident from the following properties, which it poſſeſſes in common with all other fluids: viz.—1. It yields to the leaſt force impreſſed on it.—2. Its parts are eaſily moved among one another.—3. It preſſes according to its perpendicular height.—4. And its preſſure is every way equal.



But

But the air differs from all other fluids, in the four following particulars :—1. It can be compreffed into a much lefs fpace than what it naturally poffeffes, which no other fluid can : 2. it cannot be fixed or congealed as other fluids can : 3. it is of a different denfity in every part upward from the earth's furface, its weight decreafing the higher it rifes, confequently it muft alfo decreafe in denfity : 4. it is of an elaftic nature, and the force of the fpring is equal to its weight.

It is evident, that air is a body, for it excludes all other bodies out of the fpace it poffeffes : thus, if a glafs veffel or jar be inverted, and plunged into a veffel of water with a fteady hand, ftill preffing it downwards, there will very little water get into the jar, becaufe the air, of which it is full, keeps the water out. It is upon this principle that diving-bells are conftructed.

Air being a body, muft neceffarily have gravity or weight; and its weight is determined by the following experiment :— let a bottle that holds a wine quart be emptied of its air, by means of the air pump ; then weighing the bottle, it will be found to be about 16 grains lighter than when the air is let into it again : which fhows that a quart of air weighs 16 grains. And to find the proportion of the weight of air to that of water, divide the weight of a certain quantity of water by the weight of the fame quantity of air ; thus, a quart of water weighs 14621 grains, which, divided by 16, the weight of a quart of air, quotes 914, in round numbers : which fhows that water is 914 times as heavy as air, near the furface of the earth. This is in general the greateft weight of the air ; for commonly it is reckoned only 840 times lefs denfe than water, at a mean rate ; the denfity of the air being very various, according to the fituation of the climate, feafon of the year, and many other circumftances.

The air has a different denfity as we rife from the furface of the earth, and grows continually rarer and lighter the farther it is from the earth, which is owing to its being of

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an elaſtic nature, and capable of being compreſſed into a leſs ſpace, for the lowermoſt parts of the atmoſphere, being preſſed with the weight of all that is above them, muſt conſequently be rendered more denſe and compact at the earth's ſurface than at any height above it. And that air, towards the upper part of the atmoſphere, being leſs preſſed, is conſequently leſs denſe and compact than that near the earth; for the denſity of the air is always as the force that compreſſes it. The following table given by Dr. Cotes, ſhows that the rarity of the air at a diſtance from the earth's ſurface increaſes in a geometrical proportion, while its height from the earth increaſes in an arithmetical proportion:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| At the altitude of | 7 | Miles above the ſurface of the earth the air | is | | — | 4 | times thinner and lighter than at the earth's ſurface. |
| | 14 | — | — | — | — | 16 | |
| | 21 | — | — | — | — | 64 | |
| | 28 | — | — | — | — | 256 | |
| | 35 | — | — | — | — | 1024 | |
| | 42 | — | — | — | — | 4096 | |
| | 49 | — | — | — | — | 16284 | |
| | 56 | — | — | — | — | 65536 | |
| | 63 | — | — | — | | 262144 | |
| | 70 | — | — | — | | 1048576 | |
| | 77 | — | — | — | | 4194304 | |
| | 84 | — | — | — | | 16777216 | |
| | 91 | — | — | — | | 67108864 | |
| | 98 | — | — | — | | 268435456 | |
| | 105 | — | — | — | | 1073741824 | |
| | 112 | — | — | — | | 4294967296 | |
| | 119 | — | — | — | | 17179869184 | |
| | 126 | — | — | — | | 68719476736 | |
| | 133 | — | — | — | | 274877906944 | |
| | 140 | — | — | — | | 1099511627766 | |

From this table it appears, that at the height of ſeven miles from the earth, the air is four times rarer and lighter than at the earth's ſurface; at the height of 14 miles it is 16 times rarer and lighter; at 21 miles, 64 times rarer, &c. From hence it may be proved, that a cubical inch of ſuch air as we breathe, near the earth's ſurface, would be ſo much

more

more rarefied at the height of 500 miles, that it would fill a ſphere equal in diameter to the orbit of Saturn.

The weight or preſſure of the air is determined by what is called the Toricellian experiment, which is as follows:— Fill with purified quickſilver, a glaſs tube about three feet long, and open at one end; and putting your finger upon the open end, turn that end downwards, and immerſe it in a ſmall veſſel of quickſilver, without letting in any air; then taking away your finger, the quickſilver will remain ſuſpended in the tube, about $29\frac{1}{2}$ inches above the ſurface of that in the veſſel; ſometimes more or leſs, as the weight of the air is varied. In this experiment, it is evident that the quickſilver is raiſed in the tube by the preſſure of the atmoſphere upon that in the baſon or veſſel; for if the baſon and tube be put under a glaſs, and the air be taken out of the glaſs, all the quickſilver in the tube will fall down into the baſon; and if the air be let in again, the quickſilver will riſe to the ſame height as before. Therefore the air's preſſure on the ſurface of the earth is equal to the weight of $29\frac{1}{2}$ inches depth of quickſilver all over the earth's ſurface, at a mean rate. But a ſquare column of quickſilver $29\frac{1}{2}$ inches high, and one inch thick, weighs juſt 15 pounds, which, therefore, is equal to the weight of the air upon every ſquare inch on the earth's ſurface; and the weight upon every ſquare foot, or 144 inches, amounts to 2160 pounds. According to this rate, a middle-ſized man, whoſe ſurface is generally about 14 ſquare feet, ſuſtains a preſſure of 30240 pounds, when the air is of a mean gravity. This weight could not be born, if it were not that it is equal on every part of the body, and counterbalanced by the ſpring of the air within us, which is diffuſed through the whole body, and re-acts with an equal force againſt the external preſſure.

As the earth's ſurface contains near 200,000,000 ſquare miles, in round numbers, and every ſquare mile 27,874,400 ſquare feet, there are 5,575,680,000,000,000 ſquare feet on the earth's ſurface, which, multiplied by 2160 pounds, the weight

weight on each ſquare foot, gives 12,043,468,800,000,000,000 pounds, for the preſſure of the whole atmoſphere.

All common air is impregnated with a certain kind of what is called *vivifying ſpirit*, which is eſſential to preſerve animal life; and in a gallon of air there is enough of it for one man during the ſpace of a minute, but not much longer. This ſpirit is alſo in that air which is in water, as appears by the fiſh dying, when they are excluded from freſh air, as in a pond that is frozen over.

This ſpirit in air is loſt by paſſing through the lungs of any animal, and is the reaſon why an animal dies ſo ſoon when deprived of freſh air. The little eggs of inſects, alſo, when ſtopped up in a glaſs, and excluded from the air, do not produce their young, though they be aſſiſted by warmth. The ſeeds alſo of plants, though mixed in good earth, will not grow if they be deprived of air.

The vivifying quality is alſo deſtroyed by the air paſſing through fire, particularly charcoal fire, or the flame of ſulphur.

Air may alſo become vitiated, by being cloſely confined in any place for a conſiderable time, or by being mixed with malignant ſteams: and laſtly, by the corruption of the vivifying ſpirit; as in the holds of ſhips, in oil-ciſterns, wine-cellars which have been ſhut ſome time, or brewers' vats. In any of them, the air may be ſo much vitiated as to be immediate death to any animal that enters them.

When the air has loſt its vivifying ſpirit, it is called *damp*, becauſe it abounds with humid and moiſt vapours, and becauſe it deadens fire, extinguiſhes flame, and deſtroys life. The effects of theſe damps are ſufficiently known to thoſe who work in mines.

When part of the vivifying ſpirit of air in any country begins to putrefy, the inhabitants of that country will be ſubject to an epidemical diſeaſe, which will rage till the putrefaction is over. And as the putrefying ſpirit occaſions the diſeaſe, ſo if the diſeaſed body contributes towards the

putrefaction

putrefaction of the air, the disease will then become pestilential and contagious.

## SECT. II.

### OF THE THEORY OF THE WINDS.

The wind is the consequence of the rarefaction of the air, and is no other than the air put into motion by heat, or any other cause; for when the air is rarefied by heat it will swell, and thereby affect the adjacent air; and thus, by the degrees of heat being various in different places, there will arise various winds.

When the air is heated to any degree, it will ascend upwards, and the adjacent air will rush in to supply its place; therefore, there will be a stream or current of air from all the adjacent parts towards the place where the heat is. This appears evident from the motion with which the air rushes towards any place where there is a great fire, as into a glass-house, or through the keyhole of a door in a room where there is a fire.

That wind, called the trade wind, which blows constantly from east to west about the equator, is a necessary consequence of this principle. For when the sun shines perpendicularly upon any part of the globe, the air in that part will be heated, and consequently rarefied, and will therefore ascend upwards; and, when the sun withdraws, the adjacent air, rushing in to fill the place of the rarefied air, will consequently cause a stream or current of air from all parts towards

wards that part which is moſt heated by the ſun. But the courſe of the ſun being from eaſt to weſt, with reſpect to the earth, the common courſe of the air which ſupplies the place of the rarefied air muſt be in the ſame direction, viz. from eaſt to weſt; but on the north ſide its courſe will be directed a little towards the north, and on the ſouth ſide as much towards the ſouth.

This would be the general courſe of the wind about the equator, if it were not affected by other cauſes, which change its direction: as, 1. By exhalations that ariſe out of the earth, at different times and different places, occaſioned by ſubterraneous fires, volcanoes, &c.; 2. by a ſudden inundation of rain, which cauſes a contraction of the air; 3. by the violent heat of ſome burning ſands, which cauſe an extraordinary rarefaction of the contiguous air; 4. by high mountains, which alter the direction of the wind; 5. by the declination of the ſun towards the north or ſouth, thereby cauſing a greater heat in the air on the ſame ſide of the equator.

Theſe are the principal cauſes which create ſuch a great variety and uncertainty in the winds in moſt countries diſtant from the equator; as, 1. The variations of the winds in the different parts of Europe; 2. the monſoons which are found in the Indian ſeas; 3. thoſe winds which always blow from weſt to eaſt, on the weſtern coaſt of America, and on the coaſt of Guinea; and the ſea breezes, which, in hot countries, blow from ſea to land in the daytime; and the land breezes, which blow towards the ſea in the night; and all thoſe other irregularities in the wind, as ſtorms, whirlwinds, hurricanes, &c.

## SECT. III.

### OF THE CAUSES OF THUNDER, LIGHTNING, &c.

The effluvia and vapours arifing from different bodies meet and unite together in the atmofphere, which is the common receptacle of all vaporous bodies, as the fteams from moift bodies, the fmoke from bodies burnt, and the effluvia emitted from fulphurous, nitrous, acid, and alkaline fubftances. And every volatile body rifes to a certain height in the atmofphere, according to its own fpecific gravity. And when the effluvia which arife from an acid and alkaline body meet each other in the air, there will be a conflict between thefe two vapours, or what is vulgarly called a fermentation between them. If this fermentation be great, it will produce a fire; and if the effluvia be of a combuftible nature, the fire will run from one part of the air to another, following the inflammable matter.

Thefe things may be demonftrated by the following experiment:—Mix fome oil of cloves and Glauber's fpirit of nitre together, which will immediatly produce a fudden fermentation, with a fine flame; and if the ingredients be neat, there will be a fudden explofion. Thefe are the effects of the union of an acid and alkaline fluid.

From this experiment, we may account for the effects of thunder and lightning, which is occafioned by the effluvia of fulphurous and nitrous bodies meeting each other in the air, where, affifted by the fun's heat, a fermentation, fire, and explofion enfue. When the inflammable matter is thin and light, it will afcend to the upper parts of the atmo-

fphere,

ſphere, before the fermentation takes place; but when it is more denſe, it will hover near the ſurface of the earth, where, when an exploſion takes place, the fire is viſible, and often dangerous; the exploſion alſo has a violent force; and the heat being great, will rarefy and drive away all the adjacent air, kill men and cattle, ſplit trees, rocks, &c.

Lightning differs from all other fires; for it has often been known to paſs through wood, leather, cloths, and other ſubſtances, without hurting them; at the ſame time melting iron, ſteel, ſilver, gold, and other hard bodies. It has melted or burnt aſunder a ſword, without hurting the ſcabbard; and melted money in a man's pocket, without hurting his clothes. So fine are the particles of this fire, that they paſs through ſoft, looſe bodies, without injuring them, and ſpend their force upon thoſe that are more denſe.

Any ſteel inſtruments, as knives, forks, &c. that have been ſtruck with lightning, have a ſtrong magnetic virtue, which they retain many years. The lightning ſtriking the mariner's compaſs has often turned it quite round, and made it ſtand the contrary way, that is, with the north pole towards the ſouth.

Thoſe exploſions which ſometimes happen in mines, and called fire-damps, are of the ſame nature with lightning, and occaſioned by ſulphurous and nitrous vapour riſing from the mine, which, mixing with the air, take fire from the lights uſed in the mine. This fire, when once kindled, continues to run from one part of the mine to another, as the combuſtible matter happens to be; and as the elaſticity of the air is increaſed by the heat, the air in the mine will ſwell conſiderably; and, for want of room, will at length explode, with a degree of force equal to the violence of the fire, the quantity of effluvia, and denſity of the vapours. This is ſometimes ſo ſtrong as to blow up the mine; at other times it is ſo weak, that when it has taken fire it may be eaſily blown out.

Air that will take fire from the flame of a candle, may be produced thus:—Having pumped the air out of the receiver of the air-pump, let the air run into it through the flame of the oil of turpentine; then remove the cover of the receiver, and holding a candle to that air it will take fire.

When combuſtible vapours are kindled in the bowels of the earth, where there is little or no vent, they produce earthquakes, and violent ſtorms or hurricanes of wind, as ſoon as they break forth in the open air.

An artificial earthquake may be produced thus:—Take 10 or 15 pounds of ſulphur, and as much of the filings of iron, and knead them with common water into the conſiſtence of a paſte: this, being buried under ground, will, in eight or ten hours times, burſt out into flames, and cauſe the earth to tremble around it to a conſiderable diſtance.

It is owing to ſubſtances of this nature that we have volcanoes.

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## SECT. IV.

### THE CONSTRUCTION AND USE OF THE AIR-PUMP, BAROMETER, AND AIR-GUN.

#### 1. *Of the Air-pump.*

THE air-pump is a machine to pump the air out of any veſſel, and conſtructed on the ſame principle as the water-pump.

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The air-pump, with all its apparatus, is ſhown *fig.* 1, *plate* 22, where L L is the plate, on which is placed a wet leather, and the large glaſs receiver M, placed upon the leather, ſo that the hole *i* in the plate may come within the glaſs. Then, by turning the handle F (*fig.* 2), the air will be pumped out of the receiver, which will be held down to the plate by the force of the external air or atmoſphere. For, as the handle F is turned backwards, or towards D, it raiſes the piſton *d e* in the barrel B K, by means of the wheel E, and rack work D and C; and as the piſton is leathered ſo tight, as to fit the barrel exactly, no air can get between the piſton and barrel, and therefore all the air above *d*, in the barrel, is lifted up towards B, and a vacuum is made in the barrel from *b* to *e*; upon which part of the air in the receiver M (*fig.* 1), by its ſpring, ruſhes through the hole *i* in the braſs plate L L, through the pipe G C G, which communicates with both barrels, by means of the hollow trunk I H K (*fig.* 2), and puſhing up the valve *b*, enters into the vacant place *b e* of the barrel B K. For wherever the reſiſtance or preſſure of the air is diminiſhed, the air will run to that place, if it can find a paſſage. Then, if the handle F be turned the contrary way, the piſton *d e* will be lowered in the barrel; and as the air which came laſt into the barrel cannot be puſhed back through the valve *b*, it will aſcend through a hole in the piſton, and make its eſcape through a valve at *d*; and by that valve be prevented from returning into the barrel, below the piſton. At the next raiſing of the piſton, a vacuum is again made in the barrel, between *b* and *e*, as before, when more of the air that is left in the receiver M (*fig.* 1) eſcapes by its ſpring into the barrel B K, through the valve *b*. What is here explained concerning the barrel B K muſt be underſtood with regard to the other barrel A I. And as the handle F is turned backwards and forwards, it raiſes and depreſſes the piſtons in each barrel alternately, raiſing one, while it depreſſes the other. And as there is a vacuum

vacuum made in each barrel, when the piſton is raiſed, the particles of air in the receiver M (*fig.* 1) puſh out one another by their elaſticity, through the hole *i*, and pipe G G, into the barrels, until the receiver is ſo much exhauſted of the air, and the elaſticity of the air is ſo much weakened, that it will no longer have ſufficient force to puſh through the valves *b d*; and then no more air can be taken out. It is impoſſible to make a perfect vacuum in the receiver, or to entirely exhauſt it of air; for the quantity of air taken out at any one ſtroke will always be as the denſity of the air in the receiver; and therefore it is impoſſible to take it all out: for if the receiver and barrels be both of equal capacity, there will always as much remain in the receiver, as was taken out at the laſt turn of the handle.

At *k* (*fig.* 1), juſt under the pump plate, there is a cock, by the turning of which the air may at any time be let into the receiver again, through the hole *i*, and then the receiver becomes looſe, and may be taken off the plate. The barrels (*fig.* 2) are fixed to the frame E *e e* (*fig.* 1), by the two ſcrew-nuts *f f*, which preſs the piece E upon the barrels; and the hollow trunk H (*fig.* 2) is covered by the box G H (*fig.* 1).

*l m m m n* is a glaſs tube, open at both ends, and about 34 inches long, the upper end communicating with the hole in the pump plate, and the lower end immerſed in the veſſel N, which is nearly filled with quickſilver. This tube has a wooden ruler *m m*, called the gauge, and divided into inches, and parts of an inch, from the bottom at *n*, at the ſurface of the quickſilver, and continued upwards to *m*, about 31 inches.

The uſe of this rule is, to diſcover the quantity of air that remains in the receiver M: for as the air is pumped out of the receiver M, it is alſo pumped out of the tube *l m n*, becauſe the tube opens in the receiver; and as the tube is gradually emptied of its air, the quickſilver in the veſſel N is forced up the tube by the preſſure of the atmoſphere upon the

the quickſilver in the veſſel: and if the receiver could be perfectly exhauſted of air, the quickſilver would ſtand as high in the tube, as it does at that time in the barometer; for the quickſilver in both caſes is ſupported by the ſame power, viz. the weight of the atmoſphere on the quickſilver in the open veſſel.

Every turn of the handle F exhauſts a portion of air from the receiver, and conſequently raiſes the quickſilver in the tube; and the aſcent of the quickſilver is always proportionable to the quantity of air exhauſted; and the quantity of air remaining in the receiver is proportionable to the defect of the height of the quickſilver in the gauge, from its height in the barometer.

There are ſeveral experiments made with this air-pump, to ſhow the reſiſtance, weight, and elaſticity of the air.

The reſiſtance of the air is meaſured by a ſmall machine, having two mills, *a* and *b* (*fig.* 3), which are of equal weights, and each turning freely on its own axis, independent of each other. Each mill has four thin ſails fixed to its axis; thoſe of the mill *a* have their planes at right angles to its axis; and thoſe of *b* have their planes parallel to it. Therefore, when theſe mills turn round in common air, the mill *a* has but little reſiſtance from the air, becauſe its ſails cut the air with their thin edges; but the mill *b* is greatly reſiſted by the air, becauſe it expoſes the whole plane of its ſails againſt the air. In each axle is a fine pin near the middle of the frame, which goes quite through the axle, and ſtands out a little on each ſide of it. Upon theſe pins the ſlider *d* is made to bear, to hinder the mills from going round, when the ſtrong pin C is ſet on bend againſt the lower end of the pins.

Set this machine in motion, by drawing up the ſlider *d* to the pins on one ſide, and ſetting the ſpring C on bend at the oppoſite ends of the pins; then puſhing down the ſlider *d*, the ſpring C, acting with equal force upon each mill, will ſet

ſet them both in motion; but the mill *a* will run much longer than the mill *b*, becauſe the air makes leſs reſiſtance againſt its ſails than againſt thoſe of *b*.

Again, draw up the ſlider *d*, and ſet the ſpring C againſt the pins as before; then place the machine under the receiver M, upon the pump plate; and having exhauſted the receiver of its air, puſh the wire P P, which runs through the collar of leathers on the neck *g*, upon the ſlider *d* (*fig.* 3), which will diſengage it from the pins, and thereby ſuffer the ſpring C to ſet the mills a-going; and as there is no air in the receiver to make any ſenſible reſiſtance, they will both move a conſiderable time longer than they did in the open air, and they will both ſtop at the ſame moment. This experiment ſhows the reſiſtance of air on bodies in motion; and that equal bodies meet with different degrees of reſiſtance, according as they expoſe a greater or leſs ſurface to the air in the planes of their motions.

Again, put a guinea and a feather on the braſs flap *c*, in the tall cylindrical receiver A B (*fig.* 4), which is to be placed over the *hole i*, on the pump plate; turn up the braſs flap *c*, ſo as to confine both the guinea and the feather; then, putting a wet leather *over* the *top* of the receiver, and covering it with the plate *g*, from which the guinea and feather tongs *c d* will hang within the receiver; pump the air out of this receiver, and, by means of the wire *f*, open the tongs *c d*, and the flap *c* falling down, the guinea and feather will deſcend with equal velocity in the receiver, and both fall upon the plate at the ſame inſtant.

To ſhow the weight of the air, no more is neceſſary than a thin bottle or Florence flaſk (whoſe contents are exactly known) having a braſs cap with a valve tied over it, fixed to the mouth. This braſs cap is to be ſcrewed into the hole *i* of the pump plate, and the bottle exhauſted of its air. The bottle is then to be accurately weighed; when it will be found, that for every quart the bottle contains, it will weigh

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16 grains lefs than when it was full of air, when the quickfilver ftands at $29\frac{1}{2}$ inches in the barometer.

If the receiver O (*fig.* 1), or M, be placed over the hole *i* in the pump plate, and the air be exhaufted therefrom, this fmall receiver will be preffed down to the pump plate, by the weight of the atmofphere, which will be found to be equal to as many fifteen pounds as there are fquare inches in that part of the plate which the receiver covers; and which will hold down the receiver fo faft, that it cannot be removed until the air be let into it, by turning the cock *k*, when it will be perfectly loofe.

Place the fmall glafs A B (*fig.* 5), which is open at both ends, over the hole *i*, on the pump plate L L (*fig.* 1); and having put your hand clofe upon the top of it, at B, exhauft the air out of the glafs, and your hand will be preffed down upon the glafs with a weight equal to as many times 15 pounds as the end of the glafs B contains fquare inches, as before.

If a piece of wet bladder be tied over the end of the glafs (*fig.* 6), and, when it is dry, the glafs be exhaufted of its air; the outer air will prefs upon the bladder, which will have a fpherical concave figure, and will grow more concave as more air is pumped out of the glafs, till, at length, it will break with a report as loud as that of a gun. If, inftead of the bladder, a flat piece of glafs be laid on the top of this receiver, and joined to it by a ring of wet leather between them, to exclude the air upon exhaufting the air out of the receiver, the preffure of the outward air will foon break the flat piece of glafs to pieces.

Let the two brafs cups A and B (*fig.* 7) be joined together with a wet leather between them, having a hole in the middle of it; then fix the end of the pipe D into the hole *i* of the pump plate, and exhauft the air out of them, having turned the cock E, which permits the air to come through the pipe C D. Then turn the cock E again, to

keep out the air, and unfcrew the pipe D from the pump plate, and fcrew on the handle F; then it will require a great force to pull thefe two cups afunder; for if the diameter of the cups be four inches, they will be preffed together, by the external air, with a force equal to 190 pounds. But if they be put under the large receiver M (*fig.* 1), and the air exhaufted out of the receiver, they will fall afunder, having no external air to keep them together.

Place the veffel A (*fig.* 8) on the pump plate, having fome quickfilver in it, and cover it with the receiver B, in which is inferted through the collar of leathers, in the brafs neck C, the tube *d e* open at the lower end; then exhauft the air out of the receiver, and it will alfo be exhaufted out of the tube. When the receiver is fufficiently exhaufted, pufh down the tube, fo as to immerfe the lower end into the quickfilver. In this experiment, though the tube be exhaufted of air, yet none of the quickfilver will rife in it, becaufe there is no air in the receiver, to prefs upon its furface; but if the air be let into the receiver, by the cock *k* in the pump plate, the quickfilver will immediately rife in the tube, and ftand nearly as high as it does at that time in the barometer.

This experiment fhows, that the quickfilver is fupported in the tube, merely by the preffure of the air on its furface, in the open veffel, in which the tube is immerfed; and that the more denfe and heavy the air is, the higher the quickfilver rifes; and, on the contrary, the thinner and lighter the air is, the lefs it will rife. This is the reafon why the quickfilver in the barometer falls before rain or fnow, and rifes before fair weather; for, in the former cafe, the air is too thin and light to bear up the vapours; and in the latter cafe, too denfe and heavy to let them fall.

*Note.* In all experiments made with mercury, by the air-pump, there fhould be a fhort pipe fcrewed in the hole *i* of the pump plate, fo as to rife about an inch above the plate, to prevent

prevent any quickfilver from getting into the air-pipe and barrels; for, fhould any get loofe into the pipe or barrels, it fpoils them, by loofening the folder, and corroding the brafs.

To fhow the elafticity or fpring of the air, fcrew the pipe A (*fig.* 9) into the pump plate, and place the receiver G H, upon the plate *c d*, which is fixed to the pipe, and exhauft the air out of the receiver; then turn the cock *e*, to keep out the air, and unfcrew the pipe from the pump, and fcrew it into the mouth of the copper veffel A (*fig.* 10), which is half filled with water. Then, upon opening the cock *e*, the elafticity of the air which is confined in the upper part of the copper veffel A, will force the water up through the pipe A B in a jet or fountain, into the exhaufted receiver.

There are a great number of other experiments to be made with this ufeful machine, the air-pump, as:—1. To fhow how neceffary air is for the fupport of animal life; by putting any fmall animal under a receiver, and exhaufting the air. 2. The different effects it has on different bodies; by increafing their gravity.—3. How long it will fupply flame or fuel; by putting a lighted candle under the receiver.—4. The property of air in conveying found; by putting a bell under the receiver, and ftriking it when the air is exhaufted, &c.

## 2. *Of the Barometer.*

This is an inftrument ufed for meafuring the weight of the atmofphere, foretelling the changes of weather, and meafuring the height of mountains, &c.

The common barometer is formed of a glafs tube, hermetically fealed at one end, and filled with quickfilver, defecated, and purged of its air. The open end of the tube is then immerfed in a veffel of quickfilver; and by the preffure of the atmofphere on the quickfilver, in the open veffel, the mercury in the tube will rife to the height of

 twenty-

twenty-nine inches and a half, when the weight of the atmofphere is at a mean rate. When the weight of the atmofphere is greater, then the mercury in the tube will rife higher; and when the weight of the atmofphere is lefs than its mean weight, the mercury in the tube will fall lower.

*To conftruct the Barometer.* Being provided with a glafs tube of one third or one half of an inch wide (the wider the better), and about thirty-four inches long, being clofe at the top, or hermetically fealed, pour into it well-purified quickfilver, with a fmall funnel, of either glafs or paper, till it wants about half an inch of being full; then ftopping it clofe with the finger, invert it flowly, and the air in the empty part will afcend gradually to the other end, and collect in its way any fmall air-bubbles, which will unavoidably get in, in filling the tube: then again invert it: and thus continue to invert it feveral times, turning the two ends alternately upwards, till all the air bubbles are collected, and brought up to the open end of the tube; then the tube will appear like a fine polifhed fteel rod, without a fpeck in it. Then pour in a little more quickfilver to fill the tube quite up, and ftopping the open end of the tube with the finger, invert the tube, and immerfe the finger and end of the tube, thus ftopped, into a bafon of purified quickfilver; withdraw the finger, and the mercury will defcend in the tube to fome place between 28 and 31 inches above the mercury in the open veffel, as thefe are the limits between which it always ftands in this country, on the common furface of the earth. Meafure from the furface of the quickfilver in the open veffel, to the height of twenty-eight inches, and alfo to the height of thirty-one inches, dividing the three inches between thefe two numbers, into inches and tenths of an inch, which are marked on a fcale placed againft the fide of the tube; and the tenths fubdivided into hundredth parts of an inch, by a fliding index, carrying a *vernier* or *nonius*. Thefe three inches, between twenty-eight and thirty-one, divided thus,

thus, will anſwer all the purpoſes of a ſtationary or chambe barometer; but for experiment on altitude and depths, it is neceſſary to have the ſcales continued higher up, and a great deal lower.

Several precautions are neceſſary in filling the tube, and fitting up the barometer, as:—1. The bore of the tube ſhould be pretty wide, to allow a free motion to the quickſilver.—2. The baſon at bottom ſhould alſo be pretty large, that the ſurface of the mercury in it may not ſenſibly riſe or fall with that in the tube.—3. The bottom of the tube ſhould be cut off rather obliquely, that, when it reſts on the bottom of the baſon, there may be a free paſſage for the quickſilver. And, laſtly, It is beſt to boil the quickſilver in the tube, which will expel all the air from it, and render it very pure.

This inſtrument owes its invention to *Toricelli*, the diſciple of Galileo, who lived about the beginning of the ſeventeenth century, and who having diſcovered that water could not be raiſed in a pump unleſs the ſucker was within 33 feet of the ſurface of the well, deſired Toricelli to inveſtigate the cauſe of it. After ſome time, Toricelli diſcovered, that the preſſure of the atmoſphere was the cauſe of the aſcent of the water in the pump; and that a column of water, 33 feet high, was the juſt counterpoiſe to a column of air of the ſame baſe, and which extended up to the top of the atmoſphere: and this was the true reaſon why the water did not aſcend any higher. He alſo diſcovered, that a column of quickſilver, about $2\frac{1}{2}$ feet high, would be a counterpoiſe to a column of water of the ſame baſe, and 33 feet in height; as quickſilver is nearly 14 or rather 13.6 times heavier than water. This ſuppoſition he ſoon verified, by filling a glaſs tube with quickſilver, and inverting the open end of it into a baſon of the ſame, when the mercury deſcended till its height above that in the baſon was above $2\frac{1}{2}$ feet, juſt as he expected.

It

It was not till after fome time, that it was difcovered, that the preffure of air was various at different times. This, however, was no fooner made known, than it was alfo obferved, that the variations in the mercurial column were always fucceeded by certain changes in the weather, as rain, wind, fnow, &c. Hence, this inftrument was foon ufed as the means of foretelling the change of the weather, and on this account obtained the name of *weather-glafs*, as it did that of barometer, from its being the meafure of the weight or preffure of the air.

The phenomena of the barometer are fo various, that authors have not yet agreed upon the caufes of them; nor is the ufe of it as a weather-glafs yet perfectly afcertained; though daily obfervations lead us ftill nearer to precifion. The moft general rules for judging of the weather are thofe delivered by Mr. *Patrick*, which are efteemed the beft of any, and are as follow:

1. The rifing of the mercury prefages, in general, fair weather; and its falling, foul weather, as rain, fnow, high winds, and ftorms.

2. In very hot weather, the falling of the mercury indicates thunder.

3. In winter, the rifing prefages froft: and in frofty weather, if the mercury falls three or four divifions, there will certainly follow a thaw. But in a continued froft, if the mercury rifes, it will certainly fnow.

4. When foul weather happens foon after the falling of the mercury, expect but little of it; and, on the contrary, expect but little fair weather, when it proves fair fhortly after the mercury has rifen.

5. In foul weather, when the mercury rifes much and high, and continues fo for two or three days before the foul weather is over, then expect a continuance of fair weather to follow.

6. In

6. In fair weather, when the mercury falls much and low, and continues ſo for two or three days before the rain comes, then expect a great deal of wet, and probably high winds.

7. The unſettled motion of the mercury denotes uncertain and changeable weather.

8. You are not ſo ſtrictly to obſerve the words engraved on the plate, as the mercury's riſing and falling: though, in general, it will agree with them. For, if it ſtand at the words *Much Rain*, then riſes up to *Changeable*, it preſages fair weather; though not to continue ſo long as if the mercury had riſen high: and on the contrary, if the mercury ſtood at *Fair*, and falls to *Changeable*, it preſages foul weather; though not ſo much of it as if it had ſunk lower.

Upon theſe rules of Mr. *Patrick*, Mr. *Rowning* remarks, that it is not ſo much the abſolute height of the mercury in the tube that indicates the weather, as its motion up and down; therefore, to paſs a right judgment of what weather is to be expected, we ought to know whether the mercury is actually riſing or falling; to which end the following rules ſhould be obſerved:

9. If the ſurface of the mercury be convex (ſtanding higher in the middle of the tube than at the ſides), it is a ſign that the mercury is then riſing.

10. But, if the ſurface be concave (or hollow in the middle), it is then ſinking.

11. If it be plain, or rather a little convex, the mercury is ſtationary; for, mercury being put into a glaſs tube, eſpecially a ſmall one, naturally has its ſurface a little convex; becauſe the particles of mercury attract each other more forcibly than they are attracted by the glaſs.

12. Sometimes the mercury will ſtick to the ſides of the tube; therefore, when an obſervation is to be made with ſuch a tube, the tube ſhould be ſhaken a little; then, if the air be grown heavier, the mercury will riſe about a twentieth of an inch higher: but if the air be lighter, it will ſink as

much:

much: and if it be the wheel barometer, tap it gently with the finger, which will give the mercury a free motion.

To the foregoing rules may be added the following, taken from later and clofer obfervations:

13. In winter, fpring, and autumn, the fudden falling of the mercury, and that for a large fpace, denotes high winds and ftorms; but in fummer, it denotes heavy fhowers, and often thunder; and it always finks loweft of all for great winds, though not accompanied with rain; for wind and rain together, it falls more than for either of them alone. Alfo, if, after rain, the wind change into any part of the North, with a clear and dry fky, and the mercury rife, it is a certain fign of fair weather.

14. After very great ftorms of wind, when the mercury has been low, it commonly rifes very faft. In fettled, fair, and dry weather, expect but little rain, except the barometer fink much; for a fmall finking then, only denotes a little wind, or a few drops of rain; and the mercury foon rifes again to its former ftation. In a wet feafon, fuppofe in hay-time and harveft, the fmalleft finking of the mercury muft be noticed, for, when the conftitution of the air is much inclined to fhowers, a little finking then denotes more rain, as it never then ftands very high; and if in fuch a feafon it rifes fuddenly, very faft and high, expect not fair weather more than a day or two, but rather that the mercury will fall again very foon, and rain immediately to follow: the flow gradual rifing, and keeping on for two or three days, being moft to be depended on for a week's fair weather; and the unfettled ftate of the quickfilver always denoting uncertain and changeable weather, efpecially when the mercury ftands any where about the word *Changeable* on the fcale.

15. The greateft heights of the mercury in this country are found upon eafterly and north-eafterly winds; and it may often rain or fnow, the wind being in thofe points, and the barometer fink little or none, or may even be in the rifing

rifing ftate the effect of thofe winds counteracting. But the mercury finks for wind as well as rain in all the other points of the compafs; but rifes as the wind fhifts about to the north, or to the eaft, or between thofe points; but if the barometer fhould fink, with the wind in that quarter, expect it foon to change from thence; or elfe, fhould the fall of the mercury be much, a heavy rain is likely to enfue.

16. The barometer being lower, and continuing fo longer than what can be accounted for, by immediate falls, or ftormy weather, indicates the approach of very cold weather for the feafon; and alfo cold weather, though dry, is always accompanied by a low barometer, till near its termination.

17. Warm weather is always preceded, and moftly accompanied, by a high barometer; and the rifing of the barometer, in the time of cold weather, is a fign of the approach of warmer weather: and alfo, if the wind be in any of the cold points, a fudden rife of the barometer indicates the approach of a foutherly wind, which, in winter, generally brings rain.

The barometer is alfo found to fink in a certain ratio to its diftance from the furface of the earth: it is, therefore, ufed for meafuring any acceffible heights. Various rules have been given by writers on the barometer, for applying it to this purpofe, or computing the height afcended, from the fall of the mercury in the tube, the moft accurate of which is that of Dr. Halley, now greatly improved by De Luc, by introducing into it the correction of the columns of the mercury and air, on account of heat, with other corrections and modifications. This rule is as follows, viz. $10,000 \times$ logarithm of $\frac{M}{m}$ is the altitude in fathoms, in the mean temperature of 31 degrees; and for every degree of the thermometer above that, the refult muft be increafed by fo many times its 435th part, and diminifhed when below it. In this theorem M denotes the length of the column of mercury in the barometer tube at the bottom of the hill or eminence; and $m$

 denotes

denotes the ſame at the top of the hill or eminence; and it s to be obſerved, that the reſult is always in fathoms of ſix Engliſh feet each.

As the ſcale of variation in the barometer is but ſmall, being included within three inches, viz. from 28 to 31 inches, ſeveral contrivances have been deviſed for enlarging the ſcale, to render the ſmall variations of the mercury more apparent; this has given riſe to the invention of ſo many different kinds of barometers:—a few of the moſt improved are the following:

### 1. *The Diagonal Barometer.*

This is a method of enlarging the common ſcale of three inches perpendicular height, by extending it to any length, B C (*fig.* 11), in an oblique direction. This barometer was invented by Sir Samuel Moreland. The perpendicular height of the diagonal part B C, is equal to the ſcale of variation of three inches, or C I; and conſequently, while the mercury in the common barometer riſes the whole length of the ſcale, which is three inches, and equal to I C; in this barometer it will move from B to C: thus the ſcale is enlarged in this barometer, in the proportion of B C to I C. But it is found, that the diagonal part B C cannot be bent from the perpendicular, more than in an angle of 45 degrees, which increaſes the ſcale only in the proportion of 7 to 5. This form is liable to ſome inconveniences, on account of the obliquity of the part B C, which makes the mercury frequently divide into ſeveral parts, and renders it neceſſary to fill the tube again.

### 2. *The Horizontal Rectangular Barometer.*

This barometer (*fig.* 12) was the invention of *J. Bernoulli*, and *Caſſini*, and conſiſts of a tube A C D F, ſealed at the upper end A, and bent to a right angle at D; the end F being

F being open. The mercury, in this, ſtands in both legs from E to C. The ſcale of variation from D to F is here made larger; and it is evident, in moving three inches from A to C, it will move as much more in the ſmall leg D F, as the area of the tube at A C is greater than that of D F: wherefore the motion of the mercury at E muſt be very ſenſible. Though the end of the tube F be open, yet the mercury cannot run out, being oppoſed there by the preſſure of the atmoſphere. This inſtrument is founded on that theorem in Hydroſtatics, that fluids of the ſame baſe preſs according to their perpendicular altitude, and not according to the quantity of their matter. So that the ſame preſſure ſuſtains the quickſilver, that fills the tube A D F, and the ciſtern C, as would ſupport the mercury in the tube alone. This form is, however, liable to ſome inconveniences; for the attrition of the mercury againſt the ſide of the glaſs, and the quick motion of it in the part D F, is apt to break the mercury, and render its motions unequal: it is alſo apt to be thrown out at the open end F, by any ſudden ſhock.

### 3. *Dr. Hook's Wheel Barometer* (*Fig.* 13).

This barometer tube has a large ball A at the top, and is bent up at the lower end, which is open, where an iron ball floats on the top of the mercury in the tube, and which ball is connected to another ball H, hanging freely over a pulley, and turning an index K about its centre. When the mercury riſes in the part K, it raiſes the ball, and the other ball H deſcends, and turns the pulley with the index round a graduated circle from M towards N; and the contrary way when the quickſilver and the ball ſink in the bent part of the tube. This ſcale is eaſily enlarged 10 or 12 fold, being increaſed in proportion to the axis of the pulley to the length of the index K. If this inſtrument could be conſtructed without any friction of the pulley and axis, it would anſwer

extremely well; but the friction often obstructs the motion of the quickfilver.

### 4. *Mr. Cafwell's Barofcope, or Barometer.*

This inftrument is the moft ufeful of any, for enlarging the fcale of variation, and at the fame time being the moft exact. A B C D (*fig.* 14) is a bucket of water, in which is the barofcope, *x r k u f m*, which confifts of a body *x r f m*, and a tube, *e k u o*, which are both concave cylinders, made of tin, or rather glafs, communicating with each other. The bottom of the tube, *k u*, has a leaden weight to fink it, fo that the top of the body of the barofcope, or barometer, may fwim juft even with the furface of the water, by adding the weight of a few grains to the top. When the inftrument is forced with its mouth downwards, the water afcends into the tube to the height of *n*. A fmall concave cylinder or pipe is added to the top, to keep the inftrument from finking down to the bottom. *m d* is a wire, *m S* and *d c* are two threads, oblique to the furface of the water, which anfwer as diagonals; for while the inftrument finks, more or lefs, by an alteration in the gravity of the air, where the furface of the water cuts the thread, there is formed a fmall bubble, which afcends up the thread, while the mercury of the common barofcope afcends; and *vice verfa.*

This inftrument, as the author has fhown, marks the alteration in the air 1200 times more accurately than the common barometer. The bubble on the thread will feldom ftand ftill a minute. A fmall blaft of wind, which cannot be heard in a chamber, will make it fink fenfibly; and even a cloud paffing over it always makes it defcend.

The common *barometer*, or *weather-glafs*, is ufually fitted up in a neat mahogany frame; and confifts of the common tube barometer, with a thermometer by the fide of it, and a hygrometer at the top.

The

## *The Air-gun.*

This inſtrument is an ingenious pneumatical invention, for driving a bullet, with great violence, by means of condenſed air, forced into an iron ball by a condenſor.

The condenſor (*fig.* 15) has at the end *a*, a male ſcrew, on which the hollow ball *b* is ſcrewed, in order to be filled with condenſed air. In the inſide of this ball there is a valve, to prevent the air from eſcaping (after it is injected into it), until it be forced open by a pin, *a* (*fig.* 16).

When the air is to be condenſed into the ball, place your feet on the iron croſs, *h h*, in order to hold down the piſton rod *d c*; then lift up the barrel *e a* by the handles *i i*, until the piſton *c* be brought below the hole *e*; the barrel *a c* and ball *b* will then be filled with air through the hole *e*. Then thruſt down the barrel *a e*, until the piſton *d c* reach the neck of the iron ball at *a*; then all the air between *o* and *a* will be forced up through the valve into the ball; and when the handles *i i* are again lifted up, the valve in the ball will cloſe, and ſo keep in the air: thus by rapidly continuing the ſtrokes up and down, the ball will preſently be filled: then unſcrew the ball from the condenſor, and ſcrew it upon another male ſcrew, which is connected with the barrel, and goes through the ſtock of the gun (*fig.* 16). A bullet then being depoſited in the barrel of the gun, the hammer of the lock at *a* ſtrikes againſt the pin, which opens the valve in the ball, and lets out as much air as will drive a muſket-ball to a conſiderable diſtance.

There are ſeveral kinds of air-guns; but that here deſcribed, is the moſt improved and uſeful, as the gun need not be any larger than a ſmall fowling-piece: and ſeveral balls, filled with condenſed air, may be taken to any diſtance from home with very little trouble, and which will ſave the trouble of filling the ſame ball every time it is wanted. A ball of 3½ inches diameter may be made to contain twelve penny-

penny-weights of air, which will difcharge 12 or 15 bullets with confiderable force.

There are fome air-guns that have a fmaller barrel contained within a larger one; and the fpace between the two barrels holds the condenfed air. In this inftrument there is a valve fixed at *a* (*fig.* 16), with a condenfor fixed to the barrel at *a*, and continued through the but end to *e*, where the pifton rod may be left in. Here the whole gun ferves inftead of the handles *i i* (*fig.* 15), to condenfe the air into the barrel.

The magazine air-gun differs from the others, by having a ferpentine barrel, which contains 10 or 12 bullets: thefe are brought into the barrel of the gun fucceffively, by means of a lever; and they may be difcharged as faft as if they were in feparate guns.

# CHAP. XVIII.

## OF HYDROSTATICS.

### *Definitions.*

1. HYDROSTATICS treat of the equilibrium of fluids; or the gravitation of fluid bodies remaining at reft. When this equilibrium is removed, and the fluid body fet in motion, the effects it then produces are called *Hydraulics.*

2. A *Syphon* is a bent tube (*fig.* 5, plate 23).

3. A

3. A *Valve* is a kind of flap or cover, fixed to a pipe, or to the aperture of any body, and which, by opening only one way, fuffers water or any fluid body to pafs, but not to return.

4. A *Pifton* is a fmall cylinder, fixed to the end of a rod, and fitted to the bore of a pipe, and frequently contains a valve.

*Axioms.*

1. All fluids, except air, are incompreffible, or incapable of being compreffed into a lefs fpace.

2. In a veffel of water, or any other fluid body, the preffure of the upper parts on the lower is in proportion to the depth; and is the fame at the fame depth, whatever the diameter of the veffel may be.

3. The preffure of a fluid upwards, is equal to its preffure downwards, at any given depth.

4. The bottom and fides of a veffel are preffed by the fluid it contains, in proportion to its height, without any regard to the quantity.

5. If fluids of different gravities be contained in the fame veffel, the heavieft will be at the bottom, the lighteft at the top, and the others will be farther diftant from the top in proportion to their fpecific gravities.

6. The direction of the preffure of a fluid againft the fides of the veffel which contains it, is in lines perpendicular to the fides of fuch veffel.

7. A body that is heavier than an equal quantity of a fluid, will fink in that fluid; but, if it be lighter, it will fwim at the top of the fame fluid; and if it be of the fame gravity, it will neither fink nor fwim, but will remain fufpended in any part of the fluid.

8. A folid immerfed into a fluid, is preffed by that fluid on all fides, in proportion to the height of the fluid above the folid. And bodies very deeply immerfed in any fluid, may be confidered as equally preffed on all fides.

9. Every

9. Every solid immersed in a fluid, that is specifically lighter, loses as much of its own weight as is equal to the weight of a quantity of that fluid of the same dimension with the solid.

10. And the fluid in which the solid is immersed, acquires the weight the solid loses.

As the principal fluid with which we have any concern in hydrostatics, is water, it may be necessary to mention a few of its distinguishing properties.

1. Water is a transparent, colourless, scentless fluid, which, with a certain degree of cold, turns to ice.

2. Water is one of the constituent parts of all bodies; as hath been proved by distillation; for the driest woods, earths, bones, and stones, pulverized, constantly yield a certain quantity of water.

3. Though fluidity is commonly regarded as an essential property of water, yet many philosophers, particularly *Boyle*, *Boerhaave*, and Dr. *Black*, of Edinburgh, consider it as an adventitious circumstance, and produced by a certain degree of heat; and therefore assert its natural state to be that of crystalline, as when in ice.

4. Water is a more penetrating body than air, though it be less transparent; for it will pervade bodies that air will not: as is evident from its passing through the pores of a bladder.

5. Some bodies are dissolved by water, as salts; while it conglutinates others, as bricks, stones, bones, &c.

6. Water owes its fluidity to heat, and it contains no small quantity of air; and the sediment found in all water which has not been distilled, always contains a quantity of earth. From which last element it is supposed that plants derive all their nourishment.

SECT.

## SECT. I.

### OF FLUIDITY.

A FLUID body, in Sir Iſaac Newton's definition, is *a body yielding to any force impreſſed, and which has its parts very eaſily moved one among another*. This is the definition of a perfect fluid: if the fluid require ſome ſenſible force to move its parts, it is an imperfect fluid; and the imperfection is in proportion to that force: ſuch perhaps are all the fluids with which we are acquainted.

Fluids are either elaſtic, ſuch as air; or non-elaſtic, as water, mercury, &c. The latter are incompreſſible, and occupy the ſame ſpace under all preſſures or forces; but the former dilate and expand themſelves continually, by taking off the external preſſure from them. The properties of the former fluids conſtitute the doctrine of Pneumatics, before treated of; the latter contain the principles of Hydroſtatics.

Fluidity differs from liquidity, or humidity; the latter implying wetting or adhering. Thus, air, ether, mercury, and other melted metals, and even ſmoke and flame, are fluid bodies, though not liquid ones; while water, beer, milk, &c. are both fluids and liquids.

The modern opinion concerning the original and conſtituent parts of fluids, is, that they are ſmall, ſmooth, hard, globular particles; conſequently, each particle muſt be a ſolid globular body; and conſidered ſingly, is no fluid; but becomes a fluid, by being joined with other particles of the ſame or a ſimilar kind.

That the particles of fluid bodies are very ſmall, is evident, from their texture having never been diſcovered by the fineſt microſcope: that they are ſmooth, appears from that freedom wherewith they glide over one another, when ſet in motion: that they are hard and impenetrable, is plain from their

being incapable of compreſſion: and that they are ſpherical, is obvious, from their being ſo eaſily put in motion; and from the interſtices or vacancies, which are hereafter proved to ſubſiſt between them; which could not be the caſe, unleſs they were ſpherical, and touched each other only in ſome ſingle points of their ſurfaces. For, upon mixing ſalt with water, a certain quantity of the ſalt will be diſſolved, without increaſing the dimenſions of the water; which demonſtrates the vacuities between the particles of the water. When a fluid becomes more buoyant, it is a proof that its ſpecific gravity is increaſed, and conſequently, many of its vacuities filled up; and even then it may receive a certain quantity of other diſſoluble bodies, the particles whereof are adapted to the remaining vacancies, without adding any thing to its bulk, though the abſolute weight of the whole fluid be thereby increaſed. This is demonſtrated by taking the weight of a phial of rain water with a nice balance: when the water is poured out, and ſome ſalt added to it, and the phial again filled with the water, it will be found to weigh more than when before the ſalt was put in, from the vacuities of the freſh water being filled with ſaline particles.

It has alſo been found by experiment, that the particles whereof fluids are compoſed, conſiſt of ſpheres of different diameters, whoſe interſtices may be ſucceſſively filled with proper ingredients; and where theſe interſtices are ſmaller, the gravity of the fluid will be greater, and *vice verſa.*

For example: if a barrel be filled with any large ſpherical bodies, as bullets, many ſmall ſhot may afterwards be placed in the interſtices of theſe bullets; the vacuities of the ſhot may then be filled with ſea ſand; the interſtices of which may again be filled with water, which will alſo admit of a certain quantity of ſalt in the vacuities; and thus the weight of the barrel may be greatly increaſed, without increaſing the ſpace occupied by theſe materials. This reaſoning alſo holds good in fluid bodies, as well as in thoſe which are ſolid; for river water will diſſolve a certain quantity of

ſalt;

ſalt; after which it will diſſolve a certain quantity of ſugar; and after that a certain quantity of alum; and then perhaps will receive other diſſoluble bodies, without increaſing the dimenſions of the whole.

If fluids were not compounded of ſuch primary particles, but made up of one homogeneous ſubſtance, equally denſe, without conſiſtence, there would be no difference in their ſpecific gravities, and all fluids would be of the ſame weight, which is not the caſe.

That a fluid has vacuities, is evident from the following conſideration, viz. if all ſpace were abſolutely full of matter, that matter muſt be either fluid or fixed. If it were fixed, no motion could poſſibly be therein, as is evident from reaſon and experience; it muſt therefore be fluid. But a fluid without vacuities would be denſer, and conſequently heavier, than all other fluids; and if denſer, all bodies will emerge and ſwim at the top, by hydroſtatical laws, and there would be no ſuch thing as gravity. But as gravity exiſts, all ſpace therefore cannot be filled, even with a fluid.

By the experiments of *Borcelli*, it has been demonſtrated, that the conſtituent parts of all fluids, are not fluids themſelves, but conſiſtent bodies; and that the elements of all bodies are perfectly firm and hard. The incompreſſibility of water, proved by the Florentine experiment, is a ſufficient evidence that each primary particle of this fluid is a perfect impenetrable ſolid.

This famous experiment was firſt attempted by the ingenious Lord *Verulam*, who encloſed a quantity of water in a piece of lead, and found, that the water would ſooner make its way through the pores of the lead, than be reduced to leſs compaſs, by any force that could be applied. This experiment was afterwards made at Florence, with a globe of ſilver; which being filled with water, and well cloſed, was gently preſſed, when a ſmall quantity of water iſſued through the pores of the ſilver in the form of dew.

9. The ſpecific gravity of a ſolid that is lighter than the fluid in which it is immerſed, is found by the following proceſs. To the lighter body, whoſe ſpecific gravity is required, annex another body, that is much heavier than the fluid, ſo that the compound maſs may ſink in the fluid. Weigh the heavier body, and the compound maſs, ſeparately, both in water and out of it; then find how much each loſes in water, by ſubtracting its weight in water from its weight in air; and ſubtract the leſs of theſe remainders from the greater; then ſay, as this laſt remainder is to the weight of the lighter body in air, ſo is the ſpecific gravity of the fluid to the ſpecific gravity of that body.

10. The ſpecific gravities of bodies of equal weights are reciprocally proportional to the quantities of weight loſt in the ſame fluid. Hence is found the ratio of the ſpecific gravities of ſolids, by weighing in the ſame fluid, maſſes of them that weigh equally in air, and noting the weight loſt by each.

11. A body deſcends in a fluid that is ſpecifically lighter, but aſcends in a fluid that is ſpecifically heavier, with a force equal to the difference between its weight and the weight of an equal bulk of the fluid.

12. A body ſinks in a fluid that is ſpecifically heavier, ſo far, as that the weight of the body is equal to the weight of a quantity of the fluid of the ſame bulk as the part of the body which is immerſed in the fluid. Therefore, as the ſpecific gravity of the fluid is to that of the body, ſo is the whole magnitude of the body to the magnitude of the part immerſed. By this theorem is found the abſolute weight of any body immerſed in a fluid: for the weight of that quantity of the fluid which is diſplaced by the ſolid body is always equal to the whole weight of the ſolid; thus, if a boat on the water be ſo loaded, that it diſplace three cubical feet of water, its whole weight will be 3000 ounces or $187\frac{1}{2}$ pounds, that being the weight of three cubical feet of water.

13. In equal ſolids the ſpecific gravities are as their parts immerſed in the ſame fluid.

The

The foregoing theorems have been ſufficiently demonſtrated by various authors, from the principles of mechanics: they are alſo exactly conformable to experience; as hath been ſufficiently proved from ſeveral courſes of philoſophical experiments.

Various tables have been given, by different authors, of the ſpecific gravities of many kinds of bodies. It will be ſufficient in this place to give the ſpecific gravities of ſome of the moſt uſeful bodies that have been determined with greater certainty and accuracy. The numbers in this table expreſs the number of avoirdupois ounces contained in a cubical foot of each body; that of common water being juſt 1000 ounces, or $62\frac{1}{2}$ pounds.

*Table of ſpecific Gravities.*

1. *Solids.*

| | | | |
|---|---|---|---|
| Platina, pure | 23000 | Crude antimony | 4000 |
| Fine gold | 19640 | Diamond | 3517 |
| Standard gold | 18888 | Granite | 3530 |
| Lead | 11325 | White lead | 3160 |
| Fine ſilver | 11091 | Iſland cryſtal | 2720 |
| Standard ſilver | 10535 | Marble | 2705 |
| Copper | 9000 | Pebble-ſtone | 2700 |
| Copper halfpence | 8915 | Rock cryſtal | 2650 |
| Gun-metal | 8784 | Pearl | 2630 |
| Fine braſs | 8350 | Green glaſs | 2600 |
| Caſt braſs | 8000 | Flint | 2570 |
| Steel | 7850 | Onyx ſtone | 2510 |
| Iron | 7645 | Common ſtone | 2500 |
| Pewter | 7471 | Cryſtal | 2210 |
| Caſt iron | 7425 | Clay | 2160 |
| Tin | 7320 | Oyſter-ſhells | 2092 |
| Lapis calaminaris | 5000 | Brick | 2000 |
| Loadſtone | 4930 | Common earth | 1984 |
| Mean ſpecific gravity of the whole earth | 4500 | Nitre | 1900 |
| | | Vitriol | 1880 |

Alabaſter

| | | | |
|---|---|---|---|
| Alabafter . . | 1874 | Brazil wood . . | 1031 |
| Horn . . | 1840 | Box wood . . | 1030 |
| Ivory . . | 1825 | Bees wax . . | 955 |
| Sulphur . . | 1810 | Butter . . | 940 |
| Chalk . . | 1793 | Oak . . . | 925 |
| Solid gunpowder . | 1745 | Gunpowder, fhaken | 922 |
| Alum . . | 1714 | Logwood . . | 913 |
| Dry bone . . | 1660 | Ice . . . | 908 |
| Sand . . | 1520 | Afh . . . | 800 |
| Lignum vitæ . | 1327 | Maple . . | 755 |
| Coal . . | 1250 | Beech . . . | 700 |
| Jet . . | 1238 | Elm . . . | 600 |
| Ebony . . | 1177 | Fir . . . | 550 |
| Pitch . . | 1150 | Saffafras wood . | 482 |
| Rofin . . | 1100 | Cork . . | 240 |
| Mahogany . . | 1063 | New-fallen fnow . | 86 |
| Amber . . | 1040 | | |

2. *Fluids*.

| | | | |
|---|---|---|---|
| Quickfilver . | 13600 | Ale . . . | 1028 |
| Oil of vitriol . . | 1700 | Vinegar . . | 1026 |
| Oil of tartar . . | 1550 | Tar . . . | 1015 |
| Honey . . | 1450 | Common water . | 1000 |
| Spirit of nitre . | 1315 | Diftilled water . | 993 |
| Aqua fortis . . | 1300 | Red wine . . | 990 |
| Treacle . . | 1290 | Proof fpirits . . | 931 |
| Aqua regia . . | 1234 | Olive oil . . | 913 |
| Human blood . | 1054 | Pure fpirits of wine | 866 |
| Urine . . . | 1032 | Oil of turpentine . | 800 |
| Cows milk . . | 1031 | Æther . . | 726 |
| Sea water . . | 1030 | Common air 1.232, or, | $1\frac{7}{30}$ |

As thefe numbers are the weights of a cubic foot, or 1728 cubic inches of each of the foregoing bodies in avoirdupois ounces, the quantity, in any other weight, or the weight of any other quantity, may be found by proportion.

For

For example, required the content of an irregular block of common ſtone, weighing one hundred weight, or 1792 ounces: here, as 2500, the ounces in a cubic foot of common ſtone, is to 1792, ſo is 1728, the inches in a cubical foot, to 1238¾ cubical inches, the contents.

Again, what is the weight of a block of granite, the length whereof is 63 feet, and the breadth and thickneſs each 12 feet, being the dimenſions of one of the ſtones of granite in the walls of Balbeck? Here the ſolid content of this ſtone is 9072 feet; therefore, as 1 is to 9072, ſo is 3500 ounces to 31,752,000 ounces, or 885 tons 18 cwt. 3 qrs. the weight of the ſtone.

## SECT. II.

### THE CONSTRUCTION AND USE OF THE HYDROSTATIC BALANCE, HYDROMETER, AND HYDROSTATIC BELLOWS.

#### *The Hydroſtatic Balance.*

THE hydroſtatic balance is the moſt convenient inſtrument of any hitherto invented, for diſcovering the ſpecific gravity of all ſubſtances, whether fluid or ſolid. It is conſtructed in various forms; but that which is moſt generally retained is the following:

V C G (*fig.* 1, *plate* 23) is the ſtand or pillar of the inſtrument, which is to be fixed in a table. From the top A, by two ſilken ſtrings, hangs the horizontal bar B B, from which

is fufpended by a ring *i*, the fine beam of a balance *b*, which is prevented from defcending too low on each fide by the gently fpringing piece *l*, *x*, *y*, *z*, fixed on the fupport M. The harnefs is annulated at *o*, to fhow exactly the perpendicular pofition of the examen, by the fmall-pointed index fixed above it. On each fide of the piece A is a pulley, over which paffes a ftring, which goes down to the bottom on the other fide, and hangs over the hook at V, which hook is moveable about an inch and a quarter, backward and forward, by means of the fcrew P, fo that the balance may be raifed or depreffed fo much. But when a greater elevation or depreffion is required, the fliding piece S, which carries the fcrew-pin, is readily removed to any part of the fquare brafs rod V K, and fixed by means of the fcrew.

By thefe means the motion of the balance is adjufted; the other parts of the apparatus are as follow: D is a piece to fupport the fmall board H H, fixed under the two fcales *d* and *e*, and is moveable up and down by a long flit in the pillar above C, in which D flides, having a fcrew in the back part to faften it when neceffary. From the bottom of the middle of each fcale *d* and *e*, hangs a brafs wire *a d* and *a c*, by a fine hook; thefe wires pafs through two holes, *m m*, in the table. To the wire *a d* is fufpended a curious cylindrical wire *r s*, perforated at each end for that purpofe. This wire *r s* is covered with paper, graduated by equal divifions, and is about five inches long.

In one corner of the board, at E, is a fixed tube of brafs, on which a round wire *h l* is fo adapted as to move fomewhat freely, by its flat head I. Upon the lower part of this moves another tube Q, which has fufficient friction to make it remain in any pofition required; to this is fixed an index T, which moves horizontally when the wire *h l* is turned round; and therefore may be eafily fet to the graduated wire *r s*. To the lower end of the wire *r s* hangs a weight L, which has a wire *p n*, with a fmall brafs ball *g* at the end, about a quarter of an inch in diameter. On the other wire *a c*, from the

the other ſcale, hangs, by means of a horſe-hair, a large glaſs bubble R.

To apply this inſtrument to uſe, let the weight L be taken away, and a wire *p n* be ſuſpended from the hook *s*; and let the bubble R be taken away from the other ſcale, and a weight ſuſpended in the room thereof. Suppoſe the weight to be ſufficient to keep the parts belonging to the other ſcale in equilibrium; and that the middle point of the wire *p n* is at the ſurface of the water in the veſſel N. And, *Note*, the wire *p n* is to be of ſuch a ſize, that the length of one inch ſhall weigh four grains.

It is evident, that as braſs is eight times heavier than water, for every inch the wire ſinks in the water, it will become an eighth part lighter, which is half a grain; and heavier in the ſame proportion, for every inch it riſes out of the water: thus, by ſinking it two inches below the middle point, or raiſing it as much above it, the wire will become one grain lighter or heavier. Therefore, when the middle point of the wire is at the ſurface of the water, if the balance be in equilibrium, and the index T ſet to the middle point *a* of the graduated wire *r s*, and the diſtance on each ſide *a r* and *a s* contain an hundred equal parts; then, if, in weighing bodies, the weight is required to the hundredth part of a grain, it may be eaſily found by the following method: let the body to be weighed be placed in the ſcale *d*, put the weight *x* in the ſcale *e*, and let this be ſo adjuſted, as one grain more ſhall be too much, and one grain leſs too little. Then, by moving the balance gently up or down by the ſcrew P, till the equilibrium be exactly ſhown at O; if the index T be at the middle point *a* of the wire *r s*, it ſhows that the weight put in the ſcale *e* is juſt equal to the weight of the body.

The foregoing method diſcovers the abſolute weight of the body; but to find the relative or ſpecific weight, it muſt be weighed hydroſtatically in water, as follows:—Inſtead of putting the body to be weighed into the ſcale *d* as before, let it hang by a horſe-hair, as the weight R, ſuppoſing the veſſel

 of

of water O were removed. Then the equilibrium being made, the index T ſtanding between *a* and *r* ſhows the weight of the body. As it hangs thus, let it be immerſed in the water of the veſſel O, when it will become much lighter, the ſcale *e* deſcending till the beam of the balance reſt on the ſupport *x*. Then, if 100 grains put in the ſcale *d* will exactly reſtore the equilibrium, ſo that the index T ſtands at the 36th diviſion above *a*, it is evident that the weight of an equal bulk of water would be exactly 100 grains.

In the ſame manner this balance may be applied to find the ſpecific gravities of fluids.

## *The Hydrometer.*

The hydrometer diſcovers the ſpecific gravity of fluids only; for which purpoſe it is the moſt accurate, eaſy, and expeditious inſtrument of any.

This inſtrument conſiſts of a copper ball D (*fig.* 2), to which is ſoldered a braſs wire A B, a quarter of an inch in diameter. The upper part of this wire is filed flat, and marked *Proof* at *m* (*fig.* 3), becauſe it ſinks exactly to that mark in proof ſpirits. There are two other marks at A and B (*fig.* 2), to ſhow whether the liquor be one tenth above or below proof, according as the hydrometer ſinks to A or riſes to B, when a braſs weight, C or K, is ſcrewed to its bottom *c*. There are alſo other weights to ſcrew on, which ſhow the ſpecific gravity of ſeveral different fluids as low as common water.

The round part of the wire above the ball ſhould be marked ſo as to repreſent river water, when it ſinks to R W (*fig.* 3), the weight which anſwers to that water being then ſcrewed on. When it is put into ſpring water, mineral water, ſea water, and water of ſalt ſprings, it will riſe to the marks S P, M I, S E, and S A, reſpectively. On the contrary, when it is put into Briſtol water, rain water, port wine, and moun-

tain

tain wine, it will refpectively fink to the marks *b r*, *r a*, *p o*, and *m o*. Inftruments of this kind are fometimes called Areometers.—And, *Note*, that the globe D (*fig.* 2) fhould be made of copper; for ivory imbibes fpirituous liquors, and confequently alters their gravity; and glafs globes require too much attention.

This hydrometer was the invention of Mr. Clarke, and anfwers very well to difcover the fpecific gravity of fpirituous liquors, and to fhow whether any fpirit be above or below proof, and how much.

But the moft perfect hydrometer is that reprefented in figure 4, which may be made to fhow the fpecific gravity of fluids to the greateft degree of exactnefs. It confifts of a large hollow ball B, with a fmaller ball *b*, fcrewed to its bottom, and partly filled with mercury or fmall fhot, in order to render it but little fpecifically lighter than water. In the larger ball at C, there is a fhort nick, into which is fcrewed the graduated brafs wire A C, which has a fmall weight A at the top, to caufe the inftrument to defcend in the fluid.

When this inftrument is immerfed in any fluid contained in a jar L M, the quantity of the fluid difplaced by it will be equal in bulk to that part of the inftrument which is under water, and equal in weight to the whole inftrument. Therefore, if the weight of the whole inftrument be 4000 grains, we can by thefe means compare the different dimenfions of 4000 grains weight of feveral forts of fluids; for if the weight A be fufficient to fink the inftrument in rain water, till the middle point of the ftem, marked 20, come to the furface of the water; and after that, if it be immerfed in common fpring water, and the furface of the water ftand at one tenth of an inch below the middle point 20, it is evident that the fame weight of each water differs only in bulk by the magnitude of one tenth of an inch in the ftem of the inftrument.

Then, fuppofe the ftem of the inftrument to be ten inches in length, and to weigh juft 100 grains; every tenth

of

and that even to the thirty, forty, or fifty-thoufandth part: and is infinitely fuperior to the common method ufed by excife officers and others, of fhaking the fpirits in a phial, and forming a judgment of the ftrength by the breaking of the bubbles.

## *Of the Syphon.*

The fyphon is a bent tube ufed to decant fluids from any ftanding veffel; and ferves to perform fome curious experiments. It depends upon the preffure of the air; and may be made in various forms.

If a fmall fyphon, whofe legs are of equal length, be filled with water, and the ends turned downward, the water will remain fufpended in the fyphon, as long as it is held exactly level; but when it is the leaft inclined to either leg, whereby in effect one leg is made fhorter than the other, the water will run out by the longer leg; for the air being a fluid, whofe denfity near the furface of the earth is to that of water as 1 to 850; and according to the nature of all fluid bodies preffing the furface of all things expofed to it every way equally, it muft neceffarily follow, that the weight of the atmofphere above, being kept off by the machine, and the air below bearing againft and repreffing the water, which endeavours to fall out of either leg with equal force, keeps it in fufpenfe, and prevents its falling. But if the fyphon be the leaft inclined to either leg, one of the legs is in effect fhortened, and the other prolonged; by which advantage is given to the weightier fluid in the longer leg to preponderate, or overweigh the other part of the fluid in the other leg; and the water will all defcend by the longer leg.

The leaft inclination of the fyphon to either end will be fufficient to produce this effect, which may be proved by experiment, thus:—Hang a fmall fyphon, whofe legs are of equal length, upon the edge of a jar filled with water; then from the floping of the jar, the external leg of the fyphon

ſyphon will ſomewhat incline, and the ſyphon will ſoon begin to act, and the water will deſcend from the jar, through it. But in practice, one leg of the ſyphon is uſually made longer than the other leg; and the ſhorter leg is put into the liquor, when the fluid will be decanted by the longer leg.

If the two legs of the ſyphon (*fig.* 5) were of equal length, terminating in the plane A B, and the ſyphon held exactly level, and then filled with liquor, no motion of the fluid would follow, till an advantage in point of gravity be given to one ſide, by inclining it. But inſtead of which inclination, one of the legs is lengthened in general, perhaps a few inches, as from B to C; and which, previous to the operation, is generally filled, as well as the reſt of the ſyphon, with ſome fluid, many degrees heavier than air: by the gravity of which, the oppoſite ſide becomes greatly overbalanced, and the liquor in this machine is decanted very rapidly.

The ſyphon is ſometimes diſguiſed in a cup, when no liquor will flow through it, till the fluid be raiſed therein to a certain height; and when it has once begun to flow, it will continue till the veſſel be emptied; this is called a ſyphon diſguiſed. Thus, D D (*fig.* 6) is a cup, in the centre whereof is fixed a glaſs pipe or ſyphon C B, continued through the bottom at B; over this pipe is put a glaſs tube, made air-tight at top, by the cork C, but left ſo open at the bottom, by holes made about D D, that the water may freely riſe between the two tubes, as the cup is filled. When any fluid is poured into this cup, no motion will take place through the ſyphon, till the fluid in the cup ſhall have gained the top of the innermoſt pipe at C; but when the fluid is arrived to this height, it will begin to flow through the ſyphon, which runs through the bottom of the cup, and will continue to riſe up the inſide of the outer tube, and deſcend through the inner tube, till the whole fluid in the cup be run off; which is owing to the fluid at its firſt riſing

through

through the tubes, expelling all the air from them, while the weight of the atmoſphere preſſes on the ſurface of the fluid in the cup.

This is ſometimes called Tantalus's cup, and has a hollow figure, repreſenting Tantalus, placed over the inner tube, of ſuch a length, that when the fluid is got nearly up to the mouth of the figure, the ſyphon begins to act, and empty the cup.

This is the ſame in effect as if the two legs of the ſyphon were both in the veſſel (*fig.* 7), when the water poured into the veſſel will riſe in the ſhorter leg of the ſyphon, to its own level; but when it has gained the bend of the ſyphon, it will begin to run off by the longer leg, and continue running till the veſſel be emptied as low as the extremity of the ſhorter leg of the ſyphon.

## *The Hydroſtatical Paradox.*

*Any quantity of fluid, however ſmall, may be made to counterpoiſe, and ſuſtain any weight, how large ſoever.*—This is called the hydroſtatical paradox, and depends upon the equal preſſure of the parts of fluids every where at the ſame depth.

Let A B D G (*fig.* 8) repreſent a cylindrical veſſel, to the inſide of which is fitted a cover, which, by means of leather round the edge, will eaſily ſlide up and down in the veſſel, without permitting any water to paſs between its edge and the ſurface of the veſſel. In the cover is fixed a ſmall tube E F open at the top, and extending through the cover at the bottom. Then, the veſſel being filled with water, and the cover put on, and loaded with a weight, ſuppoſe of a pound, it will be depreſſed, and the water will riſe in the tube to E, and the weight will be ſuſtained. If another pound be laid on the cover, the water will riſe to F, and the weight alſo be ſuſtained: and thus the water will riſe higher in the tube in proportion to the weight that is laid on the cover. And though the weight of the water in the tube be but a few

grains, yet its lateral pressure will sustain as much as the weight of a column of water, whose base is equal to that of the cylinder, and height equal to that in the tube. Thus, the column of water in the tube produces a pressure of water, contained in the cylinder, equal to what would have been produced by the column of water contained in A a d D; and as this pressure is every way equal, the cover will be pressed upwards, equal to the force of the column of water A a d D: consequently, if A a d D would weigh a pound, the water in the tube, from the cover to E, will sustain a pound. And the same may be observed of other weights. And by diminishing the diameter of the tube, any quantity of water, however small, will in theory sustain any weight, however large.

The same paradox may be shown by a more simple experiment: thus, let A D G B (*fig.* 9) be a hollow cylinder of wood, into which is poured some water, whose surface rises to *b g*; then, if the wooden cylinder M N be put into the hollow one, the water will rise between the outside surface of the inner cylinder, and the inner surface of the outer cylinder, to *a d*, and the wooden cylinder M N will be sustained floating. The nearer the wooden cylinder M N approaches to the size of the hollow cylinder, the less quantity of water will serve for the experiment.

### *The Hydrostatic Bellows.*

The hydrostatic bellows is the best instrument for demonstrating the upward pressure of fluids. It consists of two round or oval boards generally 16 or 18 inches in diameter (*fig.* 10), and joined to each other by leather, nailed tight round their edges, so that the two boards may open and shut like a pair of common bellows, but without any valve: and a pipe, generally three feet in length, is fixed into the side of the bellows. To prove the upward pressure of fluids, let some water be poured down the pipe of the bellows, which will run in between the two boards; then lay some weights upon

upon the upper board of the bellows; as, ſuppoſe three weights, weighing 100 pounds each, and pour mere water into the pipe, which, by running into the bellows, will raiſe up the board with all the weights upon it; and if the pipe be kept full until the weights are raiſed as high as the board can riſe, the water will remain in the pipe, and ſupport all the weights; though the water in the pipe weigh no more than a quarter of a pound, and the weights on the bellows 300 pounds.

The reaſon of this experiment appears evident from what has been ſaid of the preſſure of fluids, of equal heights, without any regard to their quantities. For if the tube be fixed in the upper board of the bellows, inſtead of the ſide, the water will riſe in it to the ſame height as it did in the pipe, in the former caſe; and if as many tubes were fixed in the upper board as it would contain, the water would riſe as high in each of them. The preſſure of the fluid upwards is thus computed:—If one pipe be fixed in the upper board, and the pipe hold juſt one quarter of a pound of water, and if a perſon put his finger upon the hole of the pipe, when the fore-mentioned weights are placed upon the bellows, he will find his finger preſſed upwards with a force equal to a quarter of a pound; and as the ſame preſſure is equal upon equal parts of the board, each part whoſe area is equal to the area of the hole of the pipe, will be preſſed upwards with an equal force, that is, with a force equal to that of a quarter of a pound; the ſum of all which preſſures againſt the under ſide of an oval board, 16 inches broad and 18 long, will amount to 300 pounds; and therefore this quantity of weight will be raiſed up, and ſuſtained by only one quarter of a pound of water in the pipe.

It is by this inſtrument that a man may raiſe himſelf upwards by his breath; for if he ſtand upon the upper board, and blow through the pipe, he will raiſe the upper board of the bellows, with himſelf upon it; and the ſmaller the bore of the pipe is, the more eaſily is the operation performed.

weight, which will then be too great for the preſſure of the water round the tube upon the column of water below it.

Again, a piece of wood, however light, may be made to lie at the bottom of the water, by not ſuffering any water to get under it. Thus, having two pieces of wood, planed quite flat and ſmooth, ſo that no water may get between them, when they are put together; and cementing one of the pieces, as *a b*, to the bottom of the veſſel A B, place the other piece upon it, and let it be held down by a ſtick, while the water is poured into the veſſel; then, upon removing the ſtick, the upper piece of wood will not riſe from the lower one, being preſſed down both by its own weight, and the weight of all the water above it, while the contrary preſſure of the water upwards is kept off by the wood placed beneath it; but if the top piece of wood be raiſed ever ſo little at any part of its edge, ſome water will get under it, which will be forced by all the weight of the water above, and will immediately preſs it upwards; and being lighter than its own bulk of water, it will float upon the ſurface of the water.

To prove that all fluids weigh juſt as much in their own elements as they do in open air, put as much ſhot in a phial as, when corked, will make it ſink in water; then let it be weighed, both in the air and in the water, and the weight in each caſe wrote down; then, as the phial hangs ſuſpended in water, and counterpoiſed by another weight, pull out the cork, that the water may run into it, when it will deſcend and pull down that end of the beam. Next, put as much weight into the oppoſite ſcale as will reſtore the equilibrium; which additional weight will be found to anſwer exactly to the additional weight of the phial, when it is again weighed in the air with the water in it.

The velocity with which water ſpouts out of a hole, or through a tube in the ſide or bottom of a veſſel, is as the ſquare root of the depth or diſtance of the hole below the ſurface of the water. Therefore, in order to make double the

of the veſſel, at equal diſtances above and below the pipe D, the perpendicular C *c* and E *e*, from theſe pipes to the ſemicircle will be equal, and the jets F and H, which ſpout from them, will each go to the ſame horizontal diſtance N K; which is double the length of either the equal perpendiculars C *c* and E *e*.

## CHAP. XIX.

## HYDRAULICS.

### SECT. I.

#### OF PUMP-WORK.

Hydraulics is that part of the doctrine of fluids which treats of the properties of fluids in motion, with a ſpecial attention to artificial water-works: and in this ſenſe it ſtands oppoſed to hydroſtatics, which concern fluids, as they remain at reſt.

The laws of fluid bodies, as given in the laſt chapter, obtain alſo in this, and therefore need not be repeated.

The greateſt benefit mankind has received from the ſcience of hydraulics is the conſtruction of the water-pump, firſt invented by Cteſebes, a mathematician in Alexandria, about 120 years before Chriſt; and it depends for its action upon the preſſure of the atmoſphere.

That the preſſure of the air on the ſurface of the water is the cauſe of the water riſing in the pump, has partly been demonſtrated in Pneumatics; for as the preſſure of the air cauſes the mercury to aſcend in the tube of the exhauſted barometer; ſo the ſame preſſure upon the ſurface of the water in a well cauſes the water to aſcend in the pump, but to a far greater height: for the mercury in the barometer riſes only to $29\frac{1}{2}$ inches at a medium; whereas the water in the tube of a pump will riſe to 33 feet at a medium, which is found equal in weight to a column of mercury of the ſame diameter, but of $29\frac{1}{2}$ inches in height; the mercury being near 14 times heavier than water.

That it is the preſſure of the atmoſphere which cauſes both the water and the mercury to aſcend, has been ſufficiently proved by numberleſs experiments; and may be ſhown by an exhauſting ſyringe, commonly termed a ſucking ſyringe. Let this be fixed in a tranſparent tube, and the lower end thereof put in a jar of mercury or water, and the whole encloſed within a tall receiver; then, if the piſton of the ſyringe be raiſed before the air is exhauſted from the receiver, the mercury or water will immediately follow it; but after the air is exhauſted, if the piſton be raiſed, the fluid will not follow.

Therefore, what is called ſuction, in hydraulic machines, is nothing more than when, by any mechanical contrivance, the preſſure of the air is in any place abated, the adjacent matter being urged on by the weight of the atmoſphere, will tend to that place; and if the matter be fluid, it will riſe ſo far above its common level, till, by its abſolute weight, a juſt equality is made: to preſerve that equilibrium which always obtains in nature.

Of

Of the pump, there are fimply three kinds, viz. the fucking, the forcing, and the lifting pump. By the former, the water is raifed by the general preffure of the atmofphere on the furface of the water in the well, and cannot be raifed to a greater height than 33 feet, as before obferved; though, in practice, it is feldom raifed above 28 feet, becaufe the air is not always denfe enough to fupport a column of water of 33 feet. By the two latter, water may be raifed to any height, having an adequate apparatus, and fufficient power.

## *Of the Sucking Pump.*

The fucking pump is that in moft common ufe, and confifts of a tube or pipe, open at each end, having within a fliding pifton, as large as the bore of the pipe, and which fits the pipe fo exactly, as to admit no air to pafs between it and the pipe. The pipe is called the barrel.

If the lower end of the barrel B be immerfed in water (*fig.* 1, *plate* 24), and the pifton D be raifed, a vacuum will be made in the barrel, by lifting up the column of upper air from A to D, and thereby permitting the air in the lower part of the barrel to expand itfelf; and the atmofphere preffing upon the furface of the water in the well, will force it to follow the pifton, and that even to the height of 33 feet, if the ftroke could be of that continued length. But when the pifton is let down again in the barrel, the water will fall with it; to prevent which, there is a valve fixed in fome convenient part of the barrel, as at C, which valve confifts of a wooden frame A (*fig.* 2), exactly fitted to the bore of the barrel, and a leather flap B, lined with lead, in order to give it fufficient weight and ftrength. This valve opening with the upward motion of the water, and again clofing when the pifton is let down, ferves to retain the water above, which flows through it: and

and at every rife of the pifton, a frefh quantity of water flows through this valve.

Befides this fixed valve, there is a moveable one placed in the pifton at D (*fig.* 1), which alfo opens the fame way, and is called the bucket.

When the bucket defcends, if the bore of the barrel be full of water, the refiftance of the water will open the bucket, and part of the water will rife above it; and when the pifton is drawn upwards, the bucket will again clofe under the incumbent weight of water, and the water will be raifed by the force applied. So that whenever the bucket rifes, and lifts up the column of both air and water, which paffes through it, the fixed valve C is difcharged of its preffure; and then a frefh quantity of water, exactly equal to that lifted up by the bucket, will, by the ordinary preffure of the atmofphere on the water in the well, be forced up through the valve C, to again fupply the barrel. This alternate motion of the two valves may be feen to great advantage in the glafs pumps.

But if there be no water in the barrel, before any water can be drawn from the well, the air in the barrel muft be exhaufted, which may be done, if the pifton valve be tight, by the ordinary motion: but it is common to pour fome water down the barrel, which is vulgarly called fetching the water, but which is of no other ufe than to fupple the leather of the valves, and render them air-tight.

The firft time the pifton is raifed in the barrel, called the firft ftroke of the pump, it will make a vacuum in the barrel, and a part of the incumbent air is lifted away, upon which the air remaining in the barrel, from its natural fpring, will become confiderably dilated; when the atmofphere preffing with a greater force on the furface of the well water, than the dilated air does on the water in the barrel, it will caufe the water in the barrel to rife therein, fo far as, together with the included air, fhall juft counterpoife the weight of the atmofphere upon the outward furface of the water.

A fimilar

A ſimilar effect will be produced at the repetition of the ſtroke, till by degrees the water ſhall have reached the moving valve or bucket, and then the proceſs will go on as before deſcribed. Thus, water, even by this machine, may be raiſed to any height whatever, provided the power be adequate to the weight, and the pipe ſtrong enough to bear the fluid's natural preſſure.

The proportion of the preſſure of the water on the pipes in pump-work is according to the height of the water above the part conſidered: but the incumbent weight on the bucket of a pump, in action, is nearly proportional to that of a column of water raiſed; for though the weight of the atmoſphere on the ſurface of the water, when the bucket riſes, be really equal to the weight of 33 feet of water; yet this weight is exactly counterbalanced by the weight of the atmoſphere, ever incumbent on the ſurface of the water thereby raiſed. Thus, all the advantage to be obtained by the hydraulic machines, is ranging matters into a convenient method of being performed; the performance itſelf depending entirely upon the moving power, with all the diſadvantages of friction.

In this pump, if both the valves be placed towards the bottom of the pipe, the pump will work as eaſy, and require no greater power than if they were fixed 30 feet, or 33 feet, above the ſurface of the water.

It is generally found to be more advantageous in practice, to place both the valve and bucket pretty low in the barrel; for ſhould a leak happen beneath the bucket, which is often the caſe, in a great length of pipe, the air getting through, would render the pipe uſeleſs; whereas, ſhould a leak happen above the bucket, it will occaſion only the loſs of ſome of the water. And by placing the valves under water, they will always be found more ſupple and pliant, and, conſequently, be in a better condition for performing their offices.

There is another advantage of placing the pump-work

means of the pipe C, and the bucket playing in the barrel B C, the water will rife as if the well had been perpendicular to the pump; becaufe the water in the well being forced by the natural preffure of the atmofphere, will replenifh the barrel B, through the pipe A C.

But when it happens that the barrel of the pump cannot go down directly to the well, as in the laft cafe, the water may be led about any other way by means of a pipe E, and thus be conveyed to the pump D. And by making this pipe of conveyance E lefs in diameter than the barrel of the pump, it will fooner be exhaufted of air, by moving the pifton, and, confequently, the water will fooner follow.

But it will always be found more eafy, in practice, to have the pipe of conveyance large, and of an equal bore throughout; becaufe the water will then have a velocity in it, and the friction will be lefs. This is the reafon why the common pumps, made by the plumbers, do not work fo eafily as thofe which are bored out of trees; for, by making the pipe fo much lefs than the bucket, they, as it were, wire-draw the water. Therefore, in pumps that have a long pipe of conveyance, the diameter of the fucking-pipe fhould be nearly equal to half the diameter of the barrel. For, if the barrel be four inches in diameter, and the pipe of conveyance only one inch, the water will move 16 times as faft through the pipe as it will through the barrel; which, confequently, requires more labour, and is attended with a greater wear and friction of the machine.

It is alfo a great fault to bore a pump conically upwards, becaufe the water cannot with freedom run off fo faft, as a vacuum may be made by the moving pifton; and the reflection of the water from the fides, will always be an hindrance in the operation.

Of

branching or forcing pipe at E. Thefe valves fhould alfo be air-tight, and fo difpofed as to let the water freely rife, but to prevent its returning back. The forcer C is leathered upwards, that it may withftand the preffure of the atmofphere from above, and that by fucking, when raifed, it may bring up the water, to fupply the barrel: and it is alfo leathered downwards, that, when repreffed, it may refift the weight of the water to be forced up.

When the forcer C is moved upwards in the barrel, it lifts up the incumbent air; and the air between that and the water having room to dilate itfelf, will be rarefied, and the water will rife from the fpring in the barrel A B, as in the fucking pump: and continuing the motion of the forcer, the water will at length rife up to the forcer, and fill the internal cavity of the pipes, between the two fixed valves D and E. And the water being prevented from defcending again by the lower valve, will, by the forcer, be preffed, and make its way through the upper valve E. When the forcer rifes, this preffure will be intermitted, and the valve at E will immediately clofe under the weight of the upper water, and thus prevent its return, while the forcer is rifing with a frefh fupply. The fame is repeated at every ftroke of the forcer.

## *M. de la Hire's Pump, which raifes Water both by the Afcent and Defcent of the Pifton in the Pump-barrel.*

B and C are two pipes (*fig.* 6), having their ends in the well of water A A. The pipe B has a valve *b* at the top, and is foldered into the pump-barrel D. The pipe C alfo has a valve at the top, and is foldered into the pipe S. The pipes E and F have each of them a valve *e* and *f* at the ends, and communicate with the pump-barrel D, and the hollow box G.

K is a folid plunger or forcer, exactly fitted to the bore of

of the barrel D. L is the rod by which it is moved up and down, through the collar of leathers M, by a pump-handle which turns upon its centre of motion I. This plunger never goes higher than K, nor lower than D.

When the plunger riſes from D to K, the weight of the atmoſphere acting upon the ſurface of the water in the well A A, forces it up the pipe B, and through the valve *b*, and thus fills the pump-barrel D with water up to the plunger, during which time the valves *e* and S on the tops of the pipes E and C remain ſhut. When the plunger has arrived to the height K, before it returns down again the valve *b* ſhuts, and thereby ſtops the mouth of the pipe B, and prevents the water from returning back: and by the motion of the plunger downwards, all the water in the barrel is forced up through the crooked pipe E, and conſequently through the valve *e*, and having filled the box G, at length riſes into the pipe N, where it diſcharges itſelf by the ſpout O. During the deſcent of the plunger K, the valve *f* ſhuts, and thereby covers the mouth of the crooked pipe F; and the plunger deſcending downwards, creates a vacuum in the upper part of the pump-barrel, and, conſequently, in the pipes C, S, and F, when the preſſure of the atmoſphere on the well water A A, forces it up the pipe C, through the valve S, and into the pump-barrel, filling all the ſpace above the plunger in the barrel with water.

Again, when the plunger has deſcended to D, before it returns up again, the valve S ſhuts; and then, by raiſing the plunger, it drives all the water above it through the crooked pipe F, and through the valve *f*, into the box G; from whence it alſo aſcends, in conjunction with the water that came through the pipe E, up the pipe N.

And thus, as the plunger deſcends, it forces the water below it up the pipe E, and alſo draws the water up the pipe C, through the valve S; and as it aſcends, it forces all the water above it up the pipe F, and alſo fills the barrel with water, through the pipe B. Therefore, there is as much

much water forced up the pipe N, to the ſpout-hole O, by the deſcent of the plunger, as by its aſcent; and in each caſe as much water diſcharged at the ſpout-hole as fills that part of the pump-barrel through which the plunger moves.

P is a cloſe air-veſſel, fixed on the top of the pipe N, which compreſſes the air, when the water riſes up the pipe N, above the ſpout O: and this condenſed air acting on the water, cauſes it to run off by the ſpout-hole nearly in an equal ſtream.

The pipe S at the top of the pipe C ſhould never be above 32 feet above the ſurface of the water in the well; becauſe if the pipe C be entirely exhauſted of air, the preſſure of the atmoſphere on the water in the well, would not force the water up the pipe to a greater height than 32 feet, at the moſt; but if S be within 24 feet of the water in the well, the pump will work ſo much the better.

The pipe N may be of any ſize required; but the pump-barrel ſhould be made in proportion to the height of the ſpout-hole above the ſurface of the water in the well; as follows:

For ten feet of perpendicular height of the ſpout-hole O, above the ſurface of the water in the well, the diameter of the bore of the barrel ſhould be 6.9 inches: for 15 feet high, 5.6 inches: for 20 feet, 4.9 inches: for 25 feet, 4.4 inches: for 30 feet, 4.0 inches: for 35 feet, 3.7 inches: for 40 feet, 3.5 inches: for 45 feet, 3.3 inches: for 50 feet, 3.1 inches: for 55 feet, 2.9 inches: for 60 feet, 2.8 inches: for 65 feet, 2.7 inches: for 70 feet, 2.6 inches: for 75 feet, 2.5 inches: for 80 feet, 2.5 inches: for 85 feet, 2.4 inches, for 90 feet, 2.3 inches: for 95 feet, 2.2 inches: and for 100 feet, 2.1 inch, or at the moſt 2.2.

In pumps of this kind the pipes B and C ſhould be made ſufficiently large: for when they are too ſmall, the velocity of the water through them being great, the water will have too much friction to ſuffer the pump to be worked with much advantage.

 *Mr.*

### *Mr. Noble's Pump.*

This pump is the moſt ſimple in its conſtruction of any of the ſame power; and may be made at a reaſonable charge, as it conſiſts only of one barrel and two piſtons, having each a bucket and valve; and it raiſes as mnch water with the ſame power, and in the ſame time, as can be raiſed by two barrels and four valves of the ſame dimenſions. The barrel conſiſts of a ſtraight tube A (*fig.* 7), in which the two buckets B and D work; the bucket B being moved by the rod C, and the bucket D by the rod E, which runs through a hole in the bucket B; the two rods and buckets are moved up and down by the two circular pieces of wood F, which are fixed to the two handles *g g*, by which means, as one bucket aſcends with its load of water, the other deſcends.

### *A Pump, or rather Engine, for raiſing Water by means of a Hair Rope. Invented by Sieur Vera.*

This pump conſiſts of the following parts:—The wheel A (*fig.* 8) is four feet in diameter, and is turned by the handle K. B B are two pullies, 14 inches in diameter, in order to keep the ropes at a proper diſtance in the well. C C is the hair rope, near one inch in diameter, which runs under the pulley I, fixed in a frame H, below the ſurface of the water G. L L L is a box made of thin boards, in order to collect the water into the reſervoir D.

When the wheel is turned by the handle K, with a conſiderable velocity, a great quantity of water will adhere to the rope C, particularly if the well be not very deep: the rope paſſes through the tube D, which is raiſed five or ſix inches higher than the bottom of the reſervoir; and thus hinders the water from returning back into the well: the water runs in a continual ſtream through the ſpout E.

If this machine be conſtructed according to the above dimenſions, it will raiſe more water than any perſon unſkilled in hydraulics would imagine.

The force required to work a pump will always be as the height to which the water is raiſed, and as the ſquare of the diameter of the pump-bore in that part where the piſton works. Thus, if two pumps be of equal heights, and one of them be twice as wide in the bore as the other, the wider will raiſe four times as much water as the narrower one; and conſequently will require four times the power to work it.

The piſton-rod of a pump is always raiſed by means of a lever, whoſe longer arm exceeds the ſhorter one, in length, generally five, or ſix times, and the power is applied at the end of the longer arm; by which means the rod is raiſed by a fifth, or ſixth part of the power, which would be required to raiſe the rod without it.

The following table ſhows the quantity of water which a common ſucking pump will diſcharge in one minute, the pump being of any given height above the ſurface of the well, from 10 to 100 feet incluſive; and the diameter of the bore of the barrel being from 6.93 inches to 2.19 inches incluſive.

| Height of the Pump above the Surface of the Well in Feet. | The Diameter of the Barrel-bore in Inches and Decimals. | Water difcharged in a Minute, Englifh Wine Meafure. | |
|---|---|---|---|
| | | *Gallons.* | *Pints.* |
| 10 | 6.93 | 81 — | 6. |
| 15 | 5.66 | 54 — | 4 |
| 20 | 4.90 | 40 — | 7 |
| 25 | 4.38 | 32 — | 6 |
| 30 | 4.00 | 27 — | 2 |
| 35 | 3.70 | 23 — | 3 |
| 40 | 3.46 | 20 — | 3 |
| 45 | 3.27 | 18 — | 1 |
| 50 | 3.10 | 16 — | 3 |
| 55 | 2.95 | 14 — | 7 |
| 60 | 2.84 | 13 — | 5 |
| 65 | 2.72 | 12 — | 4 |
| 70 | 2.62 | 11 — | 5 |
| 75 | 2.53 | 10 — | 7 |
| 80 | 2.45 | 10 — | 2 |
| 85 | 2.38 | 9 — | 5 |
| 90 | 2.31 | 9 — | 1 |
| 95 | 2.25 | 8 — | 5 |
| 100 | 2.19 | 8 — | 1 |

The foregoing table is conftructed from a pump, worked by a lever, which increafes the power five times; and the power is fuppofed to be that of a man of ordinary ftrength.

Forcing pumps are the moft ufeful machines for raifing water to any given height above the furface of a river or fpring; and machines may be conftructed to work thefe pumps, either by a running ftream, a fall of water, or by horfes.

The moft ufeful, and at the fame time moft curious application of machinery to pumps, is difplayed in the conftruction of fteam-engines, the moft improved of which is that of Mr. Watt, in the next Section.

SECT.

## SECT. II.

### OF STEAM-ENGINES:—AND A DESCRIPTION OF MR. WATT'S STEAM-ENGINE.

Of all the uſes to which ſteam has been applied, there are none where it has been uſed with greater ſucceſs than in the application of it to raiſe water from any great depth, as in that machine called the ſteam-engine, otherwiſe denominated the fire-engine, on account of the fire employed in boiling the water to produce the ſteam.

The ſteam raiſed from hot water is an elaſtic fluid like air, and has its elaſticity proportional to its denſity, when the heat is the ſame; or proportional to the heat, when the denſity is the ſame. The ſteam raiſed from boiling water of an ordinary heat is near 3000 times rarer than water, or about $3\frac{1}{2}$ times rarer than air, and its elaſticity is equal to that of common air. And it has been found by experiment, that water, when converted into very hot ſteam, will occupy 14000 times the ſpace that it occupied when in water, and, conſequently, it will become five times ſtronger than the atmoſphere. And by accidents that have happened, it has been demonſtrated that water, ſuddenly turned into ſteam by the immediate application of great heat, is vaſtly ſtronger than the atmoſphere, or even gunpowder.

The ſteam-engine is the moſt uſeful machine diſcovered in modern times; and were it not for this moſt important invention, we ſhould never have been able to work the coal-mines in England to the preſent advantage; as before the laſt century, for the want of this engine to draw the water, the attempts of our anceſtors to procure coals were always ineffectual.

Before Mr. Watt's improvement in this engine can be described, the construction of the common steam-engine should be first mentioned.

The common steam-engine is generally formed of a forcing pump, having its rod fixed to one end of a lever, which is worked by the weight or pressure of the atmosphere, upon a piston at the other end of the lever; a temporary vacuum being made below it by suddenly condensing the steam, which had been let into the cylinder, in which this piston works, by a jet of cold water thrown into it. Thus, a partial vacuum being made, the weight of the atmosphere presses down the piston, and raises the other end of the lever, with the water from the well, &c. Then a hole is immediately opened in the bottom of the cylinder, through which a fresh quantity of hot steam rushes in from a boiler of hot water, placed below it, which proves a balance for the atmosphere above the piston, upon which the weight of the pump-rods, fixed at the other end of the lever, causes that end to descend, and raises the piston of the steam-cylinder. The steam-hole is then immediately shut, and the cock opened for injecting the cold water into the steam-cylinder; the steam then condenses into water again, and thus makes another vacuum below the piston, the atmosphere above it pressing it down, and raising the pump-rods with another lift of water: and this process is continually repeated. Though this be the common principle of the steam-engine, yet there are various other methods for applying the force of steam.

The first account we have of these engines is in a small book, published in the year 1663, by the Marquis of Worcester, entitled, "A Century of Inventions," being a description of 100 famous discoveries, published that year, among which he proposes the method of raising a great quantity of water by the force of steam. And he mentions an engine of his own invention, which would raise a continual stream of water, 40 feet high, by means of two cocks, which were alternately

alternately turned by a man, in order to admit the ſteam, and to refill the veſſel with cold water.

Captain Thomas Savery having read the Marquis's book, conſtructed an engine, which, after ſeveral experiments, he brought to ſome degree of perfection; upon which he bought up, and deſtroyed, all the books of the Marquis he could procure, claimed the honour of the invention to himſelf, and obtained a patent for the ſame. His engine, however, would not raiſe water to any great height, or in quantities ſufficient to anſwer the purpoſe of draining a mine. The largeſt he ever erected was for the York Buildings Company, in London, for ſupplying the inhabitants of the Strand, and that neighbourhood, with water.

Several other gentlemen, both in England and France, attempied various improvements in the conſtruction and manner of working theſe engines; but with little ſucceſs, till the year 1705, when Mr. Newcomen, an ironmonger, and Mr. John Cowley, a glazier, both of Dartmouth, made a conſiderable improvement in theſe engines, by bringing the engine to work with a beam and piſton (which had never been then introduced), and where the ſteam, even at the greateſt depth of mines, is not required to be greater than the preſſure of the atmoſphere. Theſe gentlemen obtained a patent for the ſole uſe of this invention, for fourteen years; and the firſt engine they erected was in the year 1712, at a colliery at Griff, in Warwickſhire; the cylinder of this engine being 22 inches in diameter. The next engine they erected was in the year 1718, at a colliery in the county of Durham, which was alſo improved by Mr. Henry Beighton, F. R. S. who introduced the manner of opening and ſhutting the cocks, by the hanging beam, as at preſent uſed; and likewiſe made improvements in the pipes, valves, and ſome other parts of the machine.

When theſe engines came to be better underſtood, and their utility, particularly in draining mines, became more evident, from the great number of them every where erected,

they

pump-rods ſink by their ſuperior weight, and the piſton, at the other end of the lever, riſes; and when that ſteam is condenſed, the piſton deſcends, and the pump-rods riſe with their quantity of water; and ſo on alternately, as long as the piſton works.

As the piſton does not deſcend with a force exceeding eight or nine pounds upon every ſquare inch of its ſurface; and as, by reaſon of accidental frictions, and alterations in the denſity of the air, it is ſometimes leſs than this, it will be ſafeſt in practice to calculate the weight at ſomething leſs than eight pounds, viz. at about ſeven pounds ten ounces for every ſquare inch, or 7.64 pounds, which is ſix pounds upon every circular inch; and, as a gallon of water of 282 cubic inches weighs $10\frac{1}{5}$ pounds, we have the dimenſions of the cylinder, pumps, &c. for any ſteam-engine, as follows:

$c$=the cylinder's diameter in inches.

$p$=the pump's diameter in inches.

$f$=the depth of the pit in fathoms.

$g$=gallons drawn by a ſtroke of ſix feet.

$h$=the hogſheads drawn per hour.

$s$=the number of ſtrokes per minute.

Then $c^2$ is the area of the cylinder in circular inches; therefore $6\,c^2$ is the power of the cylinder in pounds.

And $\frac{p^2\times.7854\times72}{282}$, or $\frac{1}{5}p^2$ is $=g$, the gallons contained in one fathom, or ſix feet of any pump; which, multiplied by $f$, gives $\frac{1}{5}p^2 f$ for the gallons contained in $f$ fathoms of any pump, whoſe diameter is $p$.

Hence, $\frac{1}{5}p^2 f\times10\frac{1}{5}$ pound gives $2\,p^2 f$, nearly, for the weight in pounds of the column of water, which is to be equal to the power of the cylinder, which was before found equal to $6\,c^2$. Thus, we have the ſecond equation, viz. $6\,c^2=2\,p^2 f$, or $3\,c^2=p^2 f$; the firſt equation being $\frac{1}{5}p^2=g$, or $p^2=5g$.

From which two equations any particulars may be determined.

[illegible] and they will determine the size of [illegible] pumps of any steam-engine, capable of draw[illegible] quantity of water from any given depth, with [illegible] of the atmosphere on the pistons.

These theorems are more particularly adapted to [illegible]. But it often happens, that an engine h[illegible] pumps of different diameters, from diff[illegible] the square of the diameter of [illegible] multiplied by its depth, and double the [illegible] will be the weight of water draw[illegible] which is to be used instead of $2p^2f$, fo[illegible]

*[illegible] Theorems, for the readier computing [illegible] Powers of a Steam-Engine.*

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| [illegible] $= \frac{21c^2s}{[illegible]}$ |

*The common Steam-Engine*, (Fig. 9.)

A is the boiler.

B, the cylinder.

C, the injection-cock.

D, the steam-cock, or regulator.

E, the shifting clack.

F, the eduction pipe, or sinking pipe.

G, the eduction valve.

H, the safety valve.

I, the piston.

K, the lever beam.

L, weights to counterpoise the piston, and press down the forcer in the pump-barrel M, to drive the water through the pipe N.

O, a cistern to hold the injection water.

P, an air vessel which prevents the pipe N from bursting, and serves to keep up a regular stream.

The boiler A is filled with water to the height of *d e*, which being made to boil by a fire placed beneath it, will fill the upper part A D with a very elastic vapour or steam, which, when it is of sufficient strength, will force open the valve at H. This steam is let into the barrel or cylinder B, by turning the cock D; and by its elastic force raises the piston I, which drives the air above it through a proper clack, placed at the top: and the weights L, at the other end of the lever, cause it to descend, and drive the piston down the pump-barrel M. Then, in order to make the piston I descend, a little cold water is let into the cylinder, at the bottom, from the cistern O, by turning the cock C, which, rising in the form of a jet, condenses the hot steam in the cylinder into water, whereby it occupies about 13000 times less space than that it took up before; which creates a partial vacuum in the barrel, and thereby permits the piston to descend

212 degrees is neceſſary to produce ſteam; and the difference of heat at which water boils under different preſſures increaſes in a leſs proportion than the preſſures themſelves; ſo that a double preſſure requires leſs than a double increaſe of the heat.

There are two principal defects in the common ſteam-engine: firſt, as the vacuum in the cylinder is produced by throwing in cold water to condenſe the ſteam, the water thrown in becomes hot, and produces a ſteam from itſelf, which greatly reſiſts the motion of the piſton downwards, and thereby leſſens the power of the engine. Secondly, upon attempting to fill a cold cylinder with hot ſteam, a great part of the ſteam will be deſtroyed; and the injection water that is let in to condenſe the ſteam, not only cools the cylinder, but remains there until it be extruded at the eduction pipe by the ſteam which is afterwards let into the cylinder, which ſteam will be condenſed into water as faſt as it enters, until all the matter it comes in contact with be nearly as hot as itſelf.

The great conſumption of fuel alſo has been a material object to theſe engines; for it is well known, that a ſteam-engine of an ordinary ſize will conſume near 3000 pounds worth of coals per annum, at any part near London.

## *Mr. Watt's Steam-Engine.*

Mr. Watt has in a great meaſure, if not wholly, remedied the foregoing inconveniences: he preſerves an uniform heat in the cylinder of his engine, by ſuffering no cold water to touch it, and by protecting it from the air or other cold bodies, by a ſurrounding caſe filled with the ſteam, or with hot air, or water; and by coating it over with ſubſtances that tranſmit the heat very ſlowly. He makes his vacuum to approach nearly to that of the barometer, by condenſing the ſteam in a ſeparate veſſel, called the condenſer; which may be cooled at pleaſure, without cooling the cylinder, either

by

L, the air-pump that exhaufts the condenfer both of air and the injection water that is let in at every ftroke, and is fixed under water in the condenfing back M, which is full of water.

N, the lever beam.

O, the great water pump for clearing the mine, or raifing water for any other ufe through the pipes, &c.

This new engine differs from the common ones only in the foregoing particulars: having the cylinder, the great beam, the pumps, &c. in their ufual pofitions.

The cylinder in this new engine is fmaller than ufual in proportion to the load, and is very accurately bored; and is furrounded at a fmall diftance with another cylinder, furnifhed with a bottom and lid. The fpace between the cylinders communicates with the boiler by a large pipe C, open at both ends, fo that it is always filled with fteam, and thereby preferves the inner cylinder of the fame degree of heat with the fteam, and prevents the fteam from condenfing within it, which would be more prejudicial than an equal condenfation in the outer cylinder.

The inner cylinder has a bottom and pifton as ufual; and as it does not reach up quite to the lid of the outer cylinder, the fteam in the fpace between them has always a free accefs to the upper fide of the pifton. The lid of the outer cylinder has a hole in the middle, through which the pifton-rod moves up and down; this hole is kept tight by a collar of oakum fcrewed upon it.

There are two regulating valves at the bottom of the inner cylinder, one of which admits the fteam to pafs from the fpace between the two cylinders into the inner cylinder, below the pifton, and fhuts it out at pleafure; the other opens or fhuts the end of a pipe that leads to the condenfer. The condenfer confifts of one or more pumps furnifhed with clacks and buckets (nearly the fame as in common pumps), which are wrought by chains faftened to the great working beam of the engine. To the bottom of thefe pumps

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the piſton, which is therefore forced down by the full power of the ſteam from the boiler, which is ſomewhat greater than the preſſure of the atmoſphere.

In the common engines, when they are loaded to about ſeven pounds upon each ſquare inch of the piſton, and are of a middle ſize, the quantity of ſteam which is condenſed, in reſtoring to the cylinder the heat which it had been deprived of, by the former injection of cold water, is about one full of the cylinder, beſides what is really required to fill that veſſel; ſo that twice the full of the cylinder is employed to make it raiſe a column of water equal to ſeven pounds for each ſquare inch: or more ſimply, a cubic foot of ſteam raiſes a cubic foot of water about eight feet higher, beſides overcoming the friction of the engine, and the reſiſtance of the water to motion.

But in the improved engine of Mr. Watt, about one full and a fourth of the cylinder is required to fill it, becauſe the ſteam is one fourth more denſe than in the common engine. This engine, therefore, raiſes a load equal to $12\frac{1}{2}$ pounds upon each ſquare inch of the piſton; and each cubic foot of ſteam, of the denſity of the atmoſphere, raiſes one cubic foot of water 22 feet high.

Theſe engines work more regular and ſteady than the common ones; and the ſavings amount at leaſt to two thirds of the fuel; which is an important object where coals are dear. The new engines alſo will raiſe from 20000 to 24000 cubic feet of water, to the height of 24 feet, by only one hundred weight of good pit coals.

 CHAP.

any falſe ſtrokes of the charcoal. The black and red lead pencils are uſed to draw out the draught the ſecond time, becauſe the lines drawn with theſe will not be liable to be rubbed out with the hand, when the lines are again drawn with the pen. The pens made of crow-quills (though another good pen may anſwer the purpoſe) are to finiſh the work. Rulers are to draw the ſtraight lines, triangles, ſquares, &c. which are to be done at firſt, till practice render them needleſs. The compaſſes ſhould have ſteel points, which will take out, in order to uſe a black or red lead pencil; their uſe is to draw circles, ovals, arches, &c.; alſo to meaſure, by the help of a ſcale of equal parts, the proportions of the figures.

## *The Precepts of Drawing in general.*

There are no arts that depend leſs upon theory than thoſe of drawing and painting; in theſe it is principally practice and experience that can render any one a good artiſt. But here, as in every other art, a few rules may be of ſervice to the inexperienced ſtudent; and in attending to the following rules, the young artiſt ſhould be careful in following the outlines of the figure, which is the firſt proceſs. He muſt alſo content himſelf with copying parts of objects, before he aims at any finiſhed piece, and dwell upon each part; and never begin a ſecond till he thoroughly underſtands the proportions of all the outlines of the firſt. He muſt alſo be very ſlow in his firſt operations; and he cannot too often contemplate the length, breadth, and every other proportion of each object of his original. For this purpoſe he ſhould have it conſtantly in his eye, and cannot look at it too frequently.

1. The firſt part of drawing conſiſts of plain geometrical figures; as lines, angles, triangles, quadrangles, polygons, cones, and the like; for theſe are the foundation of the outlines of all other figures. The circle aſſiſts in all orbicular forms; as the ſun, moon, fruits, &c.: the oval, in giving a juſt proportion to the human face and mouth, the

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more exactly drawn with the lead pencil, rubbing out any falfe or imperfect ftrokes of the charcoal; then having perufed it well, mend it with the pencil, where there are any errors: this done, perufe it again, correcting by degrees all the errors of lefs magnitude, even to the leaft jot; then compare it with the original copy, ufing neither the rule nor the compafs, but giving every part its due place and proportion according to judgment.

When the artift has arrived to fome perfection in the art, he may begin to copy after life, for this is the moft correct and complete method of drawing or painting; and here only he has the largeft liberty of imitation. But there ought to be fome perfection in the art before he aims at this.

## *Particular Directions for Drawing.*

The print or painting which is to be copied, is to be placed fo, that the glofs of the colours, or fhades, may not fall upon the eye, and thereby prevent a perfect view of the piece; but it is to be fo placed, that both the light and the eye may fall obliquely upon it. It muft alfo be placed at fuch a diftance, that the whole may be taken into the eye at once; for which purpofe, the larger it is, the further diftant it muft be placed, and fet a little reclining.

Make a fmall point upon the paper, to reprefent the centre. And obferve, as a general rule, always to begin with the right fide of the piece; for by that means, what is finifhed will not be hidden by the hand or pencil. Obferve alfo the moft perfpicuous and uppermoft figures in the piece (if there be more than one), which are to be touched upon the paper in their proper places, by the charcoal; thus running over the draught, there will be had the fkeleton of the work, which is to be afterwards finifhed and filled up. But great care is to be taken in obtaining a true draught, and the more time there is beftowed upon it, the more it will improve the learner.

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profile, which is equal to the length of the eye (*fig.* 18 and 16), and the noftril in height is nearly one third of the width of the nofe. Thefe proportions are to be attended to only during the learner's firft practice: when he has arrived to a little proficiency, he is to follow his own judgment only, in the proportions, paying more regard to his original than to any verbal directions. When he is able to finifh the outline of the profile and full face, he may proceed to the other different inclinations of the face, as feen in the plate.

## LESSON III.

When the artift can eafily imitate the different features and limbs, he may begin to attempt whole length figures; which is to be done in the following manner: fketch the whole over lightly with the charcoal (or, if the learner be able, he may ufe the black lead pencil at firft), beginning with the head, next the fhoulders, then the body; after which, the arms and hands, then the hips, legs, and feet; then examine the proportion of the different parts, rubbing out any ftrokes of the charcoal where neceffary; and drawing the lines over again with the black lead pencil, to bring it as near as poffible to the original. When this is done, proceed to finifh the figure, by drawing it over again with the crow-quill pen, and Indian ink, departing from the black lead lines where it may be found neceffary. Then rub out all the marks of the pencil with India rubber. The compaffes are not to be ufed till after a very minute infpection with the eye: when, if there be any fault that cannot be eafily difcovered, by applying the compaffes, firft to the original, and then to the copy, the fault will be foon found. To afcertain the proportions of the feveral parts of the human body, a perpendicular line fhould be firft drawn through that part intended for the middle of the figure: which fhould be divided into feveral equal parts, and from fuch menfuration

a fcale

The learner ſhould pay particular attention to theſe proportions, and retain them in his memory. It is alſo neceſſary that he have ſome knowledge of anatomy, as it will enable him to judge of the proportion and diſproportion of the human figure.

## LESSON IV.

The drapery, or clothing of the figure, is next to be conſidered. Having drawn the outline of the figure faintly with charcoal, correcting every part that appears faulty; proceed to draw the outline of the drapery lightly with the charcoal, with the ſeveral folds, not ſuffering the folds to croſs each other. The quality of the drapery ſhould alſo be conſidered; as ſtuffs and woollen cloth are more harſh than ſilk, which is always flowing and eaſy. The drapery ſhould not ſtick too cloſe to the body; but ſhould appear to flow eaſily. If the drapery be ſuppoſed to be blown by a breeze of wind, it ſhould all flow one way: and the parts next the body ſhould be drawn firſt, before thoſe which fly off. The garments ſhould always bend with the figure; and the cloſer the drapery is to the body, the ſmaller muſt be the folds; and if it be quite cloſe to the body, there ſhould be no folds, but only a faint ſhadow, to repreſent that part of the body which it covers. The beſt rule in this caſe is, to remark the folds as they appear in the drapery of genteel perſons, if the figure be to have a modern dreſs; but a few particular rules may, however, greatly aſſiſt the learner:

1. Carefully avoid a ſuperfluity of drapery.—2. Let as much of the form of the body as poſſible appear under the drapery.—3. When the draperies are large, let them be thrown into large and graceful folds.—4. Drapery which is cloſe to the body, ſhould appear to be looſened by ſmall folds, judiciouſly placed: for want of this caution, the figure will have a certain ſtiffneſs, and appear as if wrapped round with a bandage, inſtead of being clothed.—5. If there be much drapery, let the greater part, if poſſible, be thrown into ſhadow.—6. Thoſe folds which fall in the light muſt have ſuch

*Length of the Fore-Arm, or upper Extremities.*

| | Apollo. | | | Venus. | | |
|---|---|---|---|---|---|---|
| | Hd. | Pts. | Min. | Hd. | Pts. | Min. |
| From the top of the ſhoulder to the elbow | 1 | 2 | 3 | 1 | 2 | 3 |
| The elbow to the hand | 1 | 1 | 2 | 1 | 0 | 6 |
| The joint of the hand to the root of the middle finger | 0 | 1 | 8 | 0 | 1 | 6 |
| The root to the tip of the middle finger | 0 | 1 | 10 | 0 | 1 | 7 |
| Length of the upper extremities | 3 | 2 | 11 | 3 | 1 | 10 |
| Breadth between the outward angles of the eyes | 0 | 1 | 6 | 0 | 1 | 7 |
| Of the face at the temples | 0 | 2 | 2 | 0 | 2 | 2 |
| Of the upper part of the neck | 0 | 2 | 0 | 0 | 1 | 11 |
| Over the ſhoulders | 2 | 0 | 0 | 1 | 3 | 8 |
| Of the body below the arm-pits | 1 | 2 | 5 | 1 | 1 | 8 |
| Between the nipples | 1 | 0 | 7 | 0 | 3 | 8 |
| From the bottom of the chin to the line croſſing the nipples | 1 | 0 | 7 | 1 | 0 | 1 |
| Of the body, at the ſmall of the waiſt | 1 | 1 | 0 | 1 | 0 | 8 |
| Over the loins | 1 | 1 | 3 | 1 | 1 | 6 |
| Over the haunches, or tops of the thigh bones | 1 | 1 | 5 | 1 | 2 | 3 |
| Of the thigh, at the top | 0 | 3 | 0 | 0 | 3 | 1 |
| Of the thigh, below the middle | 0 | 2 | 8½ | 0 | 2 | 7 |
| Of the thigh, above the knee | 0 | 1 | 8 | 0 | 2 | 0 |
| Of the leg, below the knee | 0 | 1 | 6 | 0 | 1 | 10½ |
| Of the calf of the leg | 0 | 2 | 4 | 0 | 2 | 3 |
| Below the calf | 0 | 1 | 7 | 0 | 1 | 11½ |
| Above the ankle | 0 | 1 | 2 | 0 | 1 | 2 |
| Of the ankle | 0 | 1 | 4 | 0 | 1 | 3 |
| Below the ankle | 0 | 1 | 1½ | 0 | 1 | 1 |

Middle

| | Apollo. | | | Venus. | | |
|---|---|---|---|---|---|---|
| | Hd. | Pts. | Min. | Hd. | Pts. | Min. |
| From the fore to the back part of the ſkull . . . | 0 | 3 | 6 | 0 | 3 | 4 |
| The wing of the noſe to the tip of the ear . . . | 0 | 1 | 8½ | 0 | 1 | 6 |
| The upper part of the neck . | 0 | 2 | 0 | 0 | 1 | 11 |
| The breaſt to the back, over the nipple . . . | 1 | 0 | 6 | 1 | 0 | 6 |
| The belly to the ſmall of the back . . . | 0 | 3 | 6 | 0 | 3 | 7 |
| The belly, above the navel, to the back of the loins . | 0 | 3 | 9 | 1 | 0 | 2 |
| The bottom of the belly to the round of the hip . | 1 | 0 | 0 | 1 | 0 | 5 |
| The fore part of the thigh to the bottom of the hip . | 0 | 3 | 2 | 0 | 3 | 7 |
| The middle of the thigh . | 0 | 3 | 3 | 0 | 3 | 6½ |
| The thigh, above the knee . | 0 | 2 | 1 | 0 | 2 | 3 |
| The middle of the knee . | 0 | 2 | 1 | 0 | 2 | 2 |
| The leg, above the knee . | 0 | 1 | 9 | 0 | 1 | 11 |
| The leg, at the calf . . | 0 | 1 | 8 | 0 | 1 | 9 |
| The leg, at the ankle . . | 0 | 1 | 5½ | 0 | 1 | 4 |
| The foot, at the thickeſt part | — | — | — | 0 | 1 | 3 |
| Length of the foot . . . | 1 | 0 | 6 | 1 | 0 | 4½ |
| The heel to the fore part of the bend of the foot . | — | — | — | 0 | 2 | 2 |
| The arm, over the biceps . | 0 | 2 | 0 | 0 | 1 | 9 |
| Over the elbow . . | 0 | 1 | 6 | 0 | 1 | 6 |
| Below the elbow . . | 0 | 1 | 5 | 0 | 1 | 7 |
| At the wriſt . . . | 0 | 1 | 1 | 0 | 0 | 11 |
| Below the joint of the wriſt . | 0 | 1 | 0 | 0 | 0 | 10 |
| The hand, at the roots of the fingers . . . | 0 | 0 | 5½ | 0 | 0 | 5 |
| At the roots of the nails . | 0 | 0 | 3½ | 0 | 0 | 3 |

These

plates where there is a complexity of work, though the yare generally finifhed by the graver.

The inftruments proper for etching are, needles, an oil-ftone, brufh pencils, a burnifher, fcraper, compaffes, ruler, tracer, and the graver; with the hard and foft varnifh, prepared oil, and aqua-fortis.

The needles fhould be of a fine grain, and fuch as will break without bending, of which there fhould be feveral fizes. They are to be fixed in firm round fticks, about fix inches in length, and the thicknefs of a large goofe-quill; and may be fixed in fuch fticks as have a pencil at the other end. They fhould ftand at leaft a quarter of an inch out of the ftick.

The oil-ftone is to whet the needles upon: and, *note*, if the points are to be round, they are to be whetted fhort upon the ftone, by turning them round; but if the points are to be floped, they are firft to be blunted upon the ftone, and then whetted, floping on one fide only, till they come to a fhort oval.

The brufh pencil is to cleanfe the work, wipe off the duft, and ftrike the colours, even over the ground, when laid upon the plate.

The burnifher is a piece of tempered fteel, fomewhat round at the end, for fmoothing and giving a luftre to the plate.

The fcraper is ufed for clearing the plate of any fcratches, or ftrokes, which the burnifher will not take out.

The compaffes fhould have fteel points, and are chiefly ufed in ftriking circles, meafuring diftances, &c.

The ruler is chiefly ufed to draw ftraight hatches, or lines, upon the plate.

The tracer is ufed for drawing through all the outermoft lines, or circumference of the print or drawing, which is called etching after.

The manner in which etching is performed, is, by covering the furface of the plate with a proper varnifh, or ground,

The manner of etching with the ſoft varniſh is now more frequently intermixed with the uſe of the graver than formerly: which is generally attended with great advantages, even where the whole is intended to paſs for the work of the graver; as it gives an opportunity of ſhowing the truth and ſpirit of the outline, and gives all the variety of ſhades which the different kinds of black can produce; while the exactneſs and regularity of the lines, which are requiſite for finiſhing many kinds of deſigns, are ſupplied by the graver; and by a judicious application of both, that complete finiſhing and effect is produced, which either of them alone would be incapable of affording.

### *The Preparation of the ſoft Varniſh, as directed by Mr. Lawrence, an eminent Engliſh Engraver at Paris.*

"Take of virgin wax, and aſphaltum, each two ounces; of black pitch, and Burgundy pitch, each half an ounce: melt the wax, and pitch, in a new earthenware glazed pot, and add to them by degrees the aſphaltum, finely powdered; let the whole boil till ſuch time, as that (taking a drop upon a plate) it will break, when it is cold, on bending it double two or three times betwixt the fingers. The varniſh being then boiled enough, muſt be taken off the fire; and letting it cool a little, muſt be poured into warm water, that it may work the more eaſily with the hands, ſo as to be formed into balls; which muſt be rolled up, and put into a piece of taffety for uſe."

In boiling the ingredients, it muſt be obſerved, firſt, that the fire be not too violent, leſt they burn—a ſlight ſimmering will be ſufficient: ſecondly, while the aſphaltum is putting in, and even after it is mixed with them, the ingredients ſhould be ſtirred continually with the ſpatula: and, thirdly, the water, into which this compoſition is thrown, ſhould be nearly of the ſame heat with the compoſition, to prevent a kind of cracking, which will happen when the water is too

 cold:

times two or four candles are ufed together, for difpatch; for the varnifh muft be blackened before it grows cold; for if it grows cold during the operation, the plate muft be heated again, that the varnifh be in a melted ftate when that operation is performed: great care muft be taken, not to fcorch it; which may be perceived, when it happens, by the varnifh lofing its glofs, and appearing burnt. Large plates are fometimes fufpended from the ceiling by four cords, with an iron ring about four inches diameter at the end of each cord, to hold each corner of the plate. The plate being thus fufpended with the varnifh fide downwards, may be blackened very conveniently.

In blackening the varnifh, the candle or flambeau fhould be kept at a proper diftance from the plate, that the wick may not touch the varnifh. If, after the operation, it appears that the fmoke has not penetrated the varnifh, the plate muft be again heated over the chafing-difh, and as the plate grows hot, the varnifh will gradually melt and incorporate with the fmoke, that lies above it, in fuch a manner that the whole will be equally pervaded by it.

The greateft caution is neceffary in this operation, to keep a moderate fire all the time, to move frequently the plate, and change the place of every part of it, that the varnifh may be equally melted every where, and kept from burning, and to keep the varnifh entirely free from any filth, fpark, or duft, till it be entirely cold.

### *The Method of applying the hard Varnifh.*

This is exactly the fame as that of applying the foft varnifh; being fpread equally over the warm plate with the taffety ball, and fmoked in the fame manner; but after it is fmoked, it muft be baked, or elfe dried over a gentle charcoal fire, till the fmoke of the varnifh begins to decreafe; obferving not to heat the plate too much, which would burn, and foften the varnifh.

The

The oval-pointed needle is moſt proper for making large and deep ſtrokes. It ſhould be held in the ſame manner as a pen, with the flat ſide next the thumb: though it may be uſed, with the face the other way: it muſt be held as upright and ſtraight in the hand as poſſible, ſtriking the ſtrokes freely and firmly, which render them neat and clear.

The fine needles with ſlender points, are proper for fine ſtrokes, and for the faint ſtrokes of thoſe places at the greateſt diſtance, in a landſcape; and alſo for thoſe places neareſt the light. And it is requiſite, when at work, to bruſh off all the looſe duſt which is worked up by the needles.

It is hardly neceſſary to obſerve, that the ſtudent ſhould be ſo far maſter of the art of drawing, as to be able to copy any print exactly, before he attempts etching. It is alſo neceſſary that he be able to hatch with a pen or pencil exactly, from good copies; and then he will be able to draw from plaiſter, or from the life.

In ſhading his piece, he muſt be careful to obſerve how the original is ſhadowed, how cloſe the hatches are joined, how they are laid, how they incline, and which way the light falls, which muſt always fall one way. If the light fall ſideways in the print, that ſide, which is fartheſt from the light, muſt be hatched the darkeſt.

In landſcapes the part next to the eye is to be hatched darkeſt; and the reſt to decline in its ſhadow gradually, the farther it is off from view. The ſame is to be obſerved in etching a ſky; for that which is neareſt the eye, muſt have the deepeſt ſhades; but in general, as ſoft and faint as poſſible, gradually loſing its ſhades as it comes nearer to the ground; and where they both meet, as it were, the ſky muſt be entirely loſt.

If any ſcratches, or falſe ſtrokes, happen in the working, they are to be ſtopped up with a hair pencil, dipped in the Venetian varniſh, mixed with lamp black, by which means theſe places will be defended from the aqua-fortis.

The management of the aqua-fortis is the principal matter in the whole art of etching, and on which the ſucceſs of the work chiefly depends. For the exact ſtrength of the aqua-fortis, the time it is to continue on the plate, &c. no certain rule can be given; but practice and experience alone can inform the artiſt.

## *Of etching Letters.*

To etch letters, the copper-plate is to have a ground of virgin wax, which is to be ſpread very evenly with a feather, all over the plate, while it is warm; then the letters being wrote on paper with a black lead pencil, the written ſide of the paper is to be laid upon the ground of the plate, the paper being faſtened at the four corners, as before directed. Then rub the back of the paper all over with a burniſher, taking care to rub every part of the paper; and taking the paper off the plate, the letters will all appear written on the wax, but reverſed; they are then to be drawn through the wax on the plate with a tracer, cleaning the work from the looſe wax with a linen rag, or pencil bruſh; then raiſing a border of wax, and pouring on the aqua-fortis, as before, the letters will be etched. The plate, being cleaned from the wax, is, in the next place, to be poliſhed, as follows:—take a piece of good charcoal, and pulling off the rind, put fair water on the plate, and rub it with the charcoal; and by this means the plate will be cleared from all the varniſh. But the charcoal ſhould have no knots, or roughneſs. After this, waſh the plate with a little aqua-fortis, added to twice its quantity of water. Laſtly, the plate ſhould be wiped dry, and rubbed with a little of the olive oil. Then, if any place require to be touched with the graver, it may be corrected; and the plate will be finiſhed.

### *Of holding the Graver.*

The graver ſhould not exceed the length of five inches and a half, including the handle, except it be uſed for ſtraight lines; and that part of the handle which is on the ſame line with the belly, or ſharp edge of the graver, ſhould be cut off flat, that it may be no obſtruction in working.

The handle of the graver ſhould be held in the hollow of the hand, with the fore-finger reſting upon the back of the graver, in order that it may be moved parallel to the plate. Great care muſt be taken that the fingers do not interpoſe between the plate and the graver, which would prevent the graver from being carried level with the plate, and render the ſtrokes not ſo clean.

### *Of laying the Deſign upon the Plate.*

The plate being poliſhed ſmooth, is to be heated, ſo as to melt virgin wax, with which it is to be rubbed thinly and equally all over, and ſuffered to cool. The deſign to be laid on muſt be drawn on paper, with a black lead pencil, and laid upon the plate, with the pencilled ſide upon the wax; it is then to be preſſed cloſe to the plate, and rubbed over every part with a burniſher. Then taking the paper off the plate, every line drawn with the black lead pencil will appear upon the wax, which are to be traced through the wax upon the plate with a ſharp-pointed tool. The wax being taken off, the plate is to be engraved.

### *Of whetting and tempering the Graver.*

Great care is required to whet the graver nicely; for which purpoſe, the two angles of the graver, which are to be held next the plate, are to be laid flat upon the ſtone, and rubbed ſteadily, till the belly riſes gradually above the plate, ſo that, when the graver is laid upon it, the light may be ſeen under the point; if it be not whetted in this ſhape, it will dig into the copper, and it will be impoſſible to prevent the

 point

cuſhion; preſſing more lightly, where they are to be fine, and leaning with greater force, where they are to be broad and deep. In making circular and curve lines, turn the plate upon the cuſhion againſt the graver. After ſome of the work is done, it is neceſſary to ſcrape off the roughneſs formed by the cutting of the graver, which is done with the ſcraper: or paſſing the graver over the plate in a level direction, taking care that it does not catch the copper. To render the work more viſible, it may be rubbed over with a roll of felt dipped in oil. It is neceſſary to learn to carry the graver as level as poſſible with the ſurface of the plate; for otherwiſe, if the fingers ſlip betwixt them, the line that is produced will become deeper and deeper in the progreſs of its formation, which will prevent making a ſtroke at one cut, that will be fine at the extremities, and larger in the middle; and renders it neceſſary to retouch it. Therefore it is neceſſary to acquire the habit of making ſuch ſtrokes, both ſtraight and curved, by lightening, or preſſing, the hand, according to the occaſion. And when the deſign is finiſhed, if any ſcratches or falſe ſtrokes appear in any part of the plate, they muſt be taken out by the burniſher.

In order to preſerve a due equality in the work, the principal objects of the deſign ſhould be ſketched out, before any of them be finiſhed. In working with the graver, the ſtrokes ſhould never be croſſed too much in the lozenge manner (particularly in repreſenting the fleſh of the human body), except in the caſe of a cloud, waves of the ſea, the ſkins of animals covered with hair, or the leaves of trees, where this method of croſſing may be admitted. In the diſpoſition of the ſtrokes, the action of the figures, and the diſpoſition of their parts, ſhould be conſidered; and alſo the manner in which they advance towards, or depart from, the eye of the obſerver. The graver ſhould be ſo guided, as to mark the riſing or cavities of the muſcles, making the ſtrokes wider and fainter in the light, and cloſer and bolder in the ſhades. Thus, the hand ſhould be lightened in ſuch a manner,

by perpendicular ſtrokes. When a groſs ſtroke is made, it ſhould be at right angles, and wider and thinner than the firſt ſtroke. In engraving mountains, as there are ſharp and craggy objects, the ſtrokes ſhould be frequently broken; they ſhould alſo be ſtraight, in the lozenge manner, and accompanied with long points or dots. Rocks have ſome croſs ſtrokes in them more ſquare and even. Diſtant objects towards the horizon are very ſlightly ſhaded, as in drawing. Calm, ſtill waters ſhould be repreſented by ſtraight ſtrokes, parallel to the horizon, and interlined with finer ſtrokes; omitting thoſe places where the light caſts a ſhining reflection: and the forms of objects reflected from the water at a ſmall diſtance upon it, or on the banks of the water, are expreſſed by the ſame ſtrokes, retouched more ſtrongly or faintly, as it may be neceſſary: agitated waters, as the waves of the ſea, have the firſt ſtrokes in the figure of the waves, and are interlined; and the croſs ſtrokes ſhould be very lozenge. The firſt ſtrokes in caſcades ſhould follow the fall of the water, and be interlined. In clouds, that appear thick and agitated, the graver muſt be turned every way, according to their form and agitation. In dark clouds, where two ſtrokes are neceſſary, they ſhould be croſſed more lozenge than the figures, and the ſecond ſtrokes ſhould be rather wider than the firſt. The flat clouds, that are inſenſibly loſt in a clear ſky, ſhould be formed by ſtrokes parallel to the horizon, but a little waving; if ſecond ſtrokes be required, they ſhould be more or leſs lozenge, and ſhould be ſo lightened at the extremities, as to have no outline. The flat and clear ſky is repreſented by ſtrokes parallel and perfectly ſtraight.

In all landſcapes, in general, the trees, rocks, earth, and herbage ſhould be etched as much as poſſible, leaving nothing to be done by the graver, but the perfecting, ſoftening, and ſtrengthening. And obſerve, once for all, that the dry needle produces a more delicate effect, and may be uſed to much greater advantage than the graver can, particularly

in

be neatly imitated, if a plate be provided for every colour. And if it be well done, it will form ſuch a good deception, that an able connoiſſeur cannot, from the firſt inſpection, diſtinguiſh between the original drawing and the engraved imitation; therefore, this mode of engraving is very uſeful to multiply copies of drawings left by ancient artiſts who excelled in the uſe of chalks.

## SECT. IV.

### OF MEZZOTINTO AND AQUA-TINTA SCRAPING.

Mezzotinto prints have no hatches, or ſtrokes of the graver; but the lights and ſhades are more blended together than in etchings and engravings, and appear like a drawing of India ink.

This art is of late invention, but is greatly uſed, and is admired for the amazing eaſe with which it is executed, particularly by perſons who are deficient in drawing.

The principal tools uſed in this art, beſides thoſe uſed in etching and engraving, are the grounding tool and the ſcraper.

### *General Directions for laying the Mezzotinto Ground.*

Leave a ſpace upon the bottom of the plate for the writing, coat of arms, &c. then, laying the plate upon a piece of ſwan-

where there is not to be any ſhade, are to be ſoftened or rubbed down with the burniſher, otherwiſe theſe parts will not appear clear, when the work is proved.

There is alſo another method, uſed by mezzotinto ſcrapers: which is, to etch the outlines of the original, with all the folds in the drapery, &c. marking the breadths of the ſhadows by dots; then having uſed the aqua-fortis, as in etching, and the ground being taken off the plate, the mezzotinto ground is to be laid, and the work finiſhed by ſcraping, as above.

When the work is to be proved, it is neceſſary to have ſome French paper, which has been wetted down four or five days, as no other paper will do for this work, and it is neceſſary for it to lie wet that length of time. A proof is then to be taken of the plate; when the proof is dry, correct it, by touching it with white chalk, where it ſhould be lighter; and with black chalk, where it ſhould be darker. In retouching the plate, proceed as before, where it ſhould be lighter, by uſing the ſcraper; and where it ſhould be darker uſe a ſmall grounding tool, as much as is thought neceſſary to give it its proper ſhade. Then it is to be proved again, and again corrected and retouched; and thus proceed to prove, and retouch it, till it be finiſhed.

It is to be obſerved, that the work ſhould be proved the firſt time, before it is the leaſt over-ſcraped in any part; as, by this caution, it will appear more elegant; for the ſmall grounding tool, which is uſed to deepen any ſhades that are over-ſcraped, generally gives the work a coarſe appearance.

## *Of Aqua-tinta Scraping.*

Aqua-tinta is that method lately invented of etching, by which a ſoft and beautiful ſhade is given, reſembling a drawing in water colours, or India ink.

The principal operation is as follows: the etching ground is to be laid on the plate, as in common etching, and the out-

 lines

as requifite, with the needle or point, by ftippling with dots, and biting up thofe parts; or by a rolling wheel.

The foregoing method will only ferve for prints of one fingle tint. When different colours are to be expreffed, there muft be as many different plates; each plate having only that part etched upon it, which is defigned to be charged with its proper colour, unlefs (as fometimes happens) fome of the colours are fo diftant from each other, as to allow the printer room to fill them in with his rubber, without blending them; in which cafe, two or more different colours may be printed upon the fame plate at once. When different plates are requifite, there muft be a feparate one, having a pin in each corner, to ferve as a fole or bottom to the aqua-tinta plates; and the aqua-tinta plates muft be exactly fitted, having each a fmall hole in their corners for paffing over the pins of the fole; the pins retain the plates in their due pofition, and alfo direct the printer in placing the paper exactly on each plate, fo as not to fhift; by which means each tint or colour will be exactly received on its proper place. This is the method practifed by the Paris printers. Some fubjects, however, fuch as landfcapes, may be printed off at once in the different proper colours, by painting thefe upon the plate. Here the colours muft be pretty thick in confiftence, and the plate carefully wiped in the ufual way, after laying in each tint, as well as wiped in general, when it is charged with all the tints.

In aqua-tinta plates, it muft be obferved, that the afphaltum and refin muft be finely powdered, and well incorporated together, before they be fifted on the plate; for this purpofe, it is neceffary to fift each of them through a fine muflin fieve, fifting firft a layer of one on a fheet of paper, and then a layer of the other; proceeding in this manner till the whole be finely fifted, and well incorporated together.

This art has been hitherto kept as fecret as poffible; but a ftrict attention to what has been delivered will enable the practitioner to finifh his plate with fuccefs.

flake white and white lead ſhould be wholly avoided, as the ſlighteſt touch with either of theſe will always turn black, for theſe whites will ſtand only in oils. Therefore, when white is required, I would recommend the ſtudent to make uſe of common whiting, prepared in the following manner :—

Put ſome whiting in a large veſſel of water, mixing them well together; when this has ſtood about half a minute, pour off the water into another veſſel, and throw the gritty ſediment away; after this water has reſted about a minute, pour it off as before, which will purify the whiting from all dirt and grittineſs. This being done, let the whiting ſettle, and pour the remainder of the water from it; after which lay it on the chalk to dry, and when dry, it will be fit for uſe; either for making white crayons, or for preparing tints with other colours. And, *Note*, if the ſtudent make the crayons of the whiting immediately after it is waſhed, it is not neceſſary to dry it on the chalk; for it may be mixed with any other colour inſtantly, whereby much trouble will be ſaved. All colours of a heavy or gritty nature, particularly blue verditer, muſt be waſhed in this manner.

The ſtudent muſt be provided with a large flexible pallet-knife, a large ſtone, and muller, to levigate the colours; two or three large pieces of chalk, having large ſmooth ſurfaces, to abſorb the moiſture from the colours, after they are levigated; and a piece of flat glaſs, to prevent the moiſture from being abſorbed too much, till the colours are rolled into form. Theſe implements being provided, the ſtudent may proceed to form his crayons from the following colours:

*Reds* are formed either from carmine, lake, or vermilion, or a compoſition of two or more of them; though it muſt be obſerved, that it is difficult to procure either good carmine or lake: good carmine is inclined to the vermilion tint, and ſhould be an impalpable powder: a good lake ſhould incline to the carmine tint.

The carmine crayons are prepared by mixing a ſufficient quantity of good carmine with ſpirits of wine with the levigating

Lake crayons are ſomewhat difficult to form, on account of the harſhneſs of the lake, therefore the ſtudent ſhould obſerve the following rules in the formation of theſe crayons:— Take about half the quantity of the lake intended for the crayons, and grind it very fine in ſpirits of wine; when dry, pulverize it; and take the other half, and grind it with ſpirits of wine; after which, mix it with the pulverized lake, and lay it out directly in crayons on the chalk: this colour will not bear rolling. The ſimple colour being thus prepared, proceed with the compound crayons, as before directed in the carmine crayons, and in the ſame degree of gradation.

Vermilion crayons are formed by mixing the vermilion on the ſtone with the ſpirits of wine, or even ſoft water; after which it may be rolled into crayons. The different tints are produced by mixing the ſimple colour with whiting, according to the proportions given in the carmine. And, *Note*, that theſe crayons will ſometimes be ſo ſoft, that they cannot be held in the fingers, but will break, and return to powder: which may be remedied by mixing the colour with ſome thin water-gruel, well ſtrained, which will give it ſufficient coheſion.

*Blues* are formed of Pruſſian blue, and blue verditer.

Pruſſian blue crayons are formed in the ſame manner as the lake crayons: but as the Pruſſian blue is very apt to bind, it is ſomewhat more difficult to be ſoftened than either lake or carmine. It is neceſſary to grind a large quantity of this colour, as it is chiefly uſed in draperies. The different tints may be made according to the fancy of the painter.

Blue verditer crayons are ſomewhat more difficult to form, on account of the coarſe gritty nature of the verditer, which requires ſome binding matter to unite it, otherwiſe it will never adhere together. Therefore, to a quantity of blue verditer, ſufficient to form two or three crayons, muſt be added a little ſifted plaſter of Paris, about the ſize of a pea: theſe are to be mixed well together, and the crayons formed

particularly in the light green ones, which will turn black on the pictures, if the leaſt damp come to them: though the dark colours will remain perfect.

In order to diſcover whether there be any flake white in the crayons, the following experiment may be made: having bruiſed the crayon to a powder, mix it with an equal quantity of charcoal duſt; put the whole into a crucible, which muſt be placed in a fierce fire till the charcoal duſt be conſumed; and if the crayon have any flake white in it, the lead will return to its original metallic ſtate.

*Browns* are originally produced from Cullen's earth, or umber.

Cullen's earth crayons are of a fine dark brown, and ſeveral rich tints may be produced from a mixture of this colour with carmine, in various degrees: alſo, this colour mixed with black and carmine, makes uſeful tints for painting the hair. Several gradations may be made from each of theſe, by a mixture with whiting. Roman and brown ochre alſo form an excellent colour, either mixed together, or compounded with carmine. Whiting, tinged in ſeveral degrees with either of theſe, will prove very ſerviceable. Common ſea-coal, ground to a fine powder, and mixed with carmine, forms a very fine brown.

Umber crayons are formed in the ſame manner as the above; but it is neceſſary to levigate the umber with ſpirits of wine.

*Purples* are formed by a mixture of blue and red. Good purple crayons may be formed with Pruſſian blue, ground with ſpirits, and mixed with pulverized lake. Alſo Pruſſian blue and carmine produce a deep purple of an excellent hue. From either of theſe compounds various tints may be made, by a mixture with whiting.

*Black crayons* are formed of lamp black, as no other full black can be uſed with ſafety, all others being ſubject to mildew. But as lamp black is liable to great adulteration, the

too wet it muſt be laid upon the chalk again, to abſorb more of the moiſture. They ſhould be rolled as quick as poſſible; and when finiſhed, muſt be laid upon the chalk again to abſorb the remaining moiſture. When all the crayons of one colour are formed, the chalk and grinding ſtone ſhould be well ſcraped, and waſhed with water, before they are uſed for another colour.

When a ſet of crayons is completed, they ſhould be ranged in ſome thin drawer, divided into a number of partitions, and diſpoſed according to the ſeveral gradations of light. The bottom of the partitions ſhould be covered with bran, to preſerve the crayons clean, and prevent them from breaking.

The box in which the crayons are placed for uſe, and which ſhould be held in the lap when the ſtudent paints, ſhould be about a foot ſquare, having nine partitions. In the upper corner, on the left hand, the black and gray crayons are uſually placed, as they are the moſt ſeldom uſed; in the ſecond partition are placed the blues; in the third the greens and browns: in the firſt partition, on the left hand of the ſecond row, the carmines, lakes, vermilions, and all deep reds are depoſited; the yellows and orange are in the middle partition; and in the next are placed the pearly tints, which, being of a delicate nature, muſt be kept very clean, that the different gradations of colour may be eaſily diſtinguiſhed: in the laſt row, the firſt partition contains a piece of linen rag to wipe the crayons with, while they are uſing; the ſecond partition holds the pure lake and vermilion tints; and the laſt partition contains all thoſe compounded tints, which cannot be claſſed with any colour.

### *Directions for the Artiſt.*

To arrive at excellence in this art, the ſtudent ſhould be as particular in the outline of the work, as in the diſpoſal of

when the ſubject to be imitated is in oils; but if it be a crayon picture, the following method muſt be uſed, on account of the glaſs.

The picture being placed upon the eſel, draw all the outlines upon the glaſs with a ſmall camel's hair pencil, dipped in lake, ground very fine in oils; then take a ſheet of paper, and place it on the glaſs, ſtroking over all the lines with the hand, by which means the colours will adhere to the paper, which is then to be pierced with pin-holes pretty cloſe to each other in all the outlines. The paper intended for the drawing is then to be laid upon the table, and the pierced paper to be laid upon it; then, with ſome fine powdered charcoal, tied up in a piece of lawn, rub over all the pierced outlines, which will give an exact outline of the piece, upon the paper under it. This is not to be bruſhed off till the whole is drawn over with ſketching chalk; which is a compoſition made of whiting and tobacco-pipe clay, rolled like a crayon.

But when the ſtudent paints immediately from life, it is beſt to make a correct drawing of the outlines on another paper, which he may trace by the firſt method: for if there be any falſe ſtrokes of the ſketching chalk, they will prevent the crayons from adhering to the paper.

The ſitting poſture is the moſt proper for painting with crayons, having the box of crayons in the lap. That part of the picture which the ſtudent is at work upon, ſhould be below his face; for when it is placed too high, it will fatigue the arm. The windows of the room, in which the artiſt works, ſhould be darkened to the height of ſix feet from the ground, and the ſubject to be painted ſhould be ſituated in ſuch a manner, that the light may fall on the face to the greateſt advantage, avoiding too much ſhadow, which ſeldom has a good effect in this kind of painting, particularly if the face have much delicacy.

In painting, as well as drawing, the ſtudent cannot be too attentive to the ſubject; he muſt alſo learn to appro-

priate

Whatever colour the iris of the eyes is, the eyes muſt be firſt drawn with a crayon, inclined to the carmine tint; the colour muſt be laid in brilliant at firſt, and executed lightly, not meddling with the pupil yet. The light of the eye ſhould incline very much to the blue caſt; for if a ſtaring white appearance is once introduced, it can ſeldom be altered: a broad ſhadow ſhould alſo be thrown on the upper part by the eyelaſh. The eyebrows ſhould be executed at firſt like a broad glowing ſhadow, on which is to be painted, in the finiſhing, the hair of the eyebrows, by which the former tints will ſhow themſelves through, and produce a pleaſing effect; but a black heavy tint is always to be avoided in firſt forming the eyebrows.

The lips ſhould be begun with pure carmine and lake, ſhading them with carmine and black, and laying on the ſtrong vermilion tints afterwards. Great caution is neceſſary to avoid ſtiff, harſh lines: each colour is to be gently intermixed with the neighbouring colour; the ſhadow beneath ſhould be broad, and enriched with brilliant crayons; The corner of the mouth is formed with carmine, brown ochre, and greens, variouſly intermixed. If the hair be dark, it is neceſſary to uſe a good quantity of the lake and deep carmine tints therein, which may be eaſily overpowered by the warmer hair tints, and which, as in the eyebrows, will produce a richer effect when the piece is finiſhed, than if the lake and carmine be neglected.

When the ſtudent has dead-coloured the head, he is to ſweeten the whole together, by rubbing it over with his finger, beginning at the ſtrongeſt light upon the forehead, and paſſing his finger very lightly to the next tint, to unite them together; which he muſt continue to do, till the work is ſweetened together, frequently wiping his finger on a towel, to prevent ſullying the colours. In this proceſs the ſtudent muſt be careful not to ſweeten his picture too often; as that would produce a thin and ſcanty effect, and the piece would have more of the appearance of a drawing than a ſolid

 painting,

the beauty of the face. This is requiſite even in a ſimple back ground, where there is but one object in the piece; but more attention is required in the back ground of a picture which has ſeveral objects.

A great variety of colours are uſed for back grounds; bu they ſhould always be ſuited to the complexion of the figure. A ſtrong-coloured head generally ſhould have a weak and tender-tinted ground; and, on the contrary, a delicate complexion requires ſtrong and powerful tints in the ground, by which proper contraſt between the figure and the back ground, the picture receives great force.

But when ſeveral objects are introduced into one piece, as hills, trees, buildings, &c. the general rule to be obſerved is, that each grand object be diſpoſed ſo, as to contraſt each other, not merely in their forms, but in their colour, light, ſhade, &c. For example; ſuppoſe a figure in the piece, receiving the ſtrongeſt light, and behind this figure, and near at hand, ſuppoſe there be ſtems of ſome large trees: theſe ſtems muſt have ſhade thrown over them, either from a driving cloud, or ſome other interpoſing object; behind theſe ſtems or trees, and at a diſtance, ſuppoſe there are ſeen trees on a riſing ground; theſe again ſhould receive the light, whereby they will ſerve as a contraſt to the former; and the ſame may be obſerved in all other caſes. The ſame rule holds good in an architectural back ground; as, ſuppoſe a building at a moderate diſtance, and behind this building, the figure which receives the light; a column, or ſome other object in the ſhade, intervenes to preſerve proper decorum in the piece; or, what will have the ſame effect, a ſhadow may be thrown over the lower part of the building. In a word, it muſt be remembered, that the light muſt be always placed againſt the dark, and the weak againſt the ſtrong; and *vice verſa*, in order to produce force and effect.

In finiſhing the complexion, the ſtudent ſhould be particularly attentive to Nature herſelf: for whoever carefully examines a clear and tranſparent ſkin, will diſcover a pleaſing variety of colours on the ſurface, and diſcernible, through it, which will be greatly increaſed by the effect of light and ſhade; one part will appear to incline to the vermilion, another to the carmine or lake, one to the blue, another to the green, and another to the yellow, &c. Now, in order to produce theſe effects, a good artiſt will apply thoſe colours correſponding to the tints, uſing, as often as he can, the compounded colours, inſtead of the ſimple colours: as, blue and yellow, inſtead of green, blue and carmine, inſtead of purple, and red and yellow for orange. In all other circumſtances the compounded crayons already mixed ſhould be uſed; but in this caſe no abſolute rule can be given; the ſucceſs of the piece depending upon the experience and diſcretion of the artiſt. And, obſerve, that it is impoſſible to give any ſet of rules for forming the complexion, that will hold in every caſe, the circumſtances that require different treatments are ſo many and various; but great advantage will be derived, in the commencement of this art, by an able maſter, to direct the ſtudent, and point out the deformities and beauties of a piece, as they occur in practice; which, to a good capacity, will ſoon become clear and intelligible.

In finiſhing the cheeks, uſe the pure lake tint, which will clear them from any duſt they may have contracted from the other crayons, mixing with the lake ſome bright vermilion; and laſtly, (if the ſubject require it,) give a few touches of the orange crayons, but with great caution. This being done, ſweeten the part with the finger as lightly as poſſible, leſt it produce a heavy diſagreeable effect on the cheeks; for the only method of imitating a beautiful complexion, conſiſts in one colour ſhowing itſelf through, or rather between, other colours.

The eye is next to be executed. This is generally found the moſt difficult feature in the face, as every part muſt be expreſſed

completely correct: and in the finiſhing, have a little of the ſtrong vermilion; but with great caution, as this colour is very predominant. This colour, if properly uſed, will give the lips an appearance equal, if not ſuperior, to thoſe executed in oils, notwithſtanding the great advantage the latter have, by glazing, of which the former is deſtitute.

In painting the neck, the ſtudent ſhould carefully avoid giving too much expreſſion to the muſcles, in the ſtem; and alſo be careful that the bones appear not too prominent on the cheſt, as either of them has an unpleaſing effect, and denotes a violent agitation of the body, which is ſeldom neceſſary in portrait-painting. The moſt neceſſary part to be expreſſed, is a ſtrong marking juſt above the place where the collar-bones unite. This ſhould always be expreſſed even in the moſt delicate ſubjects; and if the head be thrown much over the ſhoulders, the muſcle that riſes from behind the ear, and is inſerted into the pit, between the collar-bones, ſhould be faintly marked. But, in general, all inferior muſcles ſhould be quite avoided, and not noticed. Many artiſts, in the portraits of thin perſons, mark the muſcles of the neck too evidently. The neck ſhould, in general, have a ſmall addition to the length, as few necks are too long; and nothing is more ungraceful than a neck too ſhort; the ſtem of the neck ſhould have a pearly hue; and the light ſhould not appear too ſtrong upon the cheſt. The breaſt alſo (if any part appears) ſhould be expreſſed by pearly tints, but blended with beautiful vermilion in the upper part thereof.

## *Of Drapery.*

The drapery, by many young artiſts, is thought to require very little attention; but this is an egregious miſtake. An eminent painter being aſked, what part of the picture he thought the moſt difficult to execute? he anſwered, The drapery:—and the beſt judges of the art have univerſally allowed

# APPENDIX.

## NECESSARY RECEIPTS FOR THOSE WHO PAINT IN WATER COLOURS.

### *To make Gum Water.*

DISSOLVE one ounce of pure gum-arabic, and half an ounce of double-refined ſugar, in a quart of ſpring water: ſtrain it through a fine ſieve, or piece of fine muſlin, and bottle it up for uſe, to keep it from the duſt.

Or, ſecondly, take ſome of the whiteſt ſort of gum-arabic, bruiſe it, and tie it up in a piece of woollen cloth; and ſteep it in ſpring water till it be diſſolved. If it be too ſtiff, add more water; and if it be too thin, more gum.

With this water, moſt of the colours are to be mixed; and in ſuch a proportion, that the colour may not rub off, when dry. If the colour ſhine, it is a ſign there is too much gum in the water.

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water. This will prevent the colours from ſinking, alſo give them an additional beauty and luſtre; and likewiſe preſerve them from fading. If the paper is not good, it ſhould be waſhed three or four times with the water, drying it every time.

### *To make Size for painting Scenes, or other Candle-light Pieces.*

When the colours, mixed with gum water, are laid upon any ſurface, they are apt to produce a glare by candle-light; to prevent which, the colours ſhould be mixed with the following ſize, while it is warm:—Steep a quarter of a pound of the cuttings of white leather for ſome time in water; or for the ſpace of two or three days: then take them out, and boil them in three quarts of water, till it be conſumed to one pint, and ſtrain it through a cloth. If it feel firm under your hand when it is cold, it is a ſign it is of a ſufficient ſtrength.

### *To lay Mezzotinto Prints upon Glaſs.*

Having a clear plate of glaſs, as ſtraight as poſſible, and a little larger than the print to be laid upon it, ſoak the print in warm water for about an hour, and with a thin, flexible pallet knife, ſpread ſome Venice turpentine, or good varniſh, very thinly and evenly over one ſide of the glaſs, obſerving to keep the glaſs warm, that it ſpread the better, and taking care that there be not the leaſt ſpeck in the glaſs uncovered with the turpentine; then take the print out of the water, and ſpread it between two cloths, or ſeveral folds of ſoft paper, in order to abſorb the ſuperfluous water. Next lay the print on the glaſs by degrees, beginning at one end, and ſtroking outward that part which is faſtened

 to

*To get the Colours out of the Bladders.*

Prick a ſmall hole near the bottom, and preſs the bladder until enough run out for preſent uſe, for if they ſtand open they are apt to ſpoil.

With theſe colours any tints or ſhades whatever may be exactly imitated, by the different ways and methods of mixing them, according to judgment.

*To uſe the Colours.*

The lighter colours are to be firſt laid on the lighter parts of the print, and the darker colours are next laid over the ſhaded part, and in the regular order in which the ſhades deepen; for when the brighter colours are once laid on, it is not material if the darker colours be laid a little over them: as the colour firſt laid on will always hide thoſe laid on afterwards. The colours are not to be laid on too thick: and if any of them be too thick in conſiſtence, they ſhould be thinned, before they are uſed, with a little oil of turpentine.

If any of the colours be too ſtrong, or dark, they may be lightened to any degree, by mixing more or leſs white with them on the pallet; or if they be too light, they may be darkened to any degree, by mixing them with a deeper ſhade of the ſame colour.

*Note.* It is neceſſary to have a pencil for each colour; but that pencil which has been uſed for green ſhould never be uſed for any other colour, without firſt waſhing it well with oil of turpentine, as green will always appear predominant when the colours are dry. And it is alſo neceſſary to waſh all the pencils in oil of turpentine after uſing them.

To

### *To make the Maſtic Varniſh.*

Put two ounces of the cleareſt gum maſtic, finely powdered, into a bottle, with ſix ounces of oil of turpentine: ſtop the bottle cloſe, and ſhake them well together, in order to incorporate them with each other. Then hang the bottle in a veſſel of boiling water for half an hour, taking it out three or four times to ſhake it. If it be neceſſary to make the varniſh ſtronger, it may hang a quarter of an hour longer in the boiling water.

### *To make Camp Paper, with which a Perſon may write or draw, without Pen, Ink, or Pencil.*

Mix ſome hard ſoap with lamp-black and water, into the conſiſtence of a jelly; with this mixture bruſh over one ſide of the paper, and let it dry. When you uſe the paper, put it between two ſheets of clean paper, with its black ſide downwards: then with a pin, a ſtick, or any other ſubſtance with a ſharp point, draw, or write upon the clean paper; and where the point has touched, there will be the impreſſion upon the lowermoſt ſheet of paper, as if it had been drawn or written with a pen.

This camp paper may be made of any other colour, by mixing the ſoap with different colours.

By this paper alſo any print or drawing may be exactly copied, by laying it under the ſame, and tracing the outlines, &c.

The

## *To prepare a Plaſter Mould ſo as to take an Impreſſion from it.*

Having prepared a plaſter mould, according to the foregoing receipt, and letting it be quite dry, dip it in the following mixture: half a pint of boiled linſeed oil, and one ounce of ſpirits of turpentine; theſe are to be mixed well together in a bottle, and when wanted, the ſurface of the mould is to be dipped into it, and then ſuffered to dry. When the mould has ſucked up the oil on its ſurface, it is to be dipped again in the oil. This operation is to be repeated till the mould will imbibe no more oil, and the oil begins to ſtagnate upon it; then, with a little cotton wool, rolled up hard, wipe all the looſe oil off the mould, and put it in a dry place for a day or two, to dry, and the mould will acquire a very hard ſurface from the effect of the oil. When it is to be uſed, it muſt be oiled with oil of olives, in the ſame manner as before directed. By theſe two methods, any medal, ſeal, or impreſſion, may be ſo exactly imitated, that the new medal can ſcarcely be diſtinguiſhed from the original.

## *The Method of caſting Brimſtone, and of giving it a metallic Gloſs.*

Melt ſome ſtone brimſtone over the fire, in an iron ladle, and let it flame for about five or ſix minutes, then take it off the fire, and extinguiſh the flame, by covering the mouth of the ladle with a piece of board; when it is a little cool, ſo as not to feel gluey, or run ropy, it is then fit for uſe, and may be poured into the mould, in which it ſhould ſtand five or ſix minutes, and then be taken off; part it as before, and rub the ſurface of the impreſſion over with ſome cotton and the beſt black lead in powder, which will give it a very fine metallic gloſs.

## *Directions to the Binder for placing the Plates.*

### VOL. I.



### VOL. II.




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